# An offer you cannot refuse: obtaining efficiency and fairness in preplay negotiation games with conditional offers

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**Abstract.** We study a recently introduced extension of normal form games with a phase before the actual play of the game, where each player can make binding offers for payments of utility to the other players after the play of the game, contingent on the recipient playing the strategy indicated in the offer. Such offers transform the payoff matrix of the original game and allow for some degree of cooperation between rational players while preserving the non-cooperative nature of the game. We focus on 2-player negotiations games arising in the preplay phase when offers for payments are made conditional on a suggested matching offer of the same kind being made in return by the receiver. We study and analyze such bargaining games, obtain results describing their possible solutions and discuss the degrees of efficiency and fairness that can be achieved in such negotiation process depending on whether time is valuable or not.

## 1 Introduction

It is well-known that many solution concepts in non-cooperative games can induce outcomes that are far from being Pareto optimal. Some studies have considered various forms of preplay interaction between the players, aiming at improving the resulting payoffs. Such interaction range from cheap talk to signing contracts. Cheap talk affects neither the payoffs nor the non-cooperative rational behavior of the players, whereas by signing contracts the players pre-determine the outcome of the resulting normal form game, essentially playing as a coalition.

The problem in the focus of the present study is: *what can rational players achieve by means of interactive negotiations prior to playing a non-cooperative game?* In [3] we consider a version of preplay interaction between players, whereby they try to negotiate better outcomes in the forthcoming game by means of exchanging offers for additional (side) payments of utility conditional on the recipient playing the strategy indicated in the offer. More precisely, before the actual game is played any player, say $A$ (Ann), can make a binding offer to any other player, say $B$ (Bob), to pay him, after the end of the game, an explicitly declared amount of utility $\delta$ if $B$ plays a strategy $s$ specified in the offer by $A$. Such an offer effects a simple transformation of the payoff matrix of the game, by transferring the declared amount from the payoff of $A$ to the payoff of $B$ in every outcome corresponding to $B$ playing $\delta$. Players can exchange multiple such offers in attempt to transform the game into one where their expected payoffs, assuming rational behaviour of the other players according to a commonly adopted solution concept, would be better than those expected from the original game. Furthermore, players can make such offers *conditional on receiving desired matching offers*, which can be in turn accepted or rejected. Thus, a whole preplay negotiation phase emerges before the original normal form game is actually played, and it can be regarded as another game in which players bargain towards a mutually optimal transformation of the former.

This paper studies 2-player negotiations games arising in the preplay phase when conditional offers for payments are made on a suggested matching offer of the same kind made in return by the receiver. We study and analyze such bargaining games, obtain results describing their possible solutions and discuss the degrees of efficiency, in the sense of Pareto optimality of the resulting distribution, and fairness, in the sense of equitability of the resulting distribution, that can be achieved in such negotiation process in the cases where time is valuable or not. We focus on the ideas and intuitions behind preplay negotiation games with conditional offers, while, for space reasons, some core technical results are stated with brief proof sketches. Full proofs of those results are available in [4].

The paper is organized as follows. In Section 2 we introduce the preliminary game-theoretical notions. In Section 3 we discuss preplay offers in two-player normal form games and define the framework of preplay negotiation games. The analysis and main results are in Section 4. In Section 5 we discuss some related work and end with concluding remarks and further agenda in Section 6.

## 2 Preliminaries

Let $\mathcal{G} = (\{A, B\}, \{\Sigma_A, \Sigma_B\}, u)$ be a two player normal form game (henceforth NFG), where $\{A, B\}$ is a set of players, $\{\Sigma_A, \Sigma_B\}$ a set of finitely many strategies for each player and $u : \{A, B\} \times \Sigma_A \times \Sigma_B \to \mathbb{R}$ is a **payoff function** assigning to each player a utility for each strategy profile. The game is played by each player $i$ choosing a strategy from $\Sigma_i$. The resulting strategy profile $\sigma$ is the **outcome** of the play and $u_i(\sigma) = u(i, \sigma)$ is the associated payoff for $i$. An outcome of a play of the game $\mathcal{G}$ is called **maximal** if it is a Pareto optimal outcome with the highest sum of the payoffs of all players. Let $\mathbf{G_N}$ be the set of all normal form games for a set of players $N$. By **solution concept for $\mathbf{G_N}$** we mean a map $\mathfrak{S}$ that associates with each $\mathcal{G} \in \mathbf{G_N}$ a non-empty set $\mathfrak{S}(\mathcal{G})$ of outcomes of $\mathcal{G}$, called the **$\mathfrak{S}$-solution of the game**. For a player $i$, we denote $\mathfrak{S}_i$ the restriction of the mapping $\mathfrak{S}$ to $i$ returning only the strategies of player $i$ consistent with $\mathfrak{S}$, i.e., $\mathfrak{S}_i(\mathcal{G}) = \{\sigma_i \in \Sigma_i \mid \sigma \in \mathfrak{S}(\mathcal{G})\}$. We also use $-i$ for any $i \in \{A, B\}$ to denote $i$'s opponent. In this work we do not commit to a specific solution concept for the normal form games but we assume that the one adopted by the players satisfies the necessary condition that *every outcome in any solution prescribed* *by that solution concept must survive iterated elimination of strictly dominated strategies.* We call such solution concepts **acceptable**.

