MODELLING THE BUSINESS CYCLE IN SOUTH AFRICA: A NON-LINEAR APPROACH

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Abstract

In this paper the South African business cycle is modeled, using a simple linear method and comparing it to non-linear methods. This is useful to address the debate between the Classical and Keynesian economists regarding their views on the business cycle. They believe in a stable economy with exogenous shocks and an unstable economy with an endogenous business cycle respectively. Linear models are usually associated with the Classical view and non-linear models with the Keynesian view.

A detailed discussion on the non-linear model-building process, with particular emphasis on the family of STAR models is done. The South African GDP is used and AR, TAR, LSTAR and ESTAR models are fitted and compared.

It finds that a parameterized nonlinear model (such as the family of STAR models) outperforms the simple regression model. This is due to asymmetric behaviour in the GDP data and the possibility of a threshold between a recession and an expansion.

The results in this paper support the structural or institutional view of business cycles, which states that economic fluctuations are caused by various structural or institutional changes.

1. Introduction

In this paper the debate between the Keynesian (endogenous business cycle) and the neo-classical approach (stable business cycle with exogenous shocks) continues. This debate will be addressed to determine if the South African business cycle can be best predicted with a linear model or a non-linear model, or perhaps both.

The structuralist or institutionalist believes that economic fluctuations are caused by various structural or institutional changes. Adherents to this view do not believe

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that the market system is inherently stable or systematically unstable. According to this view the appropriate policy varies from time to time as circumstances change.

Firstly a brief discussion will follow regarding business cycle theories focusing on the Keynesian and Neo-classical views. Thereafter an outline of the family of smooth transition autoregressive (STAR) models will follow.

The non-linear model building process will be discussed, and will be followed by an application of the process to the South African GDP data.

The empirical part begins with the best linear model for the data which is an autoregressive model. A tentative threshold autoregressive model is built, and this is used to determine the delay parameter and the threshold. These are then used in the specification of the smooth transition autoregressive model. The forecasting performance of the linear model and the family of STAR models are compared.

2. Business cycle theories

While there are no built-in econo-rhythms in the market process, there are, from time to time, economy-wide disturbances of one sort or another. Business cycles can be seen as self-reversing market processes. Business cycles are an inherent part of the market process as well as disruptions of the market process. That is, both the lower turning point (the upturn) and the upper turning point (the downturn) are endogenous for those who conceive of business cycles as econo-rhythms and exogenous for those who think in terms of monetary disequilibrium (Garrison, 1989).

Non-recurring fluctuations are the essence of disequilibrium and turning points are the essence of fluctuations (Bowers, 1985). In looking at various business cycle theories, the interest is in how the turning points come about. To have a business cycle theory one must have inherent in the system a reason why each step upwards helps bring about the necessary conditions for a downward movement. Each decline into a recession must end up in a recovery.

There is a lot of debate regarding the causes of the business cycle – especially whether it is exogenous or endogenous. Mohr and Fourie (2004) describe the three broad approaches to explanations of the business cycle.

2.1 The neo-classical explanation of the business cycle

This school of thought maintains that both prices of goods and services and factors of production respond relatively quickly to any imbalances, which develop between demand and supply to counteract and remove such imbalances before serious disequilibrium develops (Cloete, 1990).

According to this view, market economies are inherently stable. Theorists subscribing to this view therefore spend considerable time and effort explaining why economic activity does not grow smoothly. Fluctuations in economic growth are regarded as a temporary phenomenon that could be ascribed to exogenous factors which originate outside the market system. The monetarists also trace the major causes of economic fluctuations to exogenous factors. These economists believe that government should leave the market system to its own devices. They believe that market forces will sort out all economic problems and government should not intervene (Mohr and Fourie, 2004).

2.2 The Keynesian explanation of the business cycle

The Keynesian school of economic thought maintains that the economy is inherently unstable. The level of economic activity overshoots and undershoots the growth path. The Keynesian school points out that the existence of a business cycle is evidence of the failure of the price mechanism to co-ordinate demand and supply in the markets for goods and services and factors of production (Cloete, 1990). It argues that prices respond with a time lag to changes in demand. This results in a level of economic activity, which tends to be continually above or below its equilibrium level. The Keynesians believe that the business cycle is mainly endogenous; the cyclical fluctuations in economic activity are generated by time lags and by the multiplier and accelerator relationships between economic variables, which are part of the internal structure of the economy. The economy reacts to stimuli because of the presence of time lags and multiplier-accelerator relationships. The result of this is cyclical fluctuations.

Keynesians believe that the government has a duty to intervene in the economy by applying appropriate monetary and fiscal policies. Keynesians believe that business cycles are part and parcel of the way in which market economies operate (Mohr and Fourie, 2004).

2.3 Structuralist or institutionalist view

Economic fluctuations are caused by various structural or institutional changes. Adherents to this view do not believe that the market system is inherently stable or systematically unstable. They focus on structural changes and unpredictable events. They do not have set ideas on economic policy. According to them the appropriate policy will vary from time to time as circumstances change.

These three broad approaches to the business cycle should be regarded as extremes rather than watertight categories. Few economists subscribe fully to any one of these approaches. Most hold an eclectic view that incorporates the three extreme views, although one of the three approaches will usually still be found to dominate (Mohr and Fourie, 2004).

