2.1) Introduction

One of the most simple optical waveguide structures is the step-index infinite planar waveguide as shown in figure 2.1.

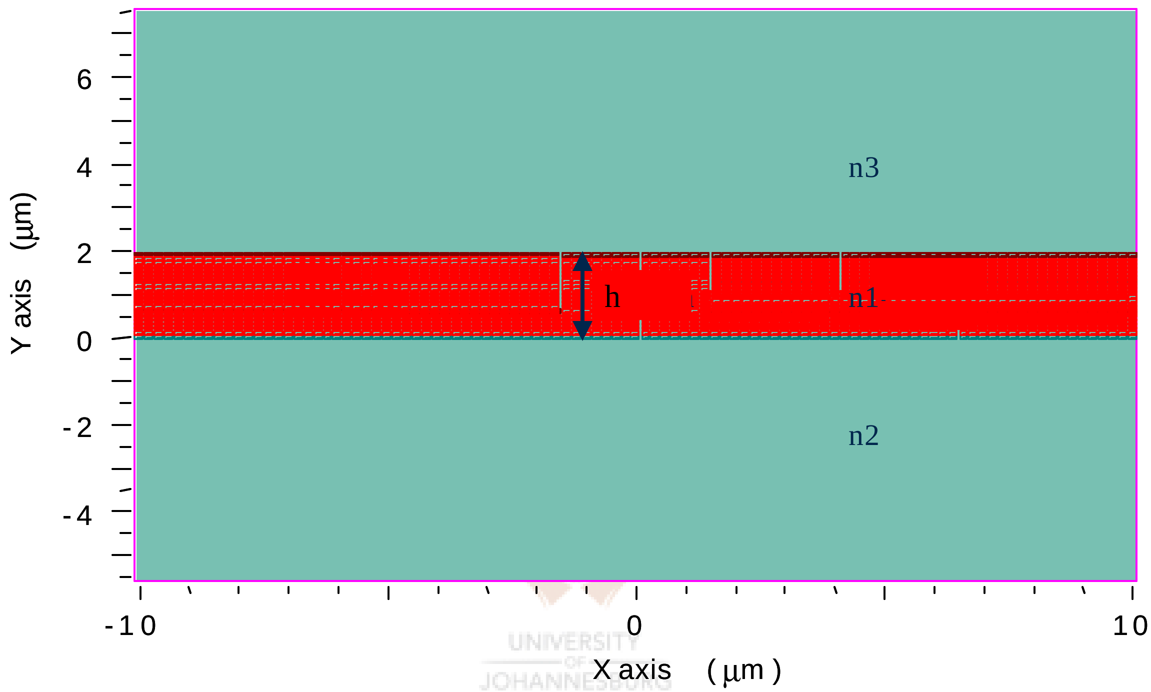

Figure 2.1: Planar slab waveguide

As can be seen, the structure consists of three regions: a higher refractive index (n_1) core layer sandwiched between two lower refractive index (n_2 and n_3) layers. In general we can state, for the guiding condition to be met, that $n_1>n_2>n_3$.

There are texts supplying exhaustive derivations for the solution of the wave equation for waveguides ([1, 2]). As a result, those derivations will not be included in this text, but references will be made to the important results as applicable to this work.

2.2) The wave vector and propagation constant

The propagation constant, denoted by b, is a coefficient resulting from the solution for the scalar wave equation. It can also be called the longitudinal wave vector. The relationship between b and k (known as the wave vector, described by $k = 2p/l$ where l is the wavelength) is best described by the triangle [1] in figure 2.2.

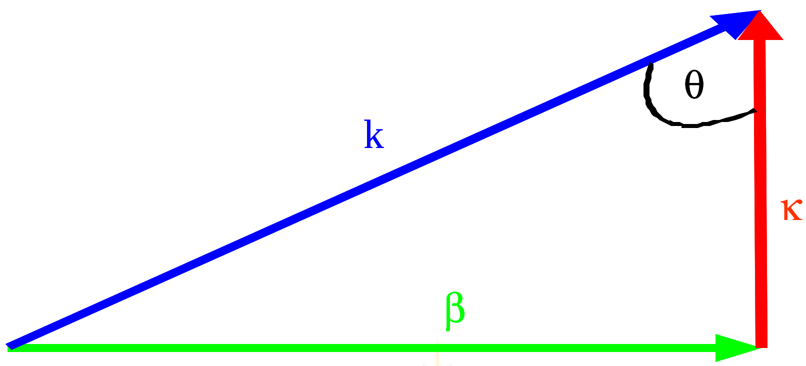

Figure 2.2: Vector triangle describing the relationship between b**,** k **and** k.