Games for which the solution concept $\mathfrak{S}$ yileds a set of payoff equivalent outcomes will be called **uniformly $\mathfrak{S}$-solvable**. Games for which $\mathfrak{S}$ yileds only maximal outcomes will be called **optimally $\mathfrak{S}$-solvable**. Games that are both **optimally $\mathfrak{S}$-solvable** and **uniformly $\mathfrak{S}$-solvable** will be called **perfectly $\mathfrak{S}$-solvable**. $\mathfrak{S}$-solvable games for which the solution concept $\mathfrak{S}$ yields a single outcome will be called **$\mathfrak{S}$-solved**. For instance, every game with a strongly dominating strategy profile is $\mathfrak{S}$-solved for any acceptable solution concept $\mathfrak{S}$. Ideally, preplay negotiation games should transform the starting NFG into a perfectly $\mathfrak{S}$-solved, or at least perfectly $\mathfrak{S}$-solvable, one.

It is necessary for the preplay negotiation phase for each player to have an **expected value** of any NFG that can be played. For sake of definiteness we adopt here a conservative, risk-averse approach and will define for every acceptable solution concept $\mathfrak{S}$, game $\mathcal{G}$ and a player $i$, the expected value of $\mathcal{G}$ for $i$ relative to the solution concept $\mathfrak{S}$ to be:

$$\mathsf{v}_i^{\mathfrak{S}}(\mathcal{G}) = \max_{\sigma_i \in \mathfrak{S}_i(\mathcal{G})} \min_{\sigma_{-i} \in \mathfrak{S}_{-i}(\mathcal{G})} u_i(\sigma)$$

We note that our further analysis does not depend essentially on this particular assumption; any other realistic notion of expected value of a NFG would yield similar results.

## 3 Two-player normal form games with preplay offers

### 3.1 Preplay offers

Following [3] we use the notation $A \xrightarrow{\delta/\sigma_B} B$ to denote an offer made by player $A$ to pay an amount $\delta$ to player $B$ after the play of the game if player $B$ plays strategy $\sigma_B$. Any preplay offer by $A$ to $B$ is assumed *binding for $A$*, upon $B$ playing the specified strategy. However, such offer *does not* create any obligation for $B$, who is still at liberty to choose his strategy when the game is actually played. In particular, after her offer $A$ does not know in advance whether $B$ will play the desired by $A$ strategy $\sigma_B$, and thus make use of the offer or not. The key observation applying here is that *after any preplay offer the game remains a non-cooperative normal form game, only the payoff matrix changes according to the offer.* We now illustrate preplay offers in a well-known scenario.

*Motivating example: Prisoners' Dilemma.* Consider a version of the Prisoner' s Dilemma (PD) game in Figure 1, left. The only Nash Equilibrium (NE) of the game is $(D, D)$, yielding a payoff of $(1, 1)$. Now, suppose player *Row* makes the offer *Row* $\xrightarrow{2/C}$ *Column* to the player *Column*. That offer transforms the game by transferring 2 utils from the payoff of *Row* to the payoff of *Column* in every entry of the column where *Column* plays $C$, as in Figure 1, middle.

| | $C$ | $D$ |
|---|---|---|
| $C$ | $3,3$ | $0,4$ |
| $D$ | $4,0$ | $1,1$ |

$Row \xrightarrow{2/C} Column$

| | $C$ | $D$ |
|---|---|---|
| $C$ | $1,5$ | $0,4$ |
| $D$ | $2,2$ | $1,1$ |

$Column \xrightarrow{2/C} Row$

| | $C$ | $D$ |
|---|---|---|
| $C$ | $3,3$ | $2,2$ |
| $D$ | $2,2$ | $1,1$ |

**Fig. 1.** From left to right: A Prisoner's Dilemma game; the game after the first offer by player Row; the game after the second offer by player Column.