3. Linear models vs non-linear models

There are two alternatives ways of modelling the business cycle – linear or non-linear. The real business cycle approach to business cycle modelling provided a dominant paradigm in the 1980s. In line with the post-war tradition, the cycle is modelled using essentially linear or log-linear relationships to propagate the energy provided by external or exogenous random or auto-correlated shocks into a cycle. It differs from the traditional Keynesian approach in that the propagation model is firmly based on neoclassical microeconomic foundations, and throughout the cycle the economy is continuously in equilibrium. A major impact of the real business cycle approach has been to require adherents to competing paradigms to provide rigorous microeconomic underpinnings to their macroeconomic theories, as the New Keynesians have done. As the 1980s progressed, momentum gathered behind a radically different approach to modelling economic fluctuations based on the relatively new mathematical theory of chaos. Chaos theory demonstrates that remarkably simple non-linear systems can yield complex dynamic paths involving cycles of various periods and seemingly random series. The actual path is highly sensitive to the starting value (Mullineux, Dickinson and Peng: 1993).

Deterministic non-linear models are capable of producing an output that is seemingly random or that displays complex cyclical behaviour – in the sense that the cycles produced appear to have no regular period and are asymmetrical. Hence, this provides an alternative to the traditional linear stochastic approach to business cycle modelling. Non-linearity is a necessary condition for chaos.

Haberler in 1937 concentrated on the endogenous causes of business cycles Zarnowitz (1999). Since then, however, economists have concentrated on exogenous disturbances, stochastic elements and policy factors as causes of the business cycle. Zarnowitz (1999) is of the opinion that the emphasis on shocks is overdone. Intense arguments rage about whether the business cycle is due to real or monetary shocks, or domestic and foreign shocks – as if the categories were well identified and the underlying models represent the economy so well that an endogenous business cycle is ruled out.

Noise in the sense of a large number of small events is often a causal factor much more powerful than a small number of large events. Noise makes trading on the financial markets possible. Noise causes markets to be somewhat inefficient and often prevents us from taking advantages of inefficiencies. Noise in the form of uncertainty about future tastes and technology by sector causes the business cycle, and makes it highly resistant to improvement by government intervention (Black, 1986).

Shocks come in different combinations and are seldom well identified. There is little agreement on which theoretical and econometric models of the economy to use. There are good reasons to accept lead-lag relationships and non-linearities as important features that account for the endogenous content of business cycles. This is missed by those concentrating on the role of shocks in linear models with little attention to timing differences, interactions between potentially self-generating movements of strongly fluctuating variables and likely asymmetries (Zarnowitz, 1999). Zarnowitz is also of the opinion that this is the core of business cycles, while the outside disturbances, whose causal role is often questionable, are more peripheral, transitory and episodic.

The broad movements of the economy, including turning points, are best seen as sequential processes rather than isolated events. Cyclical movements do not originate exclusively from supply or demand, but instead always refer to the interplay of two market sides. To categorize shocks as stemming from supply or demand alone seldom reveals anything deep or interesting about the determinants of expansions and contractions.

4. Non-linear models to forecast the business cycle

Economic theory suggests that a number of important time-series variables should exhibit non-linear behaviour. It is established that downturns in the business cycle are sharper than recoveries in that key macroeconomic variables, such as output and employment, fall more sharply than they rise.

Since the standard ARMA model relies on linear difference equations, new dynamic specifications are necessary to capture non-linear behaviour (Enders, 2004). The example Enders uses explains it best: when you are on a road trip to a new location you might take along a road atlas. Since the earth is not flat, the maps contained in the atlas are a linear approximation of the actual path of the journey. This linear approximation is extremely useful. A non-linear approximation would have been a nuisance. For other types of trips a linearity assumption will be inappropriate. For example it would be disastrous for NASA to use a flat map of the earth to plan the trajectory of a rocket launch. Similarly, the assumption that economic processes are linear can provide useful approximations to the actual time-paths of economic variables. Nevertheless, policymakers could make a serious error if they ignore the empirical evidence that unemployment increases more sharply than it decreases.

Non-linearity of business cycles is important because it has implications for business cycle theory. If indicators of business cycles are non-linear, then linear models used to describe them may yield forecasts that are inferior to those drawn from non-linear models.

However, linear structures are often adopted because they are simple and the results they yield are simple. But simplicity is sometimes more a vice than a virtue – particularly in the case of macro dynamics and economic fluctuations. It is unrealistic to assume linearity because then it means that the business cycle is only caused by exogenous shocks. The contention of many Keynesian writers is that theories of fluctuations should explain how fluctuations arise also endogenously from a working system; otherwise how is a business cycle possible at all? (Anon, 2004).

5 Smooth transition models

The assumption that the economy can only be in two states is generalized by basing the analysis on the smooth transition autoregressive (STAR) model. This model allows the business cycle indicator to alternate between two distinct regimes which represent two different phases of the business cycle, but transition between these regimes can be smooth, so that there can be a continuum of states between extreme regimes. The two-regime TAR model is a special case of the STAR model, so that the analysis accounts for the possibility that there might be only two regimes (Teräsvirta and Anderson, 1992).

Smooth transition autoregressive models allow the autoregressive parameters to change slowly. Consider the special non-linear autoregressive (NLAR) model.

$$y_t = \alpha_0 + [\alpha_1 + \beta_1 f(y_{t-1})]y_{t-1} + \varepsilon_t$$

If f() is a smooth continuous function, the autoregressive coefficient (α_1+β_1)) will change smoothly along with the value of y_{t-1} (Enders, 2004).