The additional vector, namely k, is called the transverse wave vector. k corresponds to an oscillatory solution of the scalar wave equation, so when $b < k_0 n_i$ (with i being 1, 2 or 3 in this case) we can state that

$$k = \sqrt{k_0^2 n_i^2 - b^2} \tag{2.1}$$

where k_0 denotes the wave vector in free space, i.e. for l_0.

For the situation where $b > k_0 n_i$, the solution is exponentially decaying, and we find the attenuation constant as

$$g = \sqrt{b^2 - k_0^2 n_i^2} \tag{2.2}$$

Intuitively now, it can be stated that the existence of a guided wave in a planar slab dielectric material with $n_1 > n_2 > n_3$ requires a propagation constant of

$$k_0 n_2 < b < k_0 n_1 \qquad (2.3)$$

For bounded waves in waveguides, the group velocity (in terms of the propagation constant) takes the form of [1]

$$v_g = \frac{dw}{db} \qquad (2.4)$$

2.3) Waveguide modes

At least one mode can be supported by a symmetrical waveguide. At low frequencies with large wavelengths and small wave vectors, the guide is single-mode and only supports a symmetrically even pattern. The lower the frequency, the larger the evanescent field outside the guide. As the frequency increases, the field distributes more towards the centre of the guide, and a second antisymmetrical mode develops at higher frequencies.

It can be shown that the cut-off condition for modes is described by the following formula [1]

$$kh/2 = mp/2 \text{ for } m = 1,2,\ldots \qquad (2.5)$$

where h is the height of the waveguide in a given direction, and m is the mode number. Note that at cut-off [1]

$$k = \sqrt{k_0^2 n_{co}^2 - b^2} = k_0 \sqrt{n_{co}^2 - n_{cl}^2} \qquad (2.6)$$

where n_{co} denotes the core index, whereas n_{cl} denotes the cladding index.

A dimensionless parameter V, called the normalized frequency, is defined as follows [1]:

$$V = \frac{k_0 h}{2}\sqrt{n_{co}^2 - n_{cl}^2} = \frac{k_0 h n_{co}}{2}\sqrt{2\Delta} \qquad (2.7)$$

where $\Delta = \left(n_{co}^2 - n_{cl}^2\right)/\,2n_{co}^2 \approx \left(n_{co} - n_{cl}\right)/\,n_{co}$. A waveguide is said to be multi-moded if $V >> 1$. At the opposite extreme, if V is sufficiently small so that only the two polarization states of the fundamental mode can propagate, the waveguide is said to be single-moded. By inserting equations 2.5 and 2.6 into 2.7, the normalized cut-off frequency for each mode is obtained:

$$V = \frac{k_0 h}{2}\sqrt{n_{co}^2 - n_{cl}^2} = \frac{mp}{2} \qquad (2.8)$$

Note that the value for the lowest mode is given by $m = 0$. The result is that $V = 0$ according to equation 2.8, implying that k_0 should be zero (which is unphysical), and subsequently that the lowest order mode is always supported in any symmetrical waveguide. Two other normalised parameters can be defined, and are typically used to numerically solve the eigenvalue equations. They are the asymmetry parameter a and the normalised effective index or normalised propagation vector b_n defined by [1]

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2} \qquad (2.9)$$

$$b_n = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2} \qquad (2.10)$$

where n_{eff} is the effective index of the waveguide defined as $n_{eff} = b/k_0$.

To facilitate the description of the modal fields, dimensionless modal parameters can be defined for the core and cladding regions as follows [2]:

$$U = h\sqrt{k_0^2 n_{co}^2 - b^2} \tag{2.11}$$

$$W = h\sqrt{b^2 - k_0^2 n_{cl}^2} \tag{2.12}$$

From equations 2.11 and 2.12, it is clear that

$$V^2 = U^2 + W^2 . \tag{2.13}$$

and the range of bound mode propagation constants in equation 2.3 is equivalent to

$$0 \leq U \leq V ; 0 \leq W \leq V \tag{2.14}$$

The fundamental modes are defined by the two smallest values of U (the two largest values of b) that satisfy the eigenvalue equation of the given value V.