In this game player *Row* still has the incentive to play $D$, which strictly dominates $C$ for him, but the dominant strategy for *Column* now is $C$, and thus the only NE is $(D, C)$ with payoff $(2, 2)$ – strictly dominating the original payoff $(1, 1)$. Of course, *Column* can now realize that if player *Row* is to cooperate, an extra incentive is needed. That incentive *can be created* by an offer $Column \xrightarrow{2/C} Row$, that is, if *Column*, too, makes an offer to *Row* to pay him 2 utils after the game, if player *Row* cooperates. Then the game transforms as in Figure 1, right. In this game, the only Nash equilibrium is $(C, C)$ with payoff $(3, 3)$, which is also Pareto optimal. Note that this is the same payoff for $(C, C)$ as in the original PD game, but now both players have created incentives for each other to cooperate, thus escaping[3] from the original bad Nash equilibrium $(D, D)$.

### 3.2 Conditional offers

Consider now an instance of the Battle of the Sexes (Figure 2, left), with the column player called *Him* and the row player *Her*. It has two Nash equilibria: one preferred by *Her*: (*Ballet*, *Ballet*), and the other – by *Him*: (*Soccer*, *Soccer*).

| Her\Him | *Ballet* | *Soccer* |
|---|---|---|
| *Ballet* | $5,3$ | $1,1$ |
| *Soccer* | $0,0$ | $3,5$ |

| Her\Him | *Ballet* | *Soccer* |
|---|---|---|
| *Ballet* | $5,3$ | $1,1$ |
| *Soccer* | $2,-2$ | $5,3$ |

| Her\Him | *Ballet* | *Soccer* |
|---|---|---|
| *Ballet* | $5,3$ | $0,2$ |
| *Soccer* | $2,-2$ | $4,4$ |

**Fig. 2.** From left to right: A Battle of the Sexes game; the game transformed by the offer $Him \xrightarrow{2/Soc} Her$ favouring *Her*; further transformed by an offer $Her \xrightarrow{1/Soc} Him$.

An offer $Him \xrightarrow{2/Soccer} Her$[4] would transforms the game to one in Figure 2, middle. By doing so, *Him* makes the equilibrium (*Soccer*,*Soccer*) equally beneficial for *Her* as (*Ballet*,*Ballet*) and also sends the clear message that he intends to play *Soccer*, thus essentially breaking the coordination problem and

[3] Clearly, preplay offers can only work in case when at least part of the received payoff can actually be transferred from a player to another. They obviously cannot apply to scenarios such as the original PD, where one prisoner cannot offer to the other to stay in prison for him, even if they could communicate before the play.

[4] which can be made, for example, in the form of invitation to a dinner in a luxury restaurant after the soccer match, if *Her* pitches up there.

deciding the game. However, this offer comes at a cost for *Him* and puts him in a relatively disadvantaged position: with respect to the original game, he is worse off in one of the two Nash equilibria and he is not better off in the other.

The loss and disadvantage incurred by *Him* in the example above could be partly neutralized by an offer $Her \xrightarrow{1/Soccer} Him$, which transforms the game to the one in Figure 2, right.

But, of course, *Her* has no incentive to make such an offer to *Him* in the middle game on Figure 2. So, the only realistic way for *Him* to force such matching offer by *Her* is to make his offer $Him \xrightarrow{2/Soccer} Her$ *conditional on Her making to Him the matching offer* $Her \xrightarrow{1/Soccer} Him$[5]. This conditional offer is denoted hereafter as $Him \xrightarrow{2/Soccer \mid 1/Soccer} Her$. The effect is that players reach an equitable redistribution of the payoffs in the expected (maxmin) outcome.

In practice, a conditional offer $A \xrightarrow{\alpha/\sigma_B \mid \beta/\rho_A} B$ enables the offering player to *suggest* a transformation of the game $\mathcal{G}$ into a game $\mathcal{G}(A \xrightarrow{\alpha/\sigma_B \mid \beta/\rho_A} B)$ that is updated according to the offer.

In other words, a suggested transformation updates the original game into a new game where only the payoff vectors change, according to the conditional offer that is made. At each profile, each player collects the positive reward received in the part of the conditional offer consistent with the profile, subtracting in the same fashion the payments given.

There are two possible responses to a conditional offer $A \xrightarrow{\alpha/\sigma_B \mid \beta/\rho_A} B$: it can be *accepted* or *rejected* by the receiving player. If rejected, the offer is immediately cancelled and does not commit any of the players to any payment, and therefore it does not induce any transformation of the game matrix. If accepted, the *actual transformation* induced by the offer is the suggested transformation defined above.