There are two useful forms of the STAR models that allow for a varying degree of autoregressive decay. The logistic-STAR model generalizes the standard autoregressive model such that the autoregressive coefficient is a logistic function.

The smooth transition autoregressive (STAR) model is defined as follows (Teräsvirta, Van Dijk and Medeiros, 2004)

$$y_t = \alpha_0 + \alpha_t y_{t-1} + ... + \alpha_0 y_{t-0} + \beta\lfloor \beta_0 + \beta_t y_{t-1} + ...\beta_0 y_{t-0}\beta \rfloor + \varepsilon$$

where

$$\theta = \left[1 + \exp(-y_{t-0} - c))\right]^{-1}$$

γ is the smoothness parameter, and c is the threshold. In the limit, as $\gamma \rightarrow 0$ or ∞ the LSTAR becomes a AR(p) model since the value of θ is constant. For intermediate values of γ the autoregressive decay depends on the value of y_{t-1}. The intercept and the autoregressive coefficients smoothly change between two extremes as the value of y_{t-1} change.

The other form of the STAR model is the exponential form (ESTAR).

where

$$\theta = \left[1 + \exp\left(-y(y_{t-0} - c)^2\right)\right]_{y>0}$$

For the ESTAR model as γ approached zero or infinity, the model becomes an AR(p) model since θ is constant. Otherwise the model displays non-linear behaviour. The ESTAR model has proven to be useful for periods surrounding the turning points of a series in that such periods have different degrees of autoregressive decay than others. The ESTAR is symmetrical around y_{t-1} = c so that it can approximate gravitational attraction (Enders,2004).

The use of STAR models is based on the assumption that local dynamics of the autoregressive process, characterizing the process, change with the phase of the business cycle (Teräsvirta and Granger: 1996).

6. The non-linear model-building process: the smooth transition autoregressive (star) model

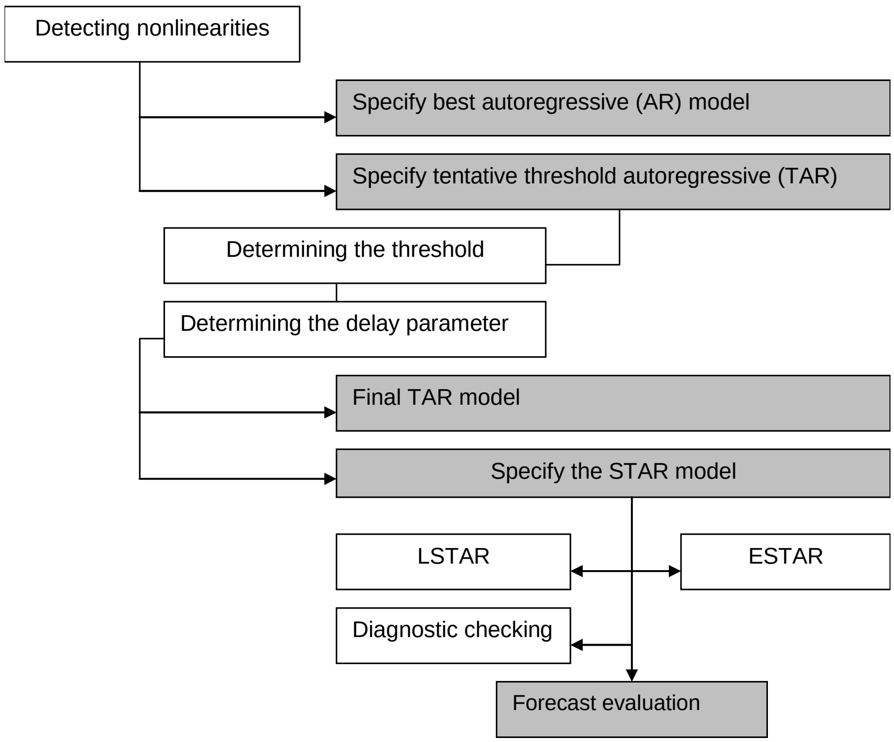


Diagram 1: The non-linear model-building process

Source: Adapted from Enders (2004)

This flow chart represents the model building process that is followed in this paper in order to arrive at the final nonlinear model for the South African business cycle. A description of this process is discussed in the next section.

7. Description of the non-linear model-building process

7.1 Testing for non-linearity

Before a model for a particular data set is specified, it is important to test for the presence of non-linearity.

7.1.1 The autocorrelation function (ACF)

The ACF assists in selecting proper values for the lag numbers p and q when an ARMA model is specified. The ACF of the residuals is also an important diagnostic. The ACF, as used with linear models, might be misleading for non-linear models. The reason for this is that the ACF measures the degree of linear association between y_t and y_{t-1}. Thus, the ACF may fail to detect important non-linear relationships present in the data. If one is interested in the non-linear relationships in the data, a useful diagnostic is to examine the ACF of the squared values of a series (Enders, 2004). The ACF of the squares of the residuals (y^2_t) can reveal a non-linear pattern. The ACF from chaos may be indicative of white noise, but the ACF of squared values of the sequence may be large. A non-explosive sequence is chaotic if it is generated from a deterministic difference equation such that it does not converge to a constant or to a repetitive cycle (Enders, 2004).