The parameter V is subject to certain limits. Firstly, when l ® μ we have V ® 0. This describes the electrostatic limit of the vector wave equation [2], and the electric field is expressible in terms of a scalar function. The second limit is described as modal cut-off, defined by

$$U = V ; W = 0 . \tag{2.15}$$

In modal cut-off, a significant fraction of a mode's power can propagate within the core at cut-off, but below cut-off these modes propagate with loss and are leaky [2]. Finally a mode is far from cut-off when $V >> U$, leading to

$$W \cong V; V \to a \qquad (2.16)$$

In this mode, all the power is concentrated in the finite region about the maximum core index. For clad profiles, this region is entirely within the core [2].

2.4) Mode solutions

The solution of modes and propagation constants in three-dimensional waveguides is an area on which exhaustive research has been done. Various methods can be implemented, each with its own advantages and disadvantages resulting from the assumptions made concerning the waveguide. It is thus important for the designer to know whether a certain solution can be used for a certain waveguide or not. A brief overview of three methods is supplied in this section, followed by a more thorough discussion on the beam propagation method, as the latter is the method that is used in this thesis.

In 1969 Marcatili described an approximate analytical solution to the wave equation whereby the corner regions of the waveguide are ignored [3]. This solution yields simple and powerful results, but care should be taken near cut-off, since the assumptions are not valid in that case.

The effective index method, developed by Hocker and Burns in 1977, converts a two-dimensional waveguide to two orthogonal oriented one-dimensional waveguides with direct interaction [1]. Here a general rectangular waveguide is decomposed into orthogonal-oriented slab waveguides. The horizontal slab maintains the same core index, whereas the vertical slab waveguide is solved by replacing the core index of refraction by the effective index defined by $n_{eff} = b_s/k_0$ (with b_s denoting the propagation constant of the slab waveguide). The procedure is to solve the characteristic equation for the TE and TM modes of the horizontal section. Once b_s is found, n_{eff} is calculated and used as the core index of the vertical waveguide section. From this the approximate propagation constant of the rectangular waveguide is calculated through the characteristic wave equation describing the vertical waveguide section.

Another approach, namely the perturbation method, replaces the profile of *n* by an approximate profile defined by $n_{app}(x,y)$ for which the analytical solution to the wave equation can be obtained. If the approximate refractive index is very close to the actual refractive index, the analytical result will be very close to the true wave equation solution. Based on the completeness of modes, the true wave mode can be expressed approximately in terms of the superposition of the complete set of analytical solutions [1]. The perturbation expressions can be applied to waveguides of an arbitrary cross-section, and the integration only needs to be carried out over the regions of interest where the approximate index profiles are different from the actual ones.

2.5) The beam propagation method (BPM)

The beam propagation method (BPM) is a powerful technique that can be used to solve optical circuits where $\Delta << 1$. In this case, the modes are governed by the scalar Helmholtz equation [4]

$$\left[\nabla^2 + \frac{w^2}{c^2} n^2(w,x,y)\right] E = 0 \tag{2.17}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{2.18}$$

h is the height of the waveguide and E is the transverse component of the electric field as a function of the circular frequency w and the dimensional coordinates x, y and z, whereas the refractive index is a function of w, x and y.

Describing E as

$$E(w,x,y,z)=x(w,x,y,z)e^{-jkz} \tag{2.19}$$

and substituting it into equation 2.17 yields

$$\frac{\partial^2 x}{\partial x^2}+\frac{\partial^2 x}{\partial y^2}+\frac{\partial^2 x}{\partial z^2}-2jk\frac{\partial x}{\partial z}-k^2 x+\frac{h^2}{c^2}n^2 x=0 \tag{2.20}$$

$$\therefore -\frac{\partial^2 x}{\partial z^2}+2jk_0\frac{\partial x}{\partial z}=\nabla_\perp^2 x+k_0^2\left\{\left[\frac{n(x,y)}{n_0}\right]^2-1\right\}x$$

where $\nabla_\perp^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ and assuming $k = w^2/c$. Neglecting the leftmost term in equation 2.20 yields the well-known parabolic or paraxial approximation of the Helmholtz equation

$$2jk_0\frac{\partial x'}{\partial z}=\nabla_\perp^2 x'+k_0^2\left\{\left[\frac{n(x,y)}{n_0}\right]^2-1\right\}x' \tag{2.21}$$

