Oligopolistic Competition in Heterogeneous Access Networks under Asymmetries of Cost and Capacity

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***Abstract*—With the rapid development of broadband wireless access technologies, multiple wireless service provider (WSPs) operating on various wireless access technologies may coexist in one service area to compete for users, leading to a highly competitive environment for the WSPs. In such a competitive heterogeneous wireless access market, different wireless access technologies used by different WSPs have different bandwidth capacities with various costs. In this paper, we set up a non-cooperative game model to study how the cost asymmetry and capacity asymmetry among WSPs affect the competition in this market. We first model such a competitive heterogeneous wireless access market as an oligopolistic price competition, in which multiple WSPs compete for a group of price- and delay-sensitive users through their prices, under cost and capacity asymmetries, to maximize their own profits. Then, we develop an analytical framework to investigate whether or not a Nash equilibrium can be achieved among the WSPs in the presence of the cost and capacity asymmetries, how the asymmetries of cost and capacity affect their equilibrium prices and what impact a new WSP with a cost and capacity advantage entering the market has on the equilibrium achieved among existing WSPs.**

***Index Terms*—Heterogeneous access network, pricing, game theory, oligopolistic competition.**

I. Introduction

With the fast evolution and proliferation of broadband wireless access technologies, i.e., wireless local area networks (WLANs) or WiFi, Worldwide Interoperability for Microwave Access(WiMAX), Wireless mesh networks (WMNS), Wide Code Division Multiple Access (WCDMA), 3G/UMTS, Beyond 3G (B3G), and so on, it has been well-recognized that in the future these wireless access technologies will be integrated to form a heterogeneous wireless access network to deliver high speed wireless data services to users [1]. For example, IP-based wireless broadband technology such as IEEE WiMAX can be integrated with 3G mobile networks or 802.11-based WLANs to provide broadband connectivity to mobile users. At present, the most common example of heterogeneous wireless access networks is the accessibility of WiFi hotspots on top of 3G cellular services [2]. Such a Heterogeneous wireless access network, where different wireless access networks operated by different wireless service providers (WSPs) coexist in the same coverage area to provide wireless service for the same group of users, will lead to a highly competitive environment for WSPs while improving user satisfaction by allowing users to seamlessly connect to the access network that offers the best possible quality of service (QoS).

With the emergence of cognitive radio technology, it can be conceivable that, in the near future, users equipped with multi-mode cognitive radio devices will have the capability to connect and switch to different networks across different spectrum bands on a more short term basis [2]. Thus, users will have increasing freedom to choose among several available WSPs who provide wireless services instead of being contractually tied to a single WSP. Dynamical selection of the most suitable WSP for their application or service requirements will become the norm [3]. For example, a user wishing to access the Internet may find himself in a zone covered by several WSPs who provide Internet access service using different wireless access technologies, e.g., WiFi, WiMAX, 3G, and so on, with different data rates at different prices. In such a heterogeneous wireless access network, WSPs with asymmetric costs and capacities have to compete for users with each other through their offered prices and QoS while maximizing their individual profits. On the other hand, from a user's perspective, he wants to achieve the highest satisfaction by choosing a WSP offering the best trade-off between QoS and price. In other words, users are relatively elastic in the demand sense since they may trade QoS for price. Therefore, in attempting to maximize their profits by attracting users who are price- and QoS-sensitive, WSPs need to price their services taking into account a wide range of factors including service cost; network capabilities and condition (e.g., network capacity, delay, response time, bit error rate and packet loss); user preferences (e.g., willingness to pay); and potential competition from other WSPs.

Different wireless access technologies have different bandwidth capacities with various costs. For example, in CDMA2000, one of major standards for 3G cellular networks operating in licensed spectrum with higher deployment costs, the nominal 1.25MHz bandwidth can achieve a data rate up to around 2Mbps for indoor office environments. On the other hand, WLANs usually operating at unlicensed frequency bands provide data services with lower cost. Meanwhile, the large bandwidth available for WLANs makes it possible to achieve higher data rates. For example, a IEEE 802.11b WLAN can achieve a bandwidth of more than 20 MHz, which offers a data rate up to 11Mbps [4]. With the introduction of new technologies, network capacity increases while network cost to deliver data traffic falls as less equipments and fewer cell sites are required due to improved data throughput, reducing operational expenses and capital investment. For instance, cost per megabyte falls dramatically from approximately $0.42

for GPRS to less than \$0.06 using CDMA2000. Meanwhile, CDMA2000 has an average throughput per cell sector of 1.1Mpbs compared to 80kpbs with GPRS [5]. Apparently, cost and capacity are key differentiators among WSPs. The higher capacity and lower network cost of certain technologies will provide an WSP with a substantial competitive advantage when competing with other WSPs.