7.1.2 McLeod-Li test

This test seeks to determine if there are significant autocorrelations in the squared residuals from a linear equation. To perform this test estimate the series using the best-fitting linear model and generate the residuals. Form the autocorrelations of the squared residuals. Use the Ljung-Box statistic to determine whether the squared residuals exhibit serial correlation. The value of the Ljung-Box or Q statistic has an asymptotic chi-square distribution with n degrees of freedom if the squared residuals are uncorrelated. Rejecting the null hypothesis would be equivalent to accepting that the model is non-linear. However, rejecting the null hypothesis of linearity does not determine the nature of the non-linearity present in the data (Enders, 2004).

7.1.3 The regression error specification test (RESET)

This test posits the null hypothesis of linearity against a general alternative hypothesis of non-linearity. If the residuals from a linear model are independent they would be correlated with neither the regressors used in the estimated equation nor with the fitted values. Hence a regression of the residuals on these values should not be statistically significant. The intuition behind this test is to regress the residuals on all the variables included in the best-fitting linear model. This regression should have little explanatory power if the model is truly linear. Thus, the sample value of F should be small. Reject linearity if the sample value of the F-statistic for the null hypothesis exceeds the critical value (Enders, 2004).

7.1.4 Lagrange multiplier tests

The tests discussed above are Portmanteau or residual-based test that do not have a specific alternative hypothesis (Enders, 2004). Lagrange multipliers (LM) tests can be used to test for a specific type of non-linearity. The LM test can help to select the proper functional form to use in non-linear estimation. This test is done routinely in software packages such as Eviews. The benefit of this model is that you need not estimate the non-linear model itself (Enders, 2004). The use of a number of LM tests can help you select the form of the non-linearity.

7.1.5 The Wald test

Enders (2004) states the Wald test computes a test statistic based on the unrestricted regression. The Wald statistic measures how close the unrestricted estimates come to satisfying the restrictions under the null hypothesis. If the restrictions are in fact true, then the unrestricted estimates should come close to satisfying the restrictions. In the case where linearity is tested, the coefficients are restricted to a certain functional form and according to that restriction linearity is rejected or not rejected.

7.2 Identifying the threshold for the data

In most instances the value of the threshold is unknown and must be estimated along with the other parameters of the TAR model (Frances and van Dijk, 2004). If the threshold is to be meaningful, the series must cross the threshold. Thus the threshold must lie between the maximum and the minimum values of the series. Usually the highest and lowest 15 percent of the values are excluded from the search to ensure an adequate number of observations on each side of the threshold. The procedure to estimate the threshold is to use every value within the band and estimate the TAR model until the smallest residual sum of squares is obtained.

7.2.1 Multiple thresholds

Another issue when determining thresholds is whether multiple thresholds exist (Frances and van Dijk, 2004). One commonly used technique is the above method of finding the consistent estimate of the threshold (Enders, 2004). As mentioned earlier, the true threshold minimizes the sum of squared residuals. The basic idea with this method is that the residual sum of squares (RSS) has several local minima if there are several thresholds. The method is followed as above and after eliminating the lowest and highest 15% of the data, it is sorted from the lowest to the highest value and a TAR model is run for each value. The respective RSSs are plotted on a graph. If there is a single threshold, there should be a single trough in the graph. If there are two troughs, there are two potential thresholds.

7.3 Determining the delay parameter

It might happen that the timing of the adjustment process is such that it takes more than one period for the regime-switch to occur. In such circumstances one could allow the regime-switch to occur according to the value of y_{t-d}. Another procedure suggested by Teräsvirta and Anderson (1992) is to test for linearity and choosing the delay parameter according to the lowest p value (rejecting linearity).

7.4 Extension of the threshold autoregressive model (TAR)

7.4.1 The smooth transition autoregressive (STAR) model

The basic TAR model, the threshold and the determined delay parameter are used to build the STAR model. STAR models allow the autoregressive parameters to change slowly. If the special NLAR model is considered:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \beta_1 y_{t-1} f(y_{t-1}) + \varepsilon_t$$

If f( ) is a smooth continuous function, the autoregressive coefficient ($\alpha_1 + \beta_1$) will change smoothly along with the value of y_{t-1}.

Once the STAR model is specified it should be determined whether the series is best modelled as an LSTAR or an ESTAR process. Teräsvirta developed a framework in 1994 that can detect the presence of non-linear behaviour (Enders,2004). This method can also be used to determine whether a series is best modelled as an LSTAR or an ESTAR process. The test is based on the Taylor series expansion of the general STAR model. Products of the regressors with the powers of y_{t-d} are formed.

$$\begin{aligned} y_t &= \alpha_0 + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} + (\beta_0 + \beta_1 y_{t-1}) \ldots \beta_p y_{t-p} (\pi_1 h_{t-d} + \pi_3 h_{t-d}^3) + \varepsilon_t \\ &= z_t + z_t y_{t-d} + z_t y_{t-d}^2 + z_t y_{t-d}^3 + \varepsilon_t \end{aligned}$$

where

$$z_t = (\alpha_0, y_{t-1}, y_{t-2}, \ldots, y_{t-p})$$

Then an auxiliary regression is estimated.

$$\begin{aligned} e_t &= a_0 + a_1 y_{t-1} + \ldots + a_p y_{t-p} + a_{11} y_{t-1} y_{t-d} + \ldots \\ &+ a_{1p} y_{t-p} y_{t-d} + a_{21} y_{t-1} y_{t-d}^2 + \ldots + a_{2p} y_{t-1} y_{t-d}^2 \\ &+ a_{31} y_{t-1} y_{t-d}^3 + \ldots + a_{3p} y_{t-p} y_{t-d}^3 + \varepsilon_t \end{aligned}$$

This auxiliary regression is used to test for the presence of LSTAR behaviour. The joint restriction that all non-linear terms are zero is tested (H_0: Linear model). A standard F-test with 3p degrees of freedom in the numerator is used.