For space reasons we do not consider the possibility of withdrawing previously made offers, which is treated in [4].

### 3.3 Preplay negotiation games

Similarly to [6], our setting for normal form games with preplay offers begins with a given 'starting' normal form game $\mathcal{G}$ and consists of two phases:

- A *preplay negotiation phase*, where players negotiate on how to transform the game $\mathcal{G}$ by making offers, accepting or rejecting conditional offers they receive.
- An *actual play* phase where, after having agreed on some transformation $X$ in the previous phase, the players play the game $\mathcal{G}$ updated with $X$.

We will call the resulting games *preplay negotiation games.*

[5] for instance, *Her* could offer to bring a coolbox with cold beer and chips to the stadium if *Him* comes there.

In order to define a PNG we need to introduce some preliminaries: moves, histories and plays. Depending on some of the optional assumptions, the players can have several possible moves in a PNG. Let us consider the case where conditional offers are allowed. Then the moves available to the player whose turn is to play depend on whether or not he/she has received since his/her previous move any conditional offers. If so, we say that the player has **pending conditional offers**. The possible moves of the player in turn are as follows.

1. A player who has no pending conditional offers can:
   (a) *Make an offer* (conditional or not).
   (b) *Pass*.
2. A player who has pending conditional offers, can for each of them:
   (a) *Accept the pending offer* , and then make an offer of his/her own or pass.
   (b) *Reject the pending offer*, and then make an offer of his/her own or pass.

The PNG is over when all players have passed at their last move, or a player has opted out.

We now define the notion of a **history** in a PNG as a sequence of moves by the players who take their turns according to an externally set protocol (a detailed discussion of the possible external protocols is provided in [3]). Every finite history in such a game is associated with the current NFG: the result of the transformation of the starting game by all offers that are so far made and accepted. The current NFG of the empty history is the input NFG of the PNG. A **play** of a PNG is any finite history at the end of which the preplay negotiations game is over, or any infinite history.

In order to eventually define realistic solution concepts for preplay negotiations games we need to endow every history in such games with a value for every player. Intuitively, **the value of a history** is the value for the player of the current NFG associated with that history, in the case of non-valuable time, and the same value accordingly discounted in the case of valuable time.

*Disagreements.* The PNG may terminate if all players pass at some stage, in which case we say that the players have reached agreement, or may go on forever, in which case the players have failed to reach agreement; we call such situation a *(passive) disagreement* and we denote any such infinite history with $D$. We will not discuss disagreements and their consequences here, but will make the explicit assumption that *any agreement is better for every player than disagreement* in terms of the payoffs, e.g. by assigning payoffs of $-\infty$ in the entire game for each player if the PNG evolves as a disagreement. In [3] we also outline a more flexible and possibly more realistic alternative, whereby players can explicitly express tentative acceptance of the current NFG – the one on which they are currently negotiating by making offers or can terminate the negotiations by explicitly opting out, which would revert to the current game to the currently accepted by everyone NFG.

A **preplay negotiation game** (PNG) starts with an input NFG $\mathcal{G}$ and either ends with a transformed game $\mathcal{G}'$ or goes on forever, which we discuss further. The **outcome of a play of the PNG** is the resulting transformed game $\mathcal{G}'$ in the former case and 'Disagreement' (briefly $D$) in the latter case. By a **solution of a PNG** we mean *the set of all transformed normal form games that can be obtained as outcomes of plays induced by subgame perfect equilibrium strategy profiles in the PNG*. Finally, we say that a strategy in a PNG is **strongly efficient** if the vector of payoffs of the outcome it attains is a redistribution of the vector of payoffs of a maximal outcome.

## 4 Preplay negotiations games with conditional offers

First, let us state a useful general result, also valid in the case of many players PNG. An extensive form game is said to have the **One Deviation Property** (ODP) [8, Lemma 98.2] if, in order to check that a strategy profile is a Nash equilibrium in (some subgame of) that game, it suffices to consider the possible profitable deviations of each player not amongst *all* of its strategies (in that subgame), but only amongst the ones differing from the considered profile in the first subsequent move (in that subgame).

**Lemma 1.** *Every PNG has the One Deviation Property.*

*Proof.* It is easy to check that a strategy profile of a PNG is a subgame perfect equilibrium if and only if it is a subgame perfect equilibrium of the same PNG without disagreement histories as, notice, strategies leading to disagreement are cannot be used as credible threats. But, a PNG without disagreement histories is an extensive game of perfect information and finite horizon. By [8, Lemma 98.2] the PNG has the one deviation property.