In general, users are more likely to choose a WSP with better network service and lower price, but more users connecting to the same WSP with a certain capacity limit will result in degradation of the corresponding network. This feature is known as a negative externality, which could deter new users from connecting to the WSP or drive users connecting the WSP to switch to other WSPs available. A well-designed dynamic pricing policy, in which price is used as a signal to induce users to utilize the network in a desirable way, allows a WSP to capture the changes of user behavior and network status, and to adjust its prices based on these dynamic changes to achieve profit maximization and better network utilization. Price-sensitive users then increase or decrease their demand as prices are dynamically changed. With a proper pricing scheme, a WSP and its users are allowed to act individually to express the values that they are willing to charge or pay, and to reach an equilibrium where their individual payoffs, which are usually expressed through utility functions, are maximized simultaneously.

In addition, in the presence of other competing WSPs, the WSPs' price setting strategies must be affected by competition among WSPs. Each WSP's price has to be dependent on other WSPs prices and the condition of their networks, which affect users behavior because utility maximizing users always choose the WSP offering the best combination of price and QoS. Such a competitive market can be modeled as an oligopoly, where all WSPs are self-interested in a sense that their actions or reactions in response to others' actions only focus on maximizing their own profits while the decision of each WSP is influenced by the actions of other WSPs, which impact the decisions of users in choosing a WSP. Since the prime concern of a WSP is cost recovery while its network status is mainly determined by its capacity, the asymmetry in cost and capacity among multiple WSPs will play an important role in the competitive interactions.

In this paper, we focus on an oligopolistic price competition, in which multiple WSPs compete for a group of price- and QoS-sensitive users through their prices in a given geographic market, under cost and capacity asymmetries. We are trying to address the following questions: Can an equilibrium be achieved among the WSPs in the presence of cost and capacity asymmetry? How does the asymmetry in cost and capacity affect equilibrium prices, under which no WSP can unilaterally improve its profit by changing its price? What impact does a new WSP with higher capacity and lower cost entering the market have on the equilibrium among existing WSPs?

Pricing, especially dynamic pricing, and price competition have received increasing attention in both the networking and economic research communities. Pricing not only has a vital impact on service provider profits but also can be used as a mechanism for congestion control and traffic management [6]. Game theory combined with market theory and price theory has been viewed as a powerful tool for modeling the economic activities of telecommunication networks. Game theory provides a sound mathematical framework to model the strategic interactions among self-interested players who must make decisions that potentially affect other players' interests. In particular, non-cooperative game theory is primarily used to analyze situations in which players' payoffs depend on the actions of other players. In this paper, we extend the non-cooperative game framework proposed in our previous work [7] to analyze the interactions among the competing WSPs and price- and QoS-sensitive users in the presence of the cost and capacity asymmetry among WSPs, characterize the Nash equilibrium for this oligopolistic competition with regard to pricing strategies of the WSPs, and determine corresponding equilibrium prices.

Note that in this paper our focus is the price setting problem among multiple WSPs instead of price discrimination among users. Thus we simply assume that the users are homogeneous in utility functions and willingness to pay. In addition, we further assume that users can dynamically choose a WSP based on the WSPs' offered prices and expected QoS, and users are associated with a WSP on a per-service or per-session basis. A similar scenario has been used in [2], where WSPs can change their price asynchronously while end-users dynamically connect to any WSP and can leave any time they want in an overlaid heterogeneous access network. Furthermore, QoS may take many different forms, such as response time, bit-error rate, packet delay, and so on. In this paper, for the purpose of facilitating analysis, we consider packet delay as a measure of QoS to determine user utility.

The paper is organized as follows. In Section II, we describe the system model, in which the interactions among the WSPs and users are modeled as a two-stage repeated game, and present the mathematical model to investigate the Nash equilibrium and corresponding equilibrium prices in this game. Numerical examples and associated simulations are provided to study how the asymmetries of cost and capacity impact on Nash equilibria in this game in Section III. Section IV finally concludes the paper.

II. Model Description

We consider a set $\mathbb{I} = \{1, 2, \ldots, I\}$ of WSPs operating their own wireless networks in a particular service area to provide wireless services to N potential users. All WSPs are able to change their prices based on the congestion status of their networks. Given the prices and QoS offered by the WSPs, a price- and QoS-sensitive user can choose to connect to one of the WSPs for his service requirements or opt out of all WSPs, and the users who are connecting to one WSP are able to switch to another WSP anytime they want. This oligopolistic price competition is modeled as a two-stage simultaneous-play game: in Stage 1, all WSPs simultaneously and independently set their prices to maximize their profits. Then, in Stage 2, given the prices quoted by the WSPs and the QoS offered by the WSPs, the users decide whether purchase the service, and if so, from which WSP. Note that the two stages are solved

sequentially and repeatedly. After Stage 2, the game moves back to Stage 1 where the WSPs adjust their optimal prices based on the decisions of the users. Consequently, the users reconsider their WSP selection decision based on the adjusted prices. The process continues until the game reaches a steady state or Nash Equilibrium, if one exists. In fact, Stage 2 can also be viewed as a Stackelberg game, with the WSPs as the leaders and potential users as the followers.

In this dynamic pricing game, each WSP only knows its own quoting prices, its own cost and the users' responses to its quoting price in real time. No WSP knows its rivals' prices, costs and the users' response to its rivals' prices. It is a realistic assumption because in practice a WSP is not allowed to disclose its private information to its competitors. Therefore, the users' reaction to the WSPs' offered prices and expected delay is the determining factor in this pricing game.