The eigen solutions of equations 2.20 and 2.21 can be expressed as

$$x(x,y,z)=u(x,y)e^{-jbz} \tag{2.22}$$

$$x'(x,y,z)=u'(x,y)e^{-jb'z} \tag{2.23}$$

Substituting equations 2.22 and 2.23 back into equations 2.20 and 2.21 provides the relationship between the eigenvalues as [4]

$$b=k_0\left[1-\left(1+2\frac{b'}{k_0}\right)^{1/2}\right] \tag{2.24}$$

The conclusion from equation 2.24 is that eigenvalues can be obtained by solving the parabolic or Fresnel equation [4].

Equation 2.24 is solved numerically by using an implicit finite difference scheme. However, a fundamental physical limitation exists resulting from the parabolic approximation to the Helmholtz equation. The effect of these limitations can be relaxed by using more accurate approximations, such as the Padé approximations found in the BeamPROP software used in this thesis. Another limitation results from the assumption of scalar waves, and prevents polarization effects from being considered. Several vector beam propagation techniques can be employed to overcome this problem, as described in [5, 6]. The third limitation is that BPM cannot account for backward reflections because the one-way equation on which it is based does not allow both positive and negative travelling waves. It is possible to introduce a bi-directional BPM algorithm, as is done in BeamPROP.

2.6) Dispersion

Dispersion can quite simply be described as the process whereby a signal pulse is spread in the time domain. This signal spreading or broadening caused by dispersion leads to the overlapping of adjacent pulses, resulting in erroneous signal transmission. It poses the most fundamental bandwidth limitation for optical communications systems.

Three main types of dispersion can be identified:

1) Material dispersion:

   Due to the dependence of the refractive index on the wavelength. Each wavelength component of a pulse is thus influenced by a different refractive index and will travel at a different velocity.

2) Modal dispersion (or inter-modal dispersion):

   The result of multi-mode propagation where each mode travels at a different group velocity. Lower order modes travel faster than higher order modes in an optical waveguide.

3) Waveguide dispersion (or intra-modal dispersion):

   Normally the smallest type of dispersion, barring the case near the vicinity of the zero dispersion point in a singlemode waveguide [1], where it plays the dominant role. If the dispersions have different signs, a careful design of the waveguide dispersion can be used to cancel the material dispersion near the zero-material dispersion point. It is a direct consequence of the dependence of the propagation constant on the wavelength.

Of the above three cases, material dispersion and waveguide dispersion play a role in single-mode waveguide devices. Another case of dispersion should be considered in the case where more than one polarization mode exists in a birefringent waveguide. It is called polarization mode dispersion and is caused by the refractive index difference between different polarizations. The birefringence is often caused due to the inevitable stress between the cladding and guiding core materials in dielectric waveguides [7].

2.6.1) Material dispersion

As material dispersion is a direct result of certain material properties, it is important to take its effect into account when designing components with waveguides, especially where filters are concerned.

The derivation of the expression for material dispersion is taken from [1] and begins with the examination of a pulse of finite bandwidth travelling at the group velocity of $v_g = dw/dk$. We recall that

$$w = 2pv \text{ and } k = \frac{2p}{l} \tag{2.25}$$

The group velocity term is written in terms of frequency and wavelength as follows:

$$v_g = \frac{dw}{dv}\frac{d\,1/l}{dk}\frac{dv}{d\,1/l} = \frac{2p}{2p}\frac{dv}{d\,1/l} = -l^2\frac{dv}{dl} \tag{2.26}$$

By substituting $v = \frac{c}{nl}$, equation 2.26 becomes

$$v_g = \frac{c}{n}\left(1 + \frac{l}{n}\frac{dn}{dl}\right). \tag{2.27}$$

From equation 2.27 the group velocity dispersion is found as follows:

$$\frac{dv_g}{dl} = \frac{cl}{n^2}\left[\frac{d^2n}{dl^2} - \frac{2}{n}\left(\frac{dn}{dl}\right)^2\right] \tag{2.28}$$

$$\therefore v_g = \frac{cl}{n^2}\left[\frac{d^2n}{dl^2} - \frac{2}{n}\left(\frac{dn}{dl}\right)^2\right]\Delta l$$