### 4.1 The case of non-valuable time

The value for a player of a history in a PNG is the value for the player of the current NFG associated with that history. When time is not valuable players assign the same value to the NFG associated with the current moment and the same game associated with any other moment in the future, which means that players can afford delaying offers at no extra cost.

To analyze equilibrium strategies of PNG when time is not valuable we consider so called **stationary acceptance strategies** where players have a minimal acceptance threshold $d$ and a minimal passing threshold $d' \geq d$ (both of which may vary among the players).

**Proposition 1.** *Every subgame perfect equilibrium strategy profile of a two-player PNG with non-valuable time consisting of stationary acceptance strategies is strongly efficient.*

*Proof.* Suppose not. Let $d^-$ be a vector of expected values that is not the redistribution of a maximal outcome of the starting game associated to some subgame perfect equilibrium strategy profile. We know that such strategy profile yields a history $h$ that ends with: 1) the proposal of $d^-$; 2) the acceptance of that proposal; 3) a pass; 4) a pass. Consider now some redistribution $d^*$ of a maximal outcome where both players get more than in $d^-$ and the history $h$ where the last four steps are substituted by the following ones: 1) the proposal of $d^*$; 2) the acceptance of that proposal; 3) pass; 4) pass. By stationarity of strategies and the ODP, the player moving at step 1) is better off deviating from $d^-$ and instead proposing $d^*$: a contradiction.

The condition of stationarity of acceptance strategies is needed if we want to avoid equilibrium strategies that lead to inefficiency. The example below provides a detailed instance of such cases.

*Example 1 (Attaining inefficiency).* Consider the following starting NFG.

| | $L$ | $R$ |
|---|---|---|
| $U$ | $2, 2$ | $4, 3$ |
| $D$ | $3, 3$ | $2, 2$ |

As there are no dominant strategy equilibria, there are acceptable solution concepts assigning 2 to each player.

We now construct a strategy profile of the PNG starting from that game, such that: (i) it is a SPE strategy profile and (ii) it attains an inefficient outcome.

1. At the root node player $A$ *proposes* outcome $(D, L)$ with payoff distribution $(3, 3)$ — i.e., makes a conditional offer where $D, L$ is dominant strategy equilibrium and yields the payoff vector $(3, 3)$ .
2. After such proposal player $B$ accepts. However, if $A$ had made a different offer (so, off the equilibrium path) $B$ would reject and keep proposing outcome $(U, R)$ with distribution of 5 for him and 2 for $A$ and accepting (and passing on) maximal outcomes guaranteeing him at least 5. $A$, on the other hand, would not have better option than proposing the same distribution (5 for $B$ and 2 for her) and accepting only maximal outcomes guaranteeing her at least 2. Notice that once they enter this subgame neither $A$ nor $B$ can profitably deviate from such distribution.
3. If, however, $B$ did not accept the $(3, 3)$ deal then $A$ would keep proposing outcome $(U, R)$ with a redistribution of $(5, 2)$ (5 for her, 2 for him) and accepting at least that much. $B$ on the other hand would also stick to the same distribution, accepting at least 2. Again, no player can profitably deviate from this stationary strategy profile starting from $B$'s rejection.
4. After player $B$ has accepted the deal $(3, 3)$, then $A$ passes. If $A$ did not pass, player $B$ would go back to his $(2, 5)$ redistribution threat. Likewise with the next round. That eventually leads to the inefficient outcome $(3, 3)$.

It is easy to check that the strategy profile described above is a subgame perfect equilibrium. No player can at any point deviate profitably by proposing the outcome $(U, L)$ with dominating payoff distribution, e.g., $(3.5, 3.5)$ .

In general, in PNG with non-valuable time every redistribution of a maximal outcome can be attained as a solution.

**Proposition 2.** *Let $\mathcal{E}$ be PNG with non-valuable time starting from a NFG $\mathcal{G}$ and let $d = (x_A, x_B)$ be any redistribution of a maximal outcome of the starting NFG. The following strategy profile $\sigma = (\sigma_A, \sigma_B)$ is a subgame perfect equilbrium:*

*For each player $i \in \{A, B\}$:*

- *if $i$ is the first player to move, he proposes a transformation of $\mathcal{G}$ where the vector of expected values in the transformed game is $d$;*
- *when $i$ can make an offer and the previously made offer has not been accepted, he proposes a transformation of the current NFG where the vector of expected values in the transformed game is $d$;*
- *when $i$ can make an offer and the previously made offer has been accepted, he passes;*
- *when $i$ has a pending offer of a suggested transformation where the vector of expected values in the transformed game is $d'$, he accepts it if and only if $x'_i \geq x_i$, and rejects it otherwise;*
- *when $i$ can pass and the other player has just passed, he passes;*
- *when $i$ can pass and the other player has not just passed, he proposes $d$;*
- *when $i$ has just accepted a proposal he passes;*

*Proof.* We have to show that there is no subgame where a player $i$ can profitably deviate from this strategy at its root. By Lemma 1 we can restrict ourselves to considering only first move deviations to the above described strategy.