Each WSP's network can be viewed as a system that can only serve a finite population of potential users. On joining WSP$_i$ ($i \in \mathbb{I}$), a user receives gross utility U_i, which is partly depend on the level of congestion of the WSP's network. Generally speaking, as more users connect the same WSP, they will experience longer delays (or response times), which negatively affects the gross utility they derive from using the network. Therefore, the larger the number of users subscribe to a WSP, the lower the gross utility the users can obtain. As defined in [8], the gross utility can be defined as:

$$U(ED) = ED^{-\theta}, \tag{1}$$

where ED is mean packet delay and $\theta > 0$. Here $U(ED)$ is assumed to be monotonically decreasing with its argument ED. To simplify the calculation, we choose $\theta = 1$. According to utility theory, a user will decide to use a service if and only if its net utility $\mathbb{U}(ED)$, which is gross utility minus price, is non-negative. If the net utilities that the user receives from all WSPs are negative, the user will choose to join none of them. If the net utilities that a user receives from some of them are positive, the user will choose the WSP from which he can derive the highest net utility. In other words, the user's strategy of network-selection or WSP-selection in this heterogeneous access network strongly depends on the price difference and the QoS performance difference among the WSPs[1].

More specifically, let $\mathbb{U}_i$ denote the user's net utility with WSP$_i$, then mathematically,

- if $\mathbb{U}_i < 0$ for all $i \in \mathbb{I}$, the user will opt out of all the WSPs;
- if $\mathbb{U}_i > 0$ and $\mathbb{U}_i > \mathbb{U}_{-i}$, where the subscript $-i$ represents all the WSPs belonging to $\mathbb{I}$ except i itself, the user will subscribe to WSP$_i$;
- only if $\mathbb{U}_1 = \cdots = \mathbb{U}_i = \cdots = \mathbb{U}_I > 0$, the user is indifferent among these WSPs.

Suppose that the gross utility that the user will derive from transmitting a packet through WSP$_i$ is $U_i(ED_i)$, where ED_i is mean packet delays for WSP$_i$, and WSP$_i$ set price p_i per packet transmission. In the situation in which the indifference relation

$$\begin{aligned} U_0(ED_1) - p_1^* &= \cdots = U_i(ED_i) - p_i^* \\ &= \cdots = U_I(ED_I) - p_I^* > 0 \end{aligned} \tag{2}$$

holds, a newly arriving user randomly selects WSP$_i$ with probability $1/i$, and there is no incentive for a user who has already connected to one WSP to unilaterally change his current strategy because he derives no benefit from switching to any other WSP. Therefore, the set of prices $(p_1^*, \cdots, p_i^*, \cdots, p_I^*)$, which maximize profits of the corresponding WSPs simultaneously, is a Nash equilibrium. In equilibrium, the distribution of users among the WSPs is stable because switching to any other WSP will not bring a higher net utility for a user. Meanwhile, in equilibrium, all WSPs also have no unilateral incentive to change their current profit maximizing prices, because changing price could lead to an increase or decrease in the users' net utility and create an incentive for the users to change their current strategies.

A. User Utility

All the N potential users will generate information packets after they connect to a WSP. The users who choose the same WSP share an M/M/1 queuing system where the information packets arrival process and the service time distributions, respectively, are Poisson and Exponential. Each user generates information packets according to a Poisson process with mean rate λ. Then the potentially total mean arrival rate of packets in the whole network is given by λN. Note that λN can be seen as an arrival rate when all potential users send out all their requests without considering price or QoS. The service times of individual packet for WSP$_i$ are i.i.d. exponentially distributed with mean μ_i^{-1}. In other words, WSP$_i$ has a capacity of serving μ_i packets per second. Then, the mean packet delay for WSP$_i$ is:

$$ED = \frac{1}{\mu_i - \lambda N E_i(p_i)}, \tag{3}$$

where $E(p_i)$ is the expectation of the acceptance for a given price p_i. Note that (3) only holds provided that $\mu_i > \lambda N E(p_i)$.

Using a Pareto distribution of customer capacity to pay, Jagannathan *et al.* suggest a parameterized customer behavior model for customer's willingness-to-pay to a given price in [9], where the expectation of acceptance for a given price p_i is given by:

$$E_i(p_i) = \begin{cases} 1 - \frac{\alpha_i}{\alpha_i+\delta_i}(\frac{p_i}{b_i})^{\delta_i} & 0 \le p_i \le b_i; \\ \frac{\delta_i}{\alpha_i+\delta_i}(\frac{b_i}{p_i})^{\alpha_i} & p_i > b_i, \end{cases} \tag{4}$$

where shape α_i, scale b_i and user-willingness elasticity δ_i can be determined by WSP$_i$ based on its own observation. Since a WSP can observe the users' acceptance to its quoted price online, these parameters can be learned using an adaptive algorithm suggested by Jagannathan *et al.* in [9] from the observed users' response (or acceptance rate) for a given price. In fact, the process of learning these parameters is a dynamic process with an aim to adjust the quoted price in line with

[1]We assume each user has to calculate his net utility to choose a WSP. According to definition, the user's net utility is a function of the response time, ED, the only information that a user needs to know is the expected delays (or response times) of the packets generated by him when making his choice. It is practically possible for a WSP to inform each user of the response time of his service request.

the change of the users' response which is driven by their net utilities. Note that different WSPs should have different values of these parameters. We will show later in Section II-B that, in our proposed pricing scheme, a WSP only need to calculate one of the three parameters, b, instead of all the three parameters based on users' response when deciding its optimal prices. This is desirable in practice.

