Weakly-bound rare isotopes with a coupled-channel approach that includes resonant levels

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Abstract. The question of how the scattering cross section changes when the spectra of the colliding nuclei have low-excitation particle-emitting resonances is explored using a multi-channel algebraic scattering (MCAS) method. As a test case, the light-mass nuclear target ^{8}Be, being particle-unstable, has been considered. Nucleon-nucleus scattering cross sections, as well as the spectra of the compound nuclei formed, have been determined from calculations that do, and do not, consider particle emission widths of the target nuclear states. The resonant character of the unstable excited states introduces a problem because the low-energy tails of these resonances can intrude into the sub-threshold, bound-state region. This unphysical behaviour needs to be corrected by modifying, in an energy-dependent way, the shape of the target resonances from the usual Lorentzian one. The resonance function must smoothly reach zero at the elastic threshold. Ways of achieving this condition are explored in this paper.

1. Introduction

The advent of radioactive ion beam (RIB) physics prompts the consideration of new theoretical challenges involving weakly-bound systems. Radioactive nuclei, especially those close to the drip lines, can have quite low particle emission thresholds and consequently have low-lying resonance states in their spectra. Low-energy scattering of RIB is a coupled-channel problem that involves such low-lying resonant states of the scattered ion. To address this, a Multi-Channel Algebraic Scattering (MCAS) formalism [1] is used, in which momentum space solutions of coupled Lippmann-Schwinger equations are found, including also all negative-energy bound states. A finite-rank separable representation using an "optimal" set of sturmian functions [2] serves to construct an input matrix of nucleon-nucleus interactions. The MCAS method has the ability to locate all compound system resonance centroids and widths, regardless of how narrow. For full details see reference [1]. Also, by use of orthogonalizing pseudo-potentials (OPP) in generating sturmians, it ensures the Pauli principle is not violated [3], despite the collective model formulation of nucleon-nucleus interactions used. Otherwise, some compound nucleus wave functions possess spurious components.

In the following sections, we first briefly summarize the MCAS method [1, 3], which has been extended to unstable target states in Ref. [4]. In section II, we display and discuss our results for neutron scattering from ^{8}Be, with and without the excited target states having non-zero widths. In section III, we look briefly at the implications of letting the target-state widths have energy dependence, to ensure that these resonances do not extend to negative energies, where they would produce unphysical behaviour for bound, sub-threshold, states. Finally, section IV gives our brief concluding remarks.

2. The MCAS formulation for unstable target states

S-matrix equations in the MCAS methodology take the form:

$$S_{cc'} = \delta_{cc'} - \mathrm{i}^{l_{c'}-l_c+1}\pi\mu \sum_{n,n'=1}^{N} \sqrt{k_c}\,\hat{\chi}_{cn}(k_c) \left([\boldsymbol{\eta} - \mathbf{G}_0]^{-1} \right)_{nn'} \hat{\chi}_{c'n'} \sqrt{k_{c'}}\,. \tag{1}$$

Traditionally, all target states are taken to have eigenvalues of zero width. Then the integrals in the (complex) Green's functions are evaluated using the method of principal parts, where, in the limit $\epsilon \to 0$, the Green's functions take the form

$$[\mathbf{G}_0]_{nn'} = \mu \left[\sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + \mathrm{i}\epsilon} \hat{\chi}_{cn'}(x)\,dx - \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2} \hat{\chi}_{cn'}(x)\,dx \right]. \tag{2}$$

$\boldsymbol{\eta}$ is a column vector of sturmian eigenvalues and $\hat{\chi}$ are form factors determined from the chosen sturmian functions. The factor μ in Eqs. 1 and 2 is $\mu = 2m_{red}/\hbar^2$, with m_{red} being the reduced mass. This method assumes time evolution of target states is given by

$$|x, t\rangle = e^{-\mathrm{i}H_0 t/\hbar}\ |x, t_0\rangle = e^{-\mathrm{i}E_0 t/\hbar}\ |x, t_0\rangle\,. \tag{3}$$

However, if states decay, they evolve as

$$|x, t\rangle = e^{-(\Gamma t/2\hbar)}\ e^{-\mathrm{i}E_0 t/\hbar}\ |x, t_0\rangle\,. \tag{4}$$

Thus, in the Green's function, channel energies become complex, as do the squared channel wave numbers,

$$\hat{k_c}^2 = \mu\left(E - \epsilon_c + \mathrm{i}\tfrac{\Gamma_c}{2}\right)\ ;\ \hat{h_c}^2 = \mu\left(\epsilon_c - E - \mathrm{i}\tfrac{\Gamma_c}{2}\right), \tag{5}$$

where $\frac{\Gamma_c}{2}$ is half the width of the target state associated with channel c. The Green's function matrix elements are then

$$[\mathbf{G}_0]_{nn'} = \mu \left[\sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + \mathrm{i}\mu\frac{\Gamma_c}{2}} \hat{\chi}_{cn'}(x)\,dx - \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2 - \mathrm{i}\mu\frac{\Gamma_c}{2}} \hat{\chi}_{cn'}(x)\,dx \right]. \tag{6}$$

When poles are moved significantly off the real momentum axis, integration along this axis is feasible.