To derive the Taylor expansion for the ESTAR model it can be written without h_{t-d} and h^3_{t-d} (Frances and van Dijk, 2004). Thus the expansion has the form:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} + (\beta_0 + \beta_1 y_{t-1} + ...\beta_p y_{t-p})(\pi_2 h_{t-d}^2) + \varepsilon_t$$
$$= z_t + z_t y_{t-d} + z_t y_{t-d}^2 + \varepsilon_t$$

The auxiliary equation for the ESTAR model is nested within that of the LSTAR model. If the ESTAR is appropriate, it should be possible to exclude all of the terms in the cubic expression.

After the alternative hypothesis is accepted when testing for LSTAR, test the restriction that the cubic expression is equal to zero using the F-test. If the null hypothesis is rejected the model has an LSTAR form. If the null hypothesis is not rejected the model has the ESTAR form (Enders, 2004).

7.5 Diagnostic testing and evaluation of forecasts

A very useful test to see if the model fits the data well is the Jarque Bera test. The residuals of the final model are tested and a good fit is suggested if the residuals are normally distributed (Enders, 2004).

To evaluate the forecasts, in-sample evaluations are done evaluating the Theil inequality coefficient and the biased, variance and covariance proportions. To evaluate the out-of-sample forecasts, the forecasts are plotted with the actual values and the root mean squared percentage error (RMSE) is calculated.

8. Using GDP to forecast the business cycle for South Africa comparing linear and non-linear methods

Teräsvirta and Anderson (1992) argue that if the possible non-linearity of business cycles is studied, it is useful to choose a business cycle indicator that shows as much cyclical variation as possible. Since the focus is on forecasting turning points, recessions and expansions, real GDP for South Africa are used. The data is transformed to the fourth-quarter differences of the quarterly logarithmic GDP. The data period selected is from the first quarter of 1961 to the first quarter of 2004. The in-sample period is from the first quarter of 1961 to the first quarter of 2002.

8.1 Autoregressive model

A simple autoregressive model was fitted to the GDP data. The lag structure was determined by using the Schwarz information criterion (SIC). The most significant lags were 1, 2, 4 and 5. The best linear model that fitted the data results are presented below. The model was tested for autocorrelation with the Breuch Godfrey test. The F-statistic was 1,36 with a p-value of 0,26, thus accepting the null hypothesis of no autocorrelation. White's test for heteroscedasticity was done on the function and the F-statistic was 1,6 with a p-value of 0,21, again accepting the null hypothesis of no heteroscedasticity. This model forms the basis for the nonlinear model.

Table 1: Results for the autoregressive model

Dependent Variable: DLOG(GDP,0,4)			
Variable	**Coefficient**	**Std. Error**	**t-Statistic**
C	0,028862	0,006223	4,638148
AR(1)	0,720651	0,074461	9,678250
AR(2)	0,229229	0,082217	2,788105
AR(4)	-0,404029	0,081529	-4,955651
AR(5)	0,239365	0,073835	3,241896
R-squared	0,639234	*Mean dependent var*	0,029605
Adjusted R-squared	0,630381	*S,D, dependent var*	0,028453
S.E. of regression	0,017298	*Akaike info criterion*	-5,247114
Sum squared resid	0,048774	*Schwarz criterion*	-5,154139
Log likelihood	445,7576	*F-statistic*	72,20418
Durbin-Watson stat	2,097587	*Prob(F-statistic)*	0,000000

Source: Eviews estimation results

8.2 Identification of non-linear models

8.2.1 Testing for non-linearity

The different tests for non-linearity were applied to the GDP data and they rendered the following results:

Table 2 Results from testing for non-linearity

	Test statistic	p-value	H_0: linear
ACF of squared residuals	Correlogram	n/a	***Reject***
McLeod-Li test	Q-statistic	0.04 (lag 26)	***Reject***
RESET	F-statistic	0,54	***Accept***
Lagrange multiplier tests	TR^2	0,25	***Accept***
Wald test	F-statistic	0	***Reject***

Source: Eviews estimation results

With the exception of the RESET test, all of the tests rejected the null hypothesis for linearity and it can only be rejected on a 46% (RESET) significance level. The Lagrange multiplier test accepts the null hypothesis, but it could reject it on a 75% significance level. With the Wald test the null hypothesis of linearity was rejected for all the coefficients. The autocorrelation function (ACF) of the sum of squared residuals was non-explosive and it did not converge to a constant or a repetitive cycle. According to Enders (2004) this is an indication of chaotic behaviour. These results are somewhat contradictory and therefore a linear and a non-linear model are fitted to the GDP data in order to determine the best forecastability.

8.2.2 Identifying the threshold for the data

The estimated autoregressive model is used as a basis for identifying the TAR model. To determine the band of the threshold the 15% percentile and the 85% percentile were calculated and all possible thresholds between these two values were tested.