Then the user's net utility $\mathbb{U}_i$ with WSP$_i$ can be written as:

$$\mathbb{U}_i = U_i(ED_i) - p_i = 1/ED_i - p_i = \mu - \lambda NE_i(p_i) - p_i. \quad (5)$$

According to the analysis of Nash equilibrium above, a set of prices $(p_1^*, \cdots, p_i^*, \cdots, p_I^*)$ is in Nash equilibrium if and only if the following condition is satisfied:

$$\begin{aligned} \mu_1 - \lambda NE_1(p_1^*) - p_1^* &= \cdots = \mu_i - \lambda NE_i(p_i^*) - p_i^* \\ &= \cdots = \mu_I - \lambda NE_I(p_I^*) - p_I^*, \\ \text{subject to} \quad & \mu_i - \lambda NE_i(p_i^*) - p_i^* > 0 \quad \text{for} \quad i \in \mathbb{I}. \end{aligned} \quad (6)$$

B. Price Selection Strategy for WSPs

A WSP's utility is expressed by its profit. Since a WSP's prime concern is cost recovery, it is reasonable to assume that a WSP will set its price greater than or at least equal to its cost c_i. The profit of WSP$_i$ is defined as the product of the expected number of packets transmitted by the users who connect to it per second and the difference between the price per packet, p_i, and the cost per packet, c_i. Thus the profit function for WSPi per second, Π_i, for a given price p_i is given by:

$$\Pi_i = \lambda NE_i(p_i)(p_i - c_i)\mu_i \quad (7)$$

The objective of each WSP is to select a price that will maximize its profit. Therefore, a strategic equilibrium $(p_1^*, \cdots, p_i^*, \cdots, p_I^*)$ for the WSPs has to satisfies the following relations first:

$$\forall p_i > c_i : \Pi_i(p_i^*, p_{-i}) \geq \Pi_i(p_i, p_{-i}) \quad (8)$$

for all $i \in \mathbb{I}$.

Mathematically, the profit of WSPi is maximized for the first order condition $\frac{\partial \Pi_i}{\partial p_i} = 0$. We have proven in [7] that, among the three parameters, while both α_i and δ_i have slight impacts on the value of the optimal price, b_i is the only factor that affects the value of the optimal price significantly. For ease of analysis, we assume $\alpha_i = 10$ and $\delta_i = 2$. Then the corresponding optimal price is given by

$$p_{i_{opt}} = \begin{cases} \frac{1}{3}(c_i + \sqrt{c_i^2 + \frac{18}{5}b_i^2}) & c_i \leq p_i \leq b_i; \\ \frac{10}{9}c_i & p_i > b_i. \end{cases} \quad (9)$$

It is straightforward to show that, for all $b_i \geq \frac{10}{9}c_i$, $p_i = \frac{1}{3}(c_i + \sqrt{c_i^2 + \frac{18}{5}b_i^2})$ is the maximum. Only when $b_i < \frac{10}{9}c_i$, $p_i = \frac{10}{9}c_i$ is the maximum. Since our attention is focused on the use of dynamic pricing in this price competition game, we will only investigate the case of $b_i \geq \frac{10}{9}c_i$ and study the Nash equilibrium prices for this case. Note that in this pricing scheme b_i is a variable, which is determined by user demand. Therefore the computed $p_{i_{opt}}$ based on b_i is a responsive price that reflects the traffic load of WSP$_i$.

Therefore, the Nash equilibrium condition in (6) can be rewritten as:

$$\begin{aligned} \mu_1 - \lambda NE(p_{1_{opt}}^*) - p_{1_{opt}}^* &= \cdots = \mu_i - \lambda NE(p_{i_{opt}}^*) - p_{i_{opt}}^* \\ &= \cdots = \mu_{I_{opt}} - \lambda NE(p_{I_{opt}}^*) - p_{I_{opt}}^* \quad (10.1) \\ \text{where} \quad E_i(p_{i_{opt}}^*) &= 1 - \frac{5}{6}(\frac{p_{i_{opt}}^*}{b_i})^2 \quad \forall i \in \mathbb{I}, \quad (10.2) \\ \text{and} \quad p_{i_{opt}}^* &= \frac{1}{3}(c_i + \sqrt{c_i^2 + \frac{18}{5}b_i^2}) \ \forall i \in \mathbb{I}. \quad (10.3) \\ \text{subject to} \quad & \sum_{i=1}^{I} E(p_{i_{opt}}^*) \leq 1 \end{aligned} \quad (10)$$

With this pricing setting, a new arrival user should randomly subscribe to a particular WSP$_i$ and the users who already connected to a WSP have no incentive to switch to a different WSP. Meanwhile, no WSP has any motivation to deviate its current profit-maximizing price. This is called Nash equilibrium and the corresponding price set $(p_{1_{opt}}^*, \cdots, p_{i_{opt}}^*, \cdots, p_{I_{opt}}^*)$ is called equilibrium prices.