3. The case of ^{9}Be

The low excitation spectrum of ^{8}Be has a 0^+ ground state that decays into two α-particles (width: 6×10^{-6} MeV), a 2^+ resonance state (centroid: 3.03 MeV; width: 1.5 MeV) and a 4^+ resonance state (centroid: 11.35 MeV; width: ∼3.5 MeV). In Table 1 are given the parameters

Table 1. The parameter values defining the nucleon-^{8}Be interaction.

	Odd parity	Even parity
$V_{\rm central}$ (MeV)	-31.5	-42.2
V_{ll} (MeV)	2.0	0.0
V_{ls} (MeV)	12.0	11.0
V_{ss} (MeV)	-2.0	0.0
Geometry $R_0 = 2.7$ fm	$a = 0.65$ fm	$\beta_2 = 0.7$
	$0s_{1/2}$	$0p_{3/2}$
0^+ $\lambda^{(OPP)}$	1000	0.0
2^+ $\lambda^{(OPP)}$	2.0	0.2
4^+ $\lambda^{(OPP)}$	0.0	0.0

in the channel-coupling potentials, as well as the λ^{OPP} values giving the strengths of Pauli hindrance. Two evaluations of the n+^{8}Be cross section are obtained using this spectrum; one with target-state widths set to zero, and the other taking into account the widths of the excited levels. The two results will be identified by the terms 'no-width' and 'width', respectively. Both cases are calculated with the same nuclear interaction, taken from a rotor model [5]. Table 2 displays ^{9}Be spectrum results from these calculations.

Table 2. Widths of resonances in n+^8Be scattering for the no-width and width cases, compared to experimental values. Boldface entries highlight the matches we find between theory and experiment to within a factor of 3.

J^π	$\Gamma_{exp.}$	$\Gamma_{no-width}$	$\frac{\Gamma_{no-width}}{\Gamma_{exp.}}$	Γ_{width}	$\frac{\Gamma_{width}}{\Gamma_{exp.}}$
$\frac{1}{2}^+$	0.217±0.001	—	—	1.595	7.350
$\frac{7}{2}^-$	1.210±0.230	2.08×10^{-5}	1.72×10^{-5}	1.641	**1.356**
$\frac{1}{2}^-$	1.080±0.110	0.495	**0.458**	1.686	**1.561**
$\frac{5}{2}^+$	0.282±0.011	0.187	**0.663**	0.740	**2.624**
$\frac{3}{2}^-$	1.330±0.360	0.466	**0.350**	3.109	**2.337**
$\frac{5}{2}^-$	7.8×10^{-4}	0.060	76.74	2.772	3554
$\frac{9}{2}^+$	1.330±0.090	0.386	0.290	2.498	**1.878**
$\frac{3}{2}^+$	0.743±0.055	3.286	4.423	5.162	6.947

Taking the excited states of ^{8}Be to be resonances gives the same spectral list as when they are treated as zero-width, but the evaluated widths of the compound nuclear states significantly increase, as reflected in the cross sections. These increases bring the theoretical ^{9}Be state widths closer, often significantly, to experimental values, as evidenced by comparison of the final columns of the two tables. Only the $\frac{5}{2}^-$, also poorly recreated in centroid, has a worse match with widths applied. The resultant elastic and reaction cross sections are shown in figure 1. The blue line depicts the results without widths, the red line with constant widths multiplied by the Heaviside (step) function: $H(E) = 0$ for $E \leq 0.0$; $H(E) = 1$ for $E > 0$. (For the green line, see section 4.)