The dispersion in time, called Dt, of an initially narrow pulse after travelling a distance L is given by

$$\Delta t = \frac{L}{v_{g2}} - \frac{L}{v_{g1}} = -L\frac{v_{g2} - v_{g1}}{v_{g2}v_{g1}} \approx -L\frac{\Delta v_g}{v_g^2} \tag{2.29}$$

By combining equations 2.27, 2.28 and 2.29, we find

$$\Delta t = -L\left(\frac{cl}{n^2}\left[\frac{d^2n}{dl^2} - \frac{2}{n}\left(\frac{dn}{dl}\right)^2\right]\Delta l\right) \Bigg/ \left(\frac{c}{n}\left(1 + \frac{l}{n}\frac{dn}{dl}\right)\right)^2 \tag{2.30}$$

Because the most waveguide materials satisfy (the refractive index varies slowly as function of the wavelength)

$$\frac{l}{n}\frac{dn}{dl} << 1 \tag{2.31}$$

and

$$\frac{d^2n}{dl^2} >> \frac{2}{n}\left(\frac{dn}{dl}\right)^2 , \tag{2.32}$$

we can simplify equation 2.30 to

$$\Delta t = -L\frac{l}{c}\frac{d^2n}{dl^2}\Delta l \ . \tag{2.33}$$

By rearranging equation 2.33, we have

$$D = \frac{\Delta t}{L\Delta l} = -\frac{l}{c}\frac{d^2n}{dl^2} . \tag{2.34}$$

The unit for parameter *D*, called the dispersion parameter, is ps/nm/km. It describes the pulse broadening in ps for each kilometre travelled by a pulsed of spectral width equal to 1 nm.

2.6.2) Waveguide dispersion

Waveguide dispersion is important because a pulse of finite temporal profile will always have a finite spectral width. It plays a critical role in single-mode waveguides, especially near the zero material dispersion region.

Consider a pulse propagating in a single-mode waveguide with a finite spectral bandwidth of Dk (or Dl). Each value of k within Dk will have a different b and thus different group velocity corresponding to it.

The dispersion of this single-mode, after travelling a distance *L*, is therefore [1]

$$\Delta t = \frac{L}{v_{g,slowest}} - \frac{L}{v_{g,\,fastest}} = \frac{L}{c}\left[\left(\frac{db}{dk}\right)_{\max} - \left(\frac{db}{dk}\right)_{\min}\right] \approx \frac{L}{c}\frac{d^2 b}{dk^2}\Delta k \qquad (2.35)$$

where b is dependent on k (because of the eigenvalue equation).

2.6.3) Total dispersion estimate

A rough estimate of the total pulse broadening can be made, assuming that the spectral shape can be approximated as a Gaussian pulse. The total effect is then described as [1]

$$\Delta t_{sum} = \sqrt{\Delta t^2_{\text{inter-modal}} + \left(\Delta t_{waveguide} + \Delta t_{material}\right)^2} \qquad (2.36)$$

Note that the effects of the material and waveguide dispersions are added directly because they both depend on wavelength, while the inter-modal dispersion depends largely on the characteristics of the modes.

2.6.4) Polarization mode dispersion (PMD)

PMD is caused by birefringence. Birefringence of a material is defined as the maximum difference in the refractive index between any two polarization states. For waveguides, then, it is $B = n_{TM} - n_{TE}$.

In the absence of mode coupling the PMD is defined as the differential delay resulting from propagation over a distance L and is found by applying equation 2.17 to the birefringence as follows [1]:

$$PMD = \frac{\Delta t}{L} = \frac{1}{c}\left|B - w\frac{dB}{dw}\right| \qquad (2.37)$$