III. Numerical Examples

Given the cost and capacity asymmetries among the WSPs, we are led to some questions: does there always exist a Nash equilibrium under the asymmetric cost and capacity in this non-cooperative pricing game? How do the cost and capacity differences among the WSPs affect equilibrium prices when there exists a Nash equilibrium? What will happen when a new WSP joins the game? In this section, we conduct some simulation experiments to address these questions.

A. Two-WSP Scenario

We first consider a heterogeneous wireless system with two WSPs competing with each other. We performed the following simulations where two WSPs, WSP$_1$ and WSP$_2$, provide wireless services in a market with $N = 50$ potential users and each user generates packets according to a Poisson process with rate $\lambda = 10$packet/sec. In order to study the impact of the asymmetries of cost and capacity on Nash equilibrium, we assume WSP$_1$'s cost and capacity are fixed while WSP$_2$'s capacity is variable. For the sake of simulation simplicity, WSP$_2$'s cost is fixed but different from that of WSP$_1$. Suppose the costs of WSP$_1$ per packet is $c_1 = 7$units and the service time per packet are i.i.d. exponentially distributed with mean 300^{-1}sec for WSP$_1$. Meanwhile, we assume WSP$_2$'s cost per packet is $c_2 = 5$units while its mean service time per packet varies from 200^{-1}sec to 450^{-1}sec.

In our simulations, a numerical search procedure is employed to obtain the equilibrium prices. We first compute the optimal prices $p_{i_{opt}}$ $(i = 1, 2)$ for both WSPs using (10.3). Note that in (10.3) variable b_i used to calculate the optimal prices represents the users' responses to the WSP$_i$ quoted prices. For the sake of convenience, in the rest of section, $p_{i_{opt}}$ and p_i are interchangeable. Let $\{p_1^1, p_1^2, \cdots, p_1^n\}$ and $\{p_1^1, p_1^2, \cdots, p_1^n\}$ be optimal price strategy forms of WSP$_1$ and WSP$_2$ respectively. We then calculate the expected

users' net utilities with both WSPs given that both WSPs set their prices according to their optimal price strategy forms respectively. With an exhaustive search, we identify the equilibrium net utilities, and the corresponding price sets $\{(p_1^{1*}, p_2^{1*}), (p_1^{2*}, p_2^{2*}), \ldots, (p_1^{n*}, p_2^{n*})\}$, which are represented as (p_1^*, p_2^*) in the rest of the section, are the equilibrium prices.

Fig. 1 plots the possible equilibrium prices under asymmetric cost and capacity. For ease of explanation, we use the equilibrium prices obtained when both WSPs have the same capacity ($\mu_1 = \mu_2 = 300^{-1}$sec) as a reference. As can be observed, when WSP_2 has a lower capacity, i.e., $\mu_2 = 200^{-1}$sec, no Nash equilibrium exists if WSP_1 choose a price lower than 8.867. This is because WSP_1 with a higher capacity attracts more users with its better QoS, namely shorter packet delay, as shown in Fig. 2. A lower price set by WSP_1 could lead to a higher user net utility that cannot be brought by WSP_2 no matter how much it lowers its price. As a result, the two WSPs can not reach equilibrium and WSP_1 could dominate over WSP_2 with its lower price. On the other hand, in equilibrium, compared to the reference case, WSP_2's equilibrium prices are much lower with the maximum price it could charge being almost only 50% more than its cost. Whereas, in the reference case, the maximum price WSP_2 could charge reaches more than twice its cost. Meanwhile, the maximum that WSP_1's price could reach is more than three times its cost while WSP_1' maximum price in the reference case is less than twice its cost.

Note that the maxima that both WSPs' prices could reach are the prices obtained under saturation conditions where $E(p_1^*) + E(p_2^*) \approx 1$. Here, acceptance rate $E(p_i^*)$ $(i = 1, 2)$ can be interpreted as the ratio of the number of users who are connecting to WSP_i to the number of total potential users. In other words, $E(p_i^*)$ reflects WSP_i's market share when they are in equilibrium. Fig. 2 demonstrates how user demand is split between the two WSPs when the WSPs and users reach equilibria. It shows that, under a saturation condition, in which almost all potential users have information packets for transmission, market shares taken up by WSP_1 and WSP_2 are 58.38% and 41.51% respectively. Fig.3 illustrates the expected revenues associated with the equilibrium prices plotted in Fig. 1. As can be seen, despite its higher cost, WSP_1 still could make higher profits due to its capacity advantage. Comparing the profit curves with $\mu_2 = 200^{-1}$sec, $\mu_2 = 250^{-1}$sec and $\mu_2 = 300^{-1}$sec, it is obvious that the capacity asymmetry has a significant impact on both WSPs' profits.