Introducing target state widths, the resonances in ^{9}Be are suppressed but still present; their

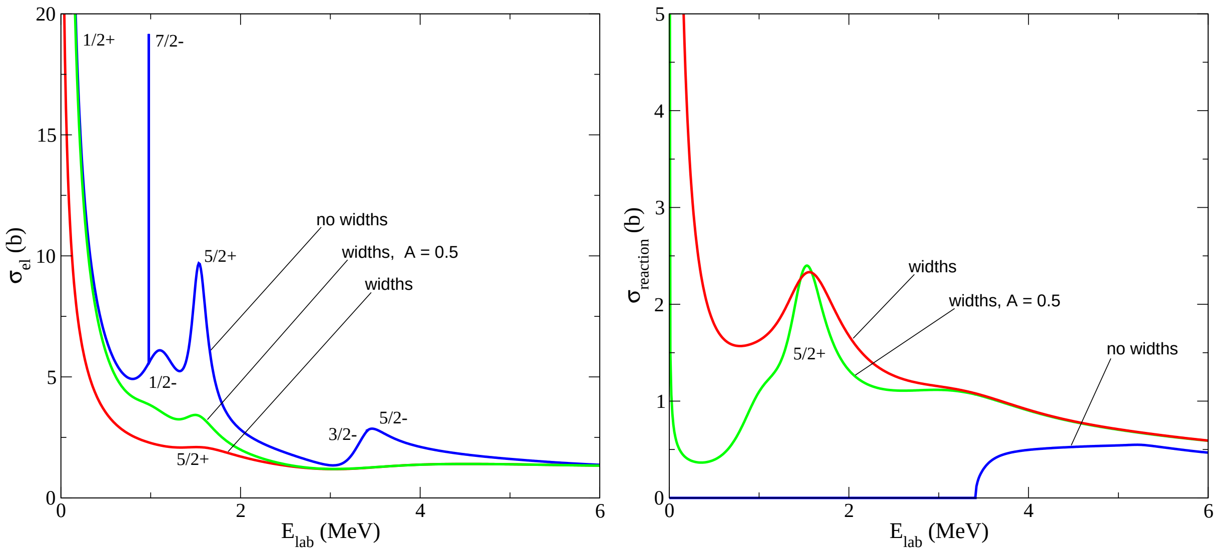

Figure 1. Elastic (left panel) and reaction (right panel) cross section as a function of neutron laboratory energy with three treatments of the widths of target states (see text).

widths increasing and magnitudes decreasing so as all but the $\left(\frac{5}{2}\right)^+_1$, and arguably $\left(\frac{5}{2}\right)^+_2$, cannot be discerned from the background. The reaction cross section found in the no-width case is zero until 3.4 MeV (3.03 MeV in the centre of mass frame), the first inelastic threshold. When the target state has width different from zero the elastic cross section has flux loss from zero projectile energy upwards. There is an asymptotic behaviour as energy approaches zero. In the reaction cross section from the width calculation, there is a broad peak at 1.5 MeV, corresponding to the $\frac{5}{2}^+$ resonance. Using widths not varying with energy (see the next section), the bound states, $E < 0.0$, of the compound ^{9}Be system are unstable due to the couplings to the decaying excited states of the ^{8}Be core. For this reason the widths have been multiplied by a Heaviside function to ensure that the compound system is stable at negative energies. An improvement of this technique is discussed in the next section.

4. Energy dependence of widths

A smooth description of the energy dependence of the decay width is needed to avoid any pathological behaviour caused by the Heaviside function. This function allows maintaining the stability of the compound system in the sub-threshold region, by setting to zero all widths for $E < 0$, but can generate "ghost" levels where the application of widths could move a sub-threshold state above $E = 0$, thus creating two copies of the same state. Also, a dramatic overestimation of the reaction cross-section occurs in the region close to threshold.

We therefore need to improve the model with a suitable parametrization of the energy-dependence of the decaying width for the excited levels, with the constraint that the width must be zero at the scattering threshold, and equal to the experimental width at the resonance centroid energy. We are presently exploring various forms of energy dependences for the widths of these excited states ($\Gamma_c(E) = \Gamma_c \times U(E)$). Examples of such expressions are

$$U(E) = \frac{(1+A)}{(1+A)^2 - 1} \left\{ \frac{1+A}{A(1-x)^2+1} - \frac{1}{A(x^2)+1} \right\} H(E) \tag{7}$$

where $x = E/E_r$, $H(E)$ is the Heaviside function defined above and A is a parameter, and the

Gaussian form

$$U(E) = e\left(\frac{E}{E_r}\right)^2 e^{-\left(\frac{E}{E_r}\right)} H(E). \tag{8}$$

In this paper we study only the first form. The second, and others, will be explored in future work. Also yet to be taken into account is causality, as needed when dealing with energy-dependent widths [6].