Two methods were used to find the significant threshold in this 70% band. The first was to order the threshold values from the lowest to the highest value and then to estimate a TAR model for each value within the 70% threshold band. The one with the lowest RSS was chosen. This turned out to be 0,041.

Another method was to use each value, within the 70% threshold band, in the TAR model and choose the one with the lowest Schwarz criterion. The threshold was again found to be 0,041. It is assumed that there is only one threshold. The reason is that there is not a significant difference between the various RSS values for the threshold values within the band. Figure 2 shows the lowest RSS which is 0,0264 (marked in red), with the threshold value of 0,041. As indicated by the black trend line, it is assumed that there is only one threshold.

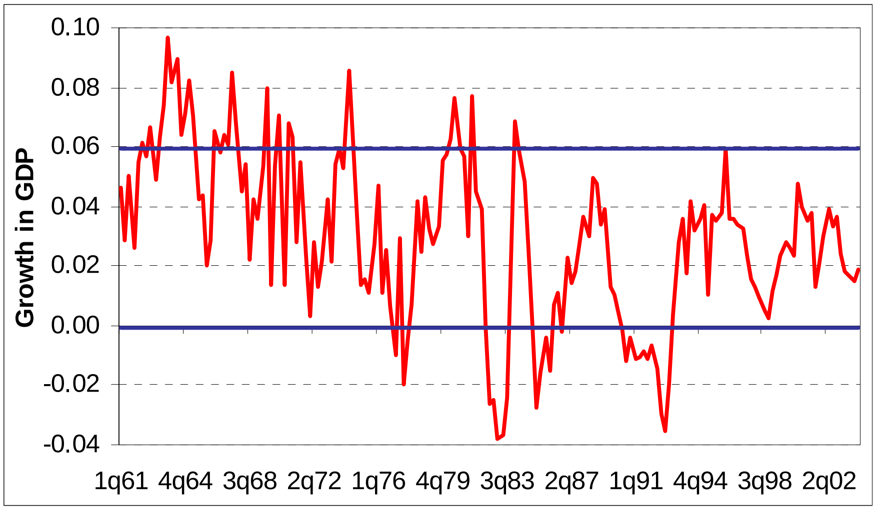


Figure 1: The GDP growth rate within the 70% threshold band to identify the TAR model
Source: South African Reserve Bank, Quarterly Bulletin (March 2004)

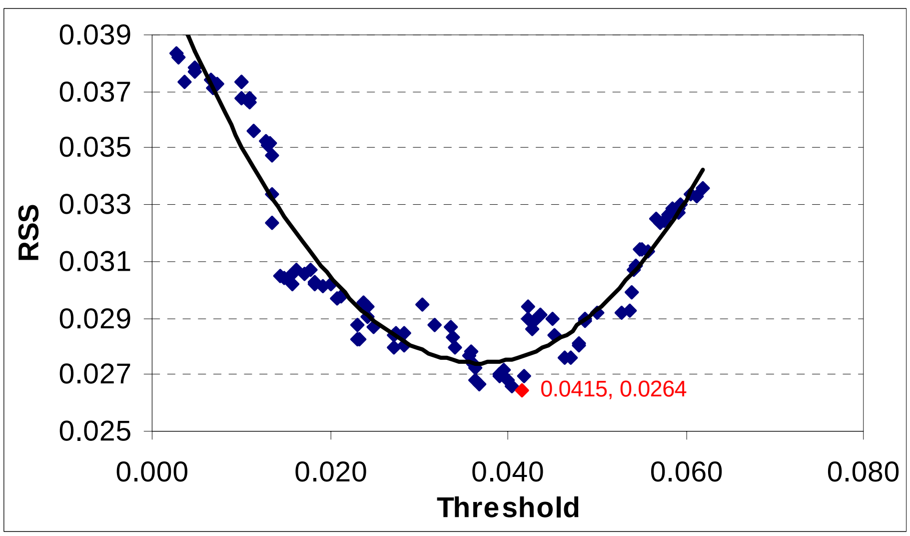


Figure 2: Residual sum of squares and the threshold
Source: Own calculations

8.2.3 Determining the delay parameter

The procedure suggested by Enders (2004) to determine a delay parameter was used. After running the TAR model with various delay assumptions the $Y_{t\ 4}$ was the most significant with the lowest Schwarz value.

Another procedure suggested by Teräsvirta and Anderson (1992) is to test for linearity and choose the delay parameter according to the lowest p value (rejecting linearity). The Wald test p-value for the F-statistic of the fourth lag was also the lowest. Thus, the delay parameter is Y_{t-4}. The final TAR model was estimated with the determined threshold and the delay parameter.