In contrast, when the lower cost WSP_2 has a higher capacity, i.e., $\mu_2 = 400^{-1}$sec, competing with WSP_1, no Nash equilibrium can be reached if WSP_2 sets its price lower than 6.427 for the same reason explained above. Similarly, compared to the reference case, in equilibrium, WSP_2 takes up higher market shares (shown in Fig. 2) at higher prices (shown in Fig. 1), which in turn results in higher profits (shown in Fig. 3). Clearly, the higher capacity and lower cost create a significant competitive advantage for WSP_2.

Furthermore, we plot the aggregated expected profits vs. the total acceptance rates, which represent the ratios of the total number of user who are connecting to a WSP for their information packet transmission to the total potential number of users. It shows that, given a total acceptance rate, the capacity asymmetry leads to higher overall profits (as well as revenues) compared to the capacity symmetry case, except for the case where $\mu_2 = 350^{-1}$sec. The explanation for this observation is as follows: in equilibrium, the capacity asymmetry between the two WSPs allows the WSP with a higher capacity to gain more market share, which results in a higher utilization of its network. Accordingly, the WSP sets a relatively high price as a best response. As a result, the WSP ends up with a higher profit, which leads to an increase in the aggregated profit.

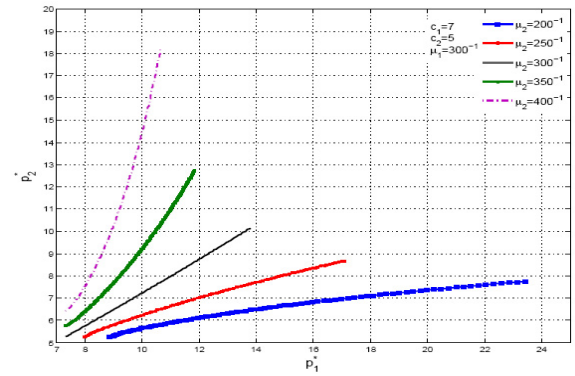

Fig. 1. Equilibrium prices under asymmetric capacity and cost in two-WSP scenario.

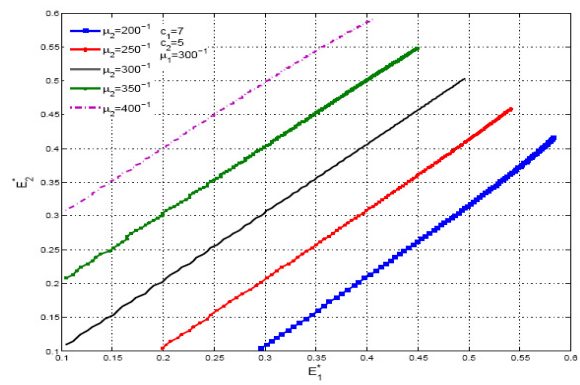

Fig. 2. Equilibrium acceptance rates under asymmetric capacity and cost.

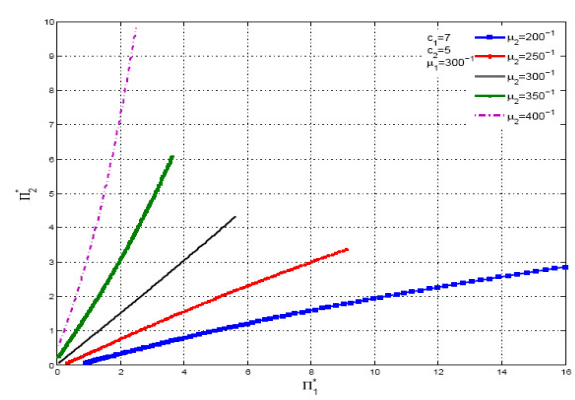

Fig. 3. Expected Profits associated with equilibrium prices in Fig. 1.

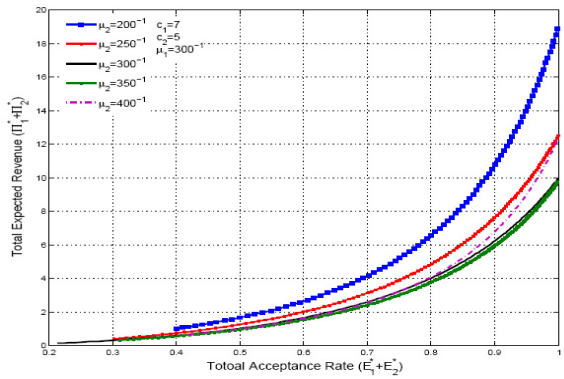

Fig. 4. Equilibrium acceptance rates under asymmetric capacity and cost in two-WSP scenario.

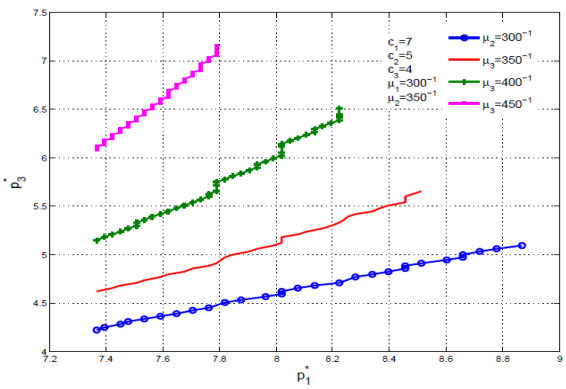

Fig. 5. Equilibrium prices under asymmetric capacity and cost in three-WSP scenario.