Suppose the player has a pending offer that induces a transformation of the current NFG where the vector of expected values is $d^*$. If she accepts it then the outcome will be $d^*$, due to the definition of the strategy profile; if she rejects it, it will be the starting offer $d$. And she will accept if and only if she will get more from $d^*$ than from $d$. So the acceptance component is optimal. For the remaining cases, if player $i$ deviates from the prescribed strategy, due to the construction of the strategy and Lemma 1, the vector of payoffs associated to the outcome of $\mathcal{E}$ will be $d$ anyway.

As a consequence of the previous proposition we obtain:

**Corollary 1.** *The game associated to the outcome of a subgame perfect equilibrium strategy profile consisting of stationary acceptance strategies in a two-player PNG with non-valuable time is optimally solvable.*

In summary, our analysis of two-player PNG with non-valuable time shows that efficiency can be attained when conditional offers are allowed and stationary acceptance strategies are followed. Indeed, any redistribution of the vector of payoffs of a maximal outcome can be made the unique solution of the final NFG by such SPE strategies. However, non-stationary acceptance strategies may lead to inefficient equilibria, as illustrated in the comment to Proposition 1.

To sum up, while SPE strategies in a two-player PNG can attain efficiency, some important issues are still remaining:

– SPE strategies with non-stationary acceptance need not be strongly efficient.
– players can keep making unfeasible moves as a part of a SPE strategy, i.e., there are forms of equilibria where some players strictly decrease their expected payoff with respect to the original game;
– even strongly efficient strategies do not always yield perfectly solved games, as there is no notion of *most fair* redistribution of the payoff vectors in the solution of the original game.

Thus, when time is of no value, even the option of making conditional offers is not sufficient to guarantee that fair and efficient outcomes are ever reached.

### 4.2 The case of valuable time

We will see here that when time is of value all the problems mentioned above can be at least partially solved. To impose value on time we introduce, for each player $i$, a *payoff discounting factor* $\delta_i \in (0,1)$ applied at every round of the PNG *associated to offers that are made* to his payoffs. These factors measure the players' impatience, i.e., how much they value time, and reduce the payoffs accordingly as time goes by. The general intuition in this case, which we will justify further, is that for the sake of time efficiency, in a SPE strategy profile:

1. If any player is ever going to make an offer, she would never make any earlier offer that gives her, if accepted, a lesser value of the resulting game.
2. If any player is ever going to accept a given offer (or any other offer, at least as good for her) she should do it the first time when she receives such offer.

To facilitate the compatison we bargaining games we however restrict players' possible strategies, imposing some additional constraints:

– every game associated with a history of a PNG does not have outcomes *in the solution* (but, possibly elsewhere) that assign negative utility to players. Notice, that we do allow payoff vectors consisting of negative reals to be present in the game matrix, only we do not allow such vectors to be associated to outcomes in the solution. This constraint that we impose has several practical consequences:
  - players' expected payoffs *decrease* in time, i.e.,the discounting factor $\delta$ has always a negative effect on the expected payoff.
  - players can make offers that redistribute the payoff vectors associated with outcomes in the solution, leaving some nonnegative amount to each player and some strictly positive amount to some.
– the expected payoff of each player at any disagreement history can be assumed 0.

We will use the following notational conventions:

– $(x,t)$ denotes the payoff vector $x$ at time $t$, where each component $x_i$ is discounted by $\delta_i^t$; $(x,t)_i$ denotes the payoff of player $i$ in vector $x$ at time $t$.

– $\mathcal{G}_X$ will denote the set of all possible redistributions of payoffs of outcomes in a NFG $\mathcal{G}$ that assign nonnegative payoffs to all players. This set is compact, but generally not connected, as in the bargaining games of [4]. However, it is a finite union of compact and connected sets, and that will suffice to generalize the results from [4] that we need.

The following properties of every 2-person PNG with valuable time starting from a given NFG $\mathcal{G}$ are the four fundamental assumptions of Osborne and Rubinstein's bargaining model [8, p.122].