8.2.4 Smooth transition autoregressive model

- **The LSTAR model**

The basic TAR model and the threshold and the determined delay parameter are used to build the LSTAR model. The logistic function is added to the model to form the LSTAR model. The following LSTAR model was the best:

Table 3: The results for the LSTAR model

Dependent Variable: DLOG(GDP,0,4)			
	Coefficient	**Std. Error**	**t-Statistic**
α_0	**0,047112**	**0,089738**	**0,524997**
α_1	0,694645	0,077803	8,928237
α_2	0,190241	0,085386	2,228014
α_3	0,212099	0,074448	2,848973
β_0	**-0,120581**	**0,178020**	**-0,677347**
β_1	0,299323	0,174418	1,716125
θ	**-16,69313**	**28,67616**	**-0,582126**
R-squared	0,644936	*Mean dependent var*	0,029605
Adjusted R-squared	0,631703	*S,D, dependent var*	0,028453
S.E. of regression	0,017267	*Akaike info criterion*	-5,239235
Sum squared resid	0,048004	*Schwarz criterion*	-5,109070
Log likelihood	447,0957	*Durbin-Watson stat*	2,036129

Source: Eviews estimation results

The best model was chosen according the lowest Schwarz criterion. The coefficients are all significant at least on a 90% level, except for the constants. Once the STAR model is specified, it should be determined whether the series is best modelled as an LSTAR or an ESTAR process. As indicated earlier, products of the regressors with the powers of y_{t-d} are formed. Then an auxiliary regression is estimated. The F-statistic for this regression was found to be significant (2,95 with a p-value of 0,0003). Therefore non-linearity is rejected in favour of the alternative of a smooth transition model. To decide if this model has the LSTAR or the ESTAR form, the coefficients to the 3rd power are restricted using the Wald test. The null hypothesis is accepted according to the F-statistic and can therefore conclude that the model has the ESTAR form.

Table 4:Results of the Wald test to determine an LSTAR or an ESTAR form

Test Statistic	Value	df	Probability
F-statistic	1,861432	(4,147)	0,1203
Chi-square	7,445728	4	0,1141

Source: Eviews estimation results

- **The ESTAR model**

The basic STAR model was used, but the logistic function was replaced by the exponential function. The Wald test in Table 4 indicates that the null hypothesis is accepted and it can be concluded that the exponential function should be used. The final results are presented in Table 5.

Again, the best ESTAR model was determined by the lowest Schwarz criterion. In both the LSTAR and ESTAR specification all the coefficients (except for the constants) are at least significant on approximately a 90% significance level.

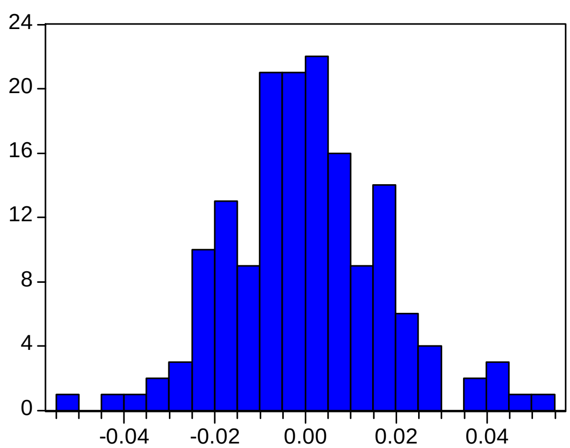
8.2.5 Diagnostic testing

The Jarque-Bera statistic in Figure 3 confirms that the residuals are normally distributed; (approximately at an 85% confidence interval). Therefore the STAR model explains movements in the real GDP growth. The little positive skewness and the 0,6 excess kurtosis might be due to the residuals corresponding to external shocks.

Table 5: Results for the ESTAR model

Dependent Variable: DLOG(GDP,0,4)			
	Coefficient	**Std. Error**	**t-Statistic**
$\boldsymbol{\alpha_0}$	**0,001465**	**0,003854**	**0,380151**
α_1	0,677382	0,076531	8,851059
α_2	0,170232	0,090429	1,882493
$\boldsymbol{\beta_0}$	**0,004716**	**0,004507**	**1,046474**
β_1	-0,552454	0,154168	-3,583454
β_2	0,267738	0,121171	2,209592
β_3	0,252310	0,131207	1,922995
$\boldsymbol{\theta}$	**-3415,845**	**3510,238**	**-0,973109**
R-squared	0,644132	*Mean dependent var*	0,029820
Adjusted R-squared	0,627743	*S.D. dependent var*	0,029071
S.E. of regression	0,017737	*Akaike info criterion*	-5,177643
Sum squared resid	0,047818	*Schwarz criterion*	-5,023884
Log likelihood	422,2114	*Durbin-Watson stat*	2,001725

Source: Eviews estimation results



Series: FINAL	
Sample 1962:2 2002:1	
Observations 160	
Mean	2.15E-12
Median	-0.000563
Maximum	0.050762
Minimum	-0.051166
Std. Dev.	0.017342
Skewness	0.239888
Kurtosis	3.607544
Jarque-Bera	3.995304
Probability	0.135653

Figure 3: Descriptive statistics for the residuals of the STAR model
Source: Eviews output

8.2.6 Forecast evaluation

A way of evaluating the estimated non-linear models is to do post-sample forecasting, even though the insight to be gained depends on what happens in the time series during the prediction period. Generally a wide range of values over the prediction period is needed to efficiently compare the forecasting performance of a STAR model to that of a linear autoregressive model. Teräsvirta and Anderson (1992) noted in their results that if the prediction period does not contain a clear recession, then the linear and non-linear forecasts are similar, except when the STAR fit is poor.

The forecasting period was from the second quarter of 2002 and the first quarter of 2004. All the models seem to fit the data well in-sample, with a static forecast. However, the out-of-sample forecast performance should be evaluated. A dynamic in-sample forecast was done. That is, the first forecasted value is used to determine the next forecast. The results are presented in the appendix to this paper.