B. Three-WSP Scenario

To study the impact of an entry of a new WSP who has a cost and capacity advantage on the existing WSPs, we then performed a simulation where a new WSP, WSP_3, with a lower cost and higher capacity joins two existing WSPs: WSP_1 whose cost and mean service time per packet are $c1 = 7$units and 300^{-1}sec respectively and WSP_2 whose cost and mean service time per packet are $c1 = 5$units and 350^{-1}sec respectively. Again, we assume WSP_3's cost per packet is fixed with $c_3 = 4$units while its mean service time per packet varies from 300^{-1}sec to 400^{-1}sec. Using the same approach presented in the two-WSP scenario above, we obtain the equilibrium prices (p_1^*, p_2^*, p_3^*), which are plotted in Fig. 5 (p_1^*, p_3^*) and Fig. 6 (p_1^*, p_2^*). As illustrated in Fig. 5, in equilibrium, WSP_3's prices rise with an increase in its capacity while the maximum equilibrium prices that WSP_1 could charge drop when WSP_3 has a higher capacity. At the same time, comparing (p_1^*, p_2^*) between the three-WSP and two-WSP scenario, Fig. 6 shows, in the three-WSP case, equilibrium prices of WSP_2, who has a capacity and cost advantage over WSP_1, are higher than those obtained in the latter case. In addition, Fig. 6 also demonstrates how the entry of the new competitor drives a dramatic drop in the maximum equilibrium prices that both WSP_1 and WSP_2 could charge.

Fig. 7 and Fig. 8 plot all WSPs' expected profits associated with the equilibrium prices plotted in Fig. 5 and Fig. 6.

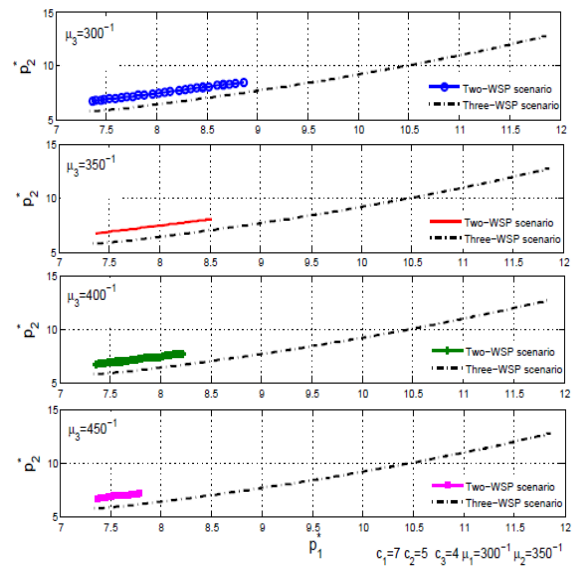

Fig. 6. Equilibrium prices under asymmetric capacity and cost in three-WSP scenario.

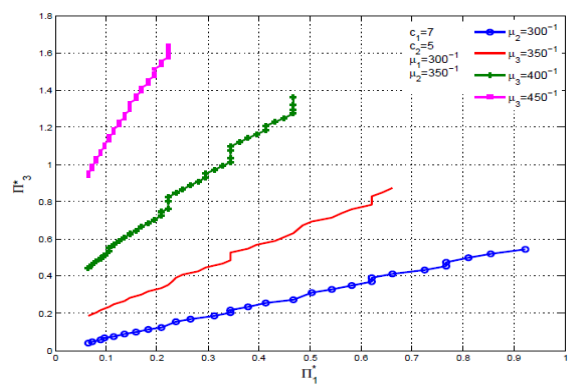

Fig. 7. Expected Profits associated with equilibrium prices in Fig. 5.

Apparently, compared to the two-WSP scenario, WSP_1 and WSP_2 gain much less profits in the three-WSP scenario, because the new WSP takes some market share from them.

In addition, Fig. 9, in which the aggregated profits of all WSPs are plotted, illustrates that the overall profits are relatively stable with respect to the capacity asymmetry compared to those in the two-WSP scenario as shown in Fig. 4. Comparing Fig. 9 with Fig. 4, we can observe that the overall profits (as well as revenue) in the three-WSP scenario is much lower than those in the two-WSP scenario. This indicates that the users pay dramatically less for their transmission due to the stronger competition driven by the new entrant. This has significant implications for users as they economically benefit from stronger competition resulting from a new entrant.