1. For each $x, y \in \mathcal{G}_X$ such that $x \neq y$, if $(x, 0)_i = (y, 0)_i$ then $(x, 0)_{-i} \neq (y, 0)_{-i}$. This holds because the set $\mathcal{G}_X$ is made by payoff vectors and subtracting some payoff to a player means adding it to the other.
2. $(b^i, 1)_{-i} = (b^i, 0)_{-i} = (D)_{-i}$, where $b^i$ is the highest payoff that $i$ obtains in $\mathcal{G}_X$ and $(D)_{-i}$ the payoff for $-i$ in any disagreement history. As $b^i$ is the best agreement for player $i$ it is also the worst one for player $-i$.
3. If $x$ is Pareto optimal amongst the payoff vectors in $\mathcal{G}_X$ then, by definition of $\mathcal{G}_X$, there is no $y$ with $(x, 0)_i \geq (y, 0)_i$ for each $i \in N$. Moreover, $x$ is a redistribution of a maximal outcome in $\mathcal{G}$.
4. There is a unique pair $(x^*, y^*)$ with $x^*, y^* \in \mathcal{G}_X$ such that $(x^*, 1)_A = (y^*, 0)_A$ and $(y^*, 1)_B = (x^*, 0)_B$ and both $x^*, y^*$ are Pareto optimal amongst the payoff vectors in $\mathcal{G}_X$.

The first three statements above are quite straightforward. To see the last one, let $x^* = (x^*_A, x^*_B)$ and $y^* = (y^*_A, y^*_B)$ and let the sum of the payoffs in any maximal outcome in $\mathcal{G}$ be $d$. Then $(x^*_A, x^*_B, y^*_A, y^*_B)$ is the unique solution of the following, clearly consistent and determined system of equations:
$y_A = \delta_A x_A$, $x_B = \delta_B y_B$, $x_A + x_B = d$, $y_A + y_B = d$.

The solution (see also [8]) is:

$$x_A = d\frac{1-\delta_B}{1-\delta_A\delta_B};\ y_A = \delta_A d\frac{1-\delta_B}{1-\delta_A\delta_B}$$

$$x_B = \delta_B d\frac{1-\delta_A}{1-\delta_A\delta_B};\ y_B = d\frac{1-\delta_A}{1-\delta_A\delta_B}.$$

**Relation with bargaining games** In the remaining part of the section we will explicitly view preplay negotiation as a bargaining process on how to play the starting normal form game. Using our observations and assumptions, we can adapt the results from [8] to show that when time is valuable not only all equilibria consisting of stationary acceptance strategies attain efficiency but they also do it by redistributing the payoff vector in relation to players' impatience. Stationary acceptance strategies will be needed to focus only on the maximal connected subspace of the set $\mathcal{G}_X$. To say it with a slogan, while in [8] efficiency and fairness can be obtained in scenarios that resemble the division of a cake, in our setting we prove similar results for a set of cakes, of possibly different size. We extend the efficiency and fairness results obtained in [8] for bargaining games of the type of 'division of a cake' to similar results for somewhat more general bargaining games of the type where players have to choose a cake from a set of cakes, of possibly different sizes and divide it. Our claim, in a nutshell, is that, when players employ stationary acceptance strategies, they immediately choose the largest cake and then bargain on how to divide it.

First, recall that in our framework time passes as new proposals are made. So, from a technical point if the PNG start with a game that is already perfectly solved, the player moving first will not be punished by passing immediately.

Then, without restriction of the generality of our analysis, we can assume a *unique* discounting factor for both players. Indeed, the discount factor of e.g., player $A$ can be made equal to that of $B$ while preserving the relative preferences of $A$ on the set of outcomes by suitably re-scaling the payoffs of $A$ in the input NFG, and therefore the expected value for $A$ of that game; for technical details see [8, p.119] following an idea of Fishburn and Rubinstein quoted there.