In figure 2 we give the induced energy dependence for the function $U(x = E/E_r)$ with respect to the renormalized energy x. The curves differ for different values of the parameter A. The green line in figure 1 shows the effect of this energy-dependent width on elastic and reaction cross sections, respectively, when $A = 0.5$. Clearly, for this value of A, the asymptotic behaviour in the reaction cross section at $E = 0.0$ is suppressed, but not resolved. The use of an energy

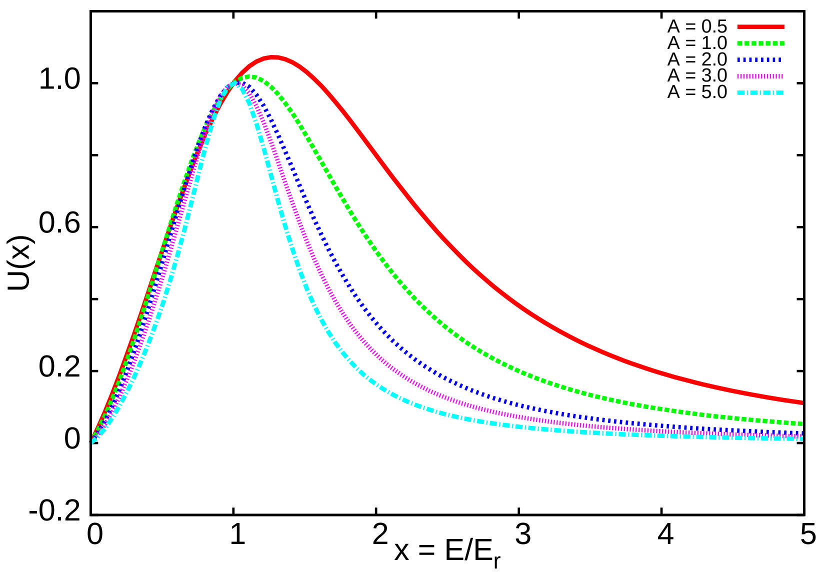

Figure 2. The scaling function, $U(E/E_r)$, for various values of the parameter A.

dependence in the widths of the excited states of the target introduces a new aspect that has to be taken into consideration. In principle, an energy dependence in the widths induces a corresponding energy-dependence in the positioning of the resonance centroids. This additional effect is a direct consequence of the principle of causality which demands that the real and imaginary part of any Green's function matrix elements are related by a dispersion integral. Such an aspect has been extensively discussed in the study of the properties of the nuclear optical potential [6]. Eq. 7 is particularly convenient in this regard, because the corresponding dispersion integral leads to an analytical energy-dependent shift of the resonance centroids. Such analytical forms have been derived in Ref. [7] for a class of functions which includes Eq. 7. The resulting energy shift function $\Delta S_c(E) = \Gamma_c \times S(E)$ is given by the analytical expression:

$$S(E) = \frac{1}{A(x-1)^2+1}\left\{c_1 - (x-1)c_2\right\} + \frac{1}{A(x)^2+1}\left\{c_3 + (x)c_4\right\} - \frac{\ln|x|}{\pi}U(x) \tag{9}$$

where

$$c_1 = \frac{(1+A)^2}{(1+A)^2-1}\frac{\ln\sqrt{1+\frac{1}{A}}}{\pi} \quad ; \quad c_2 = \frac{(1+A)^2}{(1+A)^2-1}\frac{\sqrt{A}}{\pi}\left(\frac{\pi}{2} + arctg\sqrt{A}\right) \; , \tag{10}$$

and

$$c_3 = \frac{(1+A)}{(1+A)^2-1}\frac{\ln\sqrt{A}}{\pi} \quad ; \quad c_4 = \frac{(1+A)}{(1+A)^2-1}\frac{\sqrt{A}}{2} \, . \tag{11}$$

5. Conclusions

An extension to the multi-channel algebraic scattering (MCAS) formalism that considers particle-decay widths of target nucleus eigenstates has been applied to a range of light mass nuclear targets [5], in addition to the results for n+^{8}Be shown in this paper. To ensure that any resonance aspect of target states does not have influence below the nucleon-nucleus threshold in our formalism, we have used a Heaviside function to cut off tails of the positive-energy resonances below zero energy of the compound system. This procedure, however, is too simplistic since it introduces non-physical singularities in cross sections at the approach to zero energy, and may also incorrectly overestimate reaction cross just above the threshold.

A better energy dependent scaling factor is needed: one with value one at the centroid energy and approaching zero for lower and higher energies. We propose here possible means of achieving such modified Lorentzians, and show, for one example, their effect on the elastic and reaction cross sections in the scattering of neutrons from ^{8}Be. Work is in progress to apply these and other forms to the light-nuclear systems considered in our previous work. Work is in progress, also, for preserving causality in the presence of energy-dependent widths by the use of appropriate dispersion relations.

Acknowledgments

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