IV. Conclusions

In this paper, we study the impact of the cost asymmetry and capacity asymmetry among WSPs on a competitive heterogeneous wireless access market. We first model such a competitive heterogeneous wireless access market as an oligopolistic

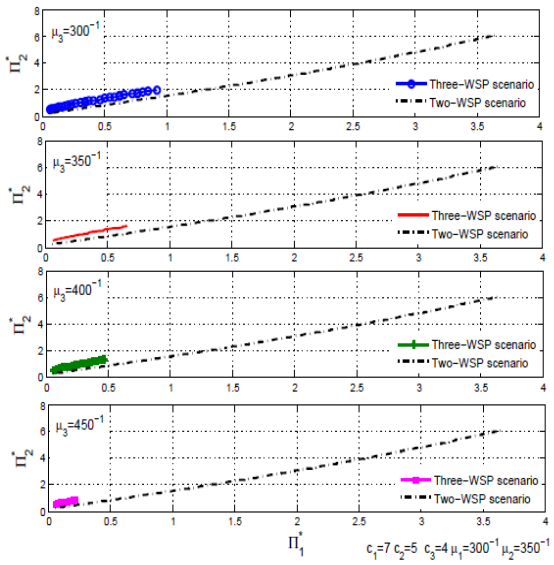

Fig. 8. Equilibrium prices under asymmetric capacity and cost in three-WSP scenario.

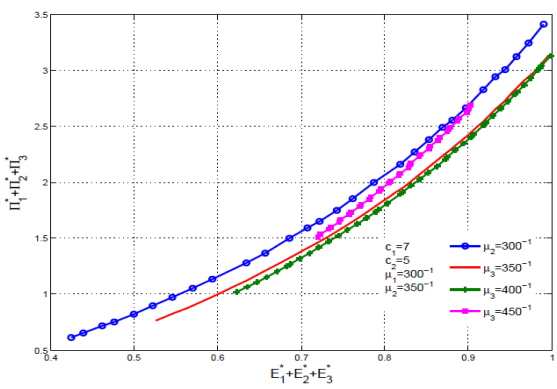

Fig. 9. Equilibrium acceptance rates under asymmetric capacity and cost in two-WSP scenario.

price competition, in which multiple WSPs compete for a group of price- and delay-sensitive users through their prices to maximize their own profit under cost and capacity asymmetries. Then, we formulate this oligopolistic price competition as a non-cooperative game model and develop an analytical framework to investigate the Nash equilibrium and corresponding equilibrium prices in this game. Through numerical examples, we study the role that the asymmetries of cost and capacity play in this game, in terms of the impacts on Nash equilibria, by addressing the following questions: whether an Nash equilibrium can be achieved among the WSPs in the presence of the cost and capacity asymmetries? how do the cost and capacity asymmetries affect equilibrium prices? what impact does a new WSP with a cost and capacity advantage entering the market has on the equilibrium achieved among existing WSPs. Our results show that, under the assumption that all WSPs adopt the responsive pricing scheme described in Section II, while higher capacity and lower cost create a significant competitive advantage for a WSP, which leads to a higher profit for it, Nash equilibria can still be achieved between the WSP with the competitive advantage and other WSPs due to users' price- and delay- sensitivity when demand is not too low. In addition, we further discuss the impact of a new entrant with a cost and capacity advantage on the existing WSPs' equilibrium prices, market shares, profits as well as the aggregated profit of all WSPs. Due to the stronger competition driven by the new entrant, the maximum prices that the existing WSPs could charge decrease, which will economically benefit the users.

References

[1] R. Beaubrun, "Integration of Heterogeneous Wireless Access Networks," in *Heterogeneous Wireless Access Networks: Architectures and Protocols*, E. Hossain, Ed. Springer, 2008, pp. 1–18.

[2] S. Sengupta, S. Anand, M. Chatterjee, and R. Chandramouli, "Dynamic Pricing for Service Provisioning and Network Selection in Heterogeneous Networks," *Physical Communication*, vol. 2, no. 1-2, pp. 138–150, March-June 2009.

[3] T. Kanter, "Going Wireless: Enabling an Adaptive and Extensible Environment," *ACM Journal on Mobile Networks and Applications*, vol. 8, no. 1, pp. 37–50, February 2003.

[4] W. Song and W. Zhuang, "Interworking of 3G Cellular Networks and Wireless LANs," *International Journal of Wireless and Mobile Computing*, vol. 3, no. 4, pp. 237–247, 2007.

[5] Qualcomm, "The Economics of Wireless Mobile Data." [Online]. Available: www.wirelessdevnet.com/library/WirelessMobileData.pdf

[6] C. Courcoubetis and R. Weber, *Pricing Communication Networks Economics, Technology and Modeling*. John Wiley and Sons Ltd, 2003.

[7] H. Zhu, A. Nel, and W. Clarke, "A Duopoly Pricing Model for Wireless Mesh Networks under Congestion-sensitive Users," *South African Institute of Electrical Engineers Africa Research Journal*, vol. 100, no. 2, pp. 48–58, June 2009.

[8] M. Mandjes, "Pricing Strategies and Service Differentiation," *Netnomics*, vol. 6, no. 1, pp. 59–81, April 2004.

[9] S. Jagannatha, J. Nayak, K. Almeroth, and M. Hofmann, "A Model for Discovering Customer Value for E-content," in *Proceedings of the 8th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, Edmonton, Canada, 23-26 July 2002.