Now we are ready to state the main result for this case:

**Theorem 1.** *Let $(x^*, y^*)$ be the unique pair of payoff vectors defined above. Then, in a PNG with valuable time starting from a NFG $\mathcal{G}$ with a unique discounting factor $\delta$ for both players, the strategy of player $A$ in every subgame perfect equilibrium consisting of stationary acceptance strategies satisfies the following (to obtain the strategy for $B$ simply swap $x^*$ and $y^*$):*

- *if $A$ is the first player to move she 'proposes' outcome $x^*$, i.e., makes a conditional offer that, if accepted, would update the game into one with is dominant strategy equilibrium yielding the Pareto maximal outcome $x^*$ as payoff vector;*
- *when $A$ has a pending offer $y'$, she accepts it if and only if the payoff she gets in $y'$ is at least as much as in $y^*$;*
- *when $A$ can pass, she passes if and only if the expected value associated to the proposed game $y'_A$ is at least as much as $y^*_A$; otherwise she proposes $x^*$.*

*Proof idea* To prove the claim we use a variant of the argument given in [8] for bargaining games, summarized as follows. [6] We first show [Step 1] that the best SPE payoff for player $A$ in any subgame $\mathcal{G}'_A$ starting with her proposal and where $\mathcal{G}'$ is the currently accepted game — let us denote it by $M_A(G'_A)$ — yields the same utility as the worst one — $m_A(\mathcal{G}'_A)$ — which, in turn, is the payoff of $A$ at $x^*$. The argument for $B$ is symmetric. Then we show [Step 2] that in every SPE the initial proposal is $x^*$, which is immediately accepted by the other player, followed by each player passing. Finally, we show [Step 3] that the acceptance and the passing conditions are shared by every SPE strategy profile.

To summarize, when time is valuable and players' value of time (impatience) is measured by a vector of discount factors $\delta$ the SPEs following stationary acceptance strategies are essentially unique, efficient and redistribute a maximal payoff vector in a *fair* way, depending on players' impatience, viz. in each SPE

[6] A full proof can be found in [4].

play, players agree as soon as possible and divide (almost) evenly any of the maximal outcomes in the game. Thus, introducing value of time solves both problems of efficiency and fairness at once.

If a PNG starts with a game that is uniformly solvable but not optimally solvable (i.e. there is space for improvement), the player moving first can improve on the expected outcome of the initial game only on the condition to ensure to her opponent at least his expected value in the initial game.

## 5 Related work

The present study has a rich pre-history, related to earlier work on bargaining and various pre-play negotiation procedures, notable examples of which are [2, 5, 7, 9, 10]; see [3] for a broad discussion. Here we only mention the most relevant recent work. To our knowledge, Jackson and Wilkie [6] are the first to have studied arbitrary transfers from a player to a player in a normal form game. Their framework bears essentiial similarities with ours, as it studies a two-stage transformation on a normal form game where players announce transfers of payments between each other on the initial normal form game and then play the updated game. Yet, there are substantial differences with our framework, the most important ones being that in [6] players make positive side payments to other players *conditional on the entire outcome of the game*, by announcing their offers simultaneously, and therefore time and its value do not play a role in the negotiations phase. Ellingsen and Paltseva [1] generalize [6] in several ways. In their framework each player specifies a (possibly negative) transfer to the other players for each (possibly mixed) strategy profile $\sigma$ and, at the same time, specifies a signing decision for each contract of the other players. The authors show that their more general contracting game always has efficient equilibria. In particular, they show that all efficient outcomes guaranteeing to each player at least as much as the worst Nash-equilibrium payoff in the original game can be attained in some equilibrium. The message conveyed by this stream of works is that efficiency can be reached if the structure of players' offers is complex enough. Instead, we focus on the effects that additional factors in the preplay negotiation game, such as valuable time, withdrawals, and opting out have on attaining outcomes with desirable properties, such as efficiency and fairness.

## 6 Conclusion

We have analyzed the role and effect of conditional offers in preplay negotiation games under various assumptions concerning players' rationality, their value of time, possibility of revoking previous commitments and opting out from the negotiations. We have shown that when time is of no value efficiency can be attained, provided some coherence in players' behaviour is assumed (Prop. 1). Yet, there are cases in which inefficiency can occur in equilibrium outcomes (comment to Prop. 1). When time is not valuable, the outcomes reached in the PNG can be extremely unfair, even if efficient (Prop. 2). However, when time is valuable, under some natural assumptions players reach efficient and relatively fair outcomes (Theorem 1). The latter result draws an explicit connection between our preplay negotiation games and bargaining games studied in [8].

Some related issues remain still open, in particular the possibility of ruling out the existence of non-stationary acceptance strategies in all SPE profiles of PNGs with valuable time. As for the potential future developments, the framework can be extended in various ways, as also discussed in [3]. In particular, we conjecture that when players can only make *unconditional* offers, not contingent on matching offers by the recipients, the analysis of the preplay negotiations phase changes radically and in a way becomes even more challenging. Its complete analysis is the subject of an ongoing work. Lastly, the analysis of preplay negotiation phase in games with three and more players is substantially more complicated and is one of the main future directions in this research.

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