In isotropic materials, any asymmetrical stress will induce a birefringence through the photoelastic effect. Stress-induced birefringence caused by stresses inherent in deposited waveguides can be compensated for by etching deep grooves in close proximity to the waveguides, as discussed in section 4.3. Waveguides can be designed to be polarization insensitive by choosing the correct physical dimensions [8]. The basic idea is to compensate for polarization dispersion due to the refractive index contrast in the transversal direction by applying sufficient refractive index in the lateral direction. It has been demonstrated that the waveguide width needed to obtain polarization independence in a strip waveguide is given by [9]

$$W = l\left(\frac{\left(\Delta_1\Delta_2\right)^{1/2}}{p\left(n_{core}^2 - n_{clad}^2\right)^{1/2}\left(N_{TE}^2 - N_{TM}^2\right)}\right)^{1/3} \text{ with} \tag{2.38}$$

$$\Delta_1 = \frac{n_{core}^2 - n_{clad}^2}{2n_{core}^2} \quad \text{and} \quad \Delta_2 = \frac{n_{core}^2 - 1}{2n_{core}^2}$$

where N_{TE} and N_{TM} are the vertical effective indexes of the middle region for the TE and TM modes respectively.

2.7) Loss and signal attenuation

Attenuation is inherent in the waveguide itself, and results in the loss of signal. The effect of attenuation needs to be considered in order to design an effective waveguide.

Three fundamental types of loss can be identified [1]:

1) Material absorption loss
2) Scattering loss
3) Radiation loss.

Mechanical losses, such as coupling loss, also occur.

2.7.1) Fundamental absorption loss

Fundamental absorption loss occurs via atomic or molecular absorption and band-to-band electronic absorption of electromagnetic energy. The amplitude of a wave attenuates and the power intensity (power per cross-sectional area) is described by

$$I(z) = I_0 e^{-az} \tag{2.39}$$

where α is related to the imaginary index of n′′ and is called the attenuation constant (the imaginary index is described in detail in [1]). The abovementioned relation is described by

$$a = 2k_0 n'' \tag{2.40}$$

2.7.2) Fundamental scattering loss

Scattering loss occurs when the electromagnetic waves interact with scattering centres with a size smaller than the wavelength (but still in the order of the wavelength). These scattering centres represent impurity clusters and localised dielectric fluctuations. Two dominant scattering mechanisms exist in dielectric waveguides, namely Rayleigh scattering and surface scattering. The latter is a result of surface roughness of waveguides, and can be significant in rib/ridge waveguides if the etched surfaces are too rough. The interface scattering loss increases as the square of the refractive index difference [8].

Most dielectric waveguide materials are amorphous in nature. This disorder can be structural (where some basic molecular units are connected in a random way) and/or compositional (where chemical composition varies from place to place) in nature, and causes Raleigh scattering. Raleigh scattering decreases rapidly as the wavelength is increased [1], but plays a role where impurity absorption takes place at certain wavelength bands. A good example of this is given in [8], where silicon oxynitride layers exhibited losses because of N-H bonds absorbing in the 1500nm wavelength range.

2.7.3) Waveguide bending loss

Waveguide bending is essential for integrated optical circuits. Each curved path involves two types of loss: pure bending loss and transition loss. Pure bending loss occurs in regions where the waveguide has a constant radius of curvature. Transition loss occurs when the waveguide has a discontinuity in its bending curvature.

A quantative expression for pure bending loss, α, has been derived by Marcuse [1]. Assuming that the power attenuates as given in equation 31 after propagating a distance z in a bend of radius R, it has been shown that α is exponentially proportional to R. More specifically, then, the bending loss increases exponentially as the bending radius R decreases. When designing waveguide bends, it is advisable to introduce as safety factor in the minimum bending radius, so as to ensure that slight manufacturing anomalies will not be detrimental to the circuit.

Transition losses occur at locations where the waveguide's curvature changes abruptly, because of the mode mismatch between the two sections with different curvatures. These losses can be minimized by introducing lateral offsets between the waveguides, resulting in a compensation for the mode offsets. This method of compensation is introduced in section 4.4.

2.8) Conclusion

In this chapter, a broad overview of waveguide theory was presented. Although some of the theory is not used directly in this work, it is included in order to view waveguides in a holistic manner. All of the discussed topics play some role in waveguide structures, and should be kept in mind by the designer.

BPM is used extensively in this thesis by means of the BeamPROP software. It is an extensive and powerful numerical solution technique that is capable of solving waveguides of various shapes and material classes. The analytical results presented in this chapter assist the designer to verify the results obtained by BeamPROP to within an order.

2.9) References

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