When dynamic forecasts are done, the non-linear models far outperform the linear model in-sample. Most of the bias of the non-linear models lies within the covariance proportion, where with the linear model most of the bias is within the bias proportion (how far the mean of the forecast is from the mean of the actual series). For a forecast to be good, the bias and variance proportions should be small so that most of the bias is concentrated in the covariance proportions, which is the remaining unsystematic forecasting errors.

Moreover, to get the true forecasting performance, the forecast performance out-of-sample should be compared between the models. The results are presented in Figure 4.

Initially, all the models underestimated (second quarter 2002 and fourth quarter 2002) the actual value. From the graph it seems as if all the models predict the turning point well (fourth quarter 2003), but the RMSE for the ESTAR model was the lowest. The ESTAR model learns after 5 periods and is able to forecast the turning point. This is a good performance, as the out-of-sample period is quite extended (8 quarters). Even if the first forecast was out of line, it learned quickly and corrected itself to reflect the underlying economic conditions. The linear model, however, was not able to learn. The LSTAR and the TAR are both good forecasters and all three non-linear methods outperform the linear model. The accuracy of the non-linear models suggests that the dynamics of the process during a recession are different from those during an expansion (Teräsvirta and Anderson, 1992).

Clements and Franses (2003) posited that, if the world is considered to be inherently non-linear, then as computational capabilities increase more complex models become amenable to analysis, allowing the possibility that future generation of models will significantly outperform linear models, especially if such models become truly multivariate. It remains to be seen how this new model will perform in forecasting any subsequent South African recession.

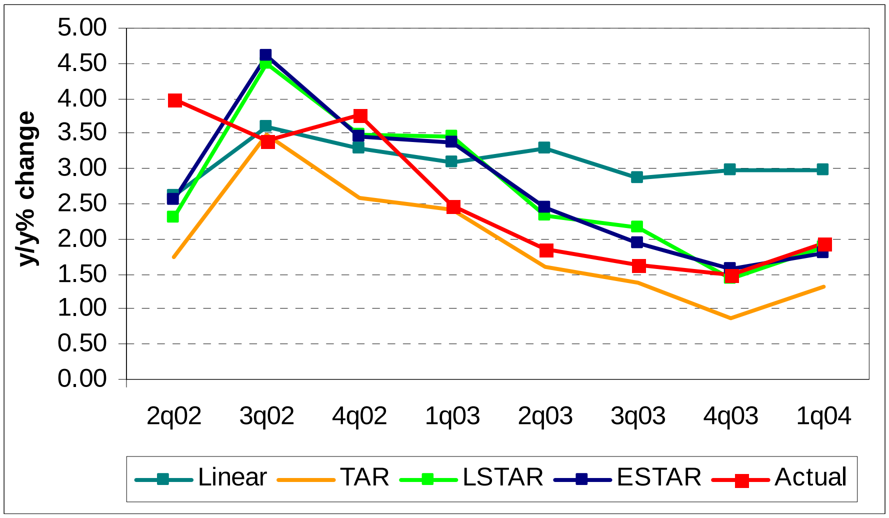


Figure 4: Comparison of models – out-of-sample
Source: South African Reserve Bank, Quarterly Bulletin (March 2004) and forecast results

From the root mean squared percentage errors in Table 6 it can be seen that the ESTAR outperforms the other models, since its RMSE% is the lowest (0.96).

Table 6: Root mean squared errors of various models

Models	Linear	TAR	LSTAR	ESTAR
RMSE	1687,942	2351,440	1852,017	1619,831
RMSE (%)	1,0021	1,3960	1,0995	0,9617

Source: Own calculations

9. Conclusion

In this paper the focus was on the nonlinear model building process concentrating especially on the family of STAR models. The South African GDP was used and AR, TAR, LSTAR and ESTAR models were fitted. The forecasting performance of the linear model and the STAR family non-linear models were compared. The non-linear models outperformed the linear model when the GDP was forecasted. The ESTAR model fitted the data best (lowest RMSE) and was able to forecast (out-of-sample) the turning point in the business cycle.

Thus to forecast the business cycle in South Africa a parameterized nonlinear model such as the family of STAR models, outperforms the simple regression model. This is due to asymmetric behaviour in the GDP data and the possibility of a threshold between a recession and an expansion which implies a certain state of the economy.

The business cycle in South Africa can be best modeled by parameterized non-linear methods.

These results support the structural or institutional view. They believe economic fluctuations are caused by various structural or institutional changes. Adherents to this view do not believe that the market system is inherently stable or systematically unstable (Classical vs. Keynesian view). They focus on structural changes and unpredictable events. They do not have set ideas on economic policy. According to them the appropriate policy will vary from time to time as circumstances change.

The following can be recommended. The form of the STAR model has not changed, but for future research the insignificant constants might be dropped from the STAR specification to get a better fit.

The STAR model can be expanded to include other explanatory variables and not only lags of the variable itself. In other words, a STAR-ADL (Smooth transition autoregressive distributed lag) model could be developed.

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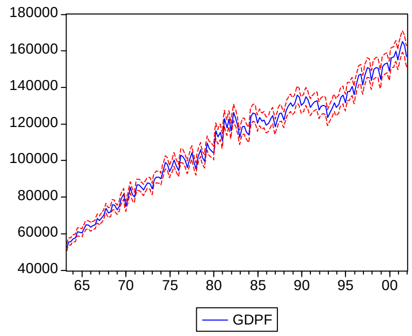
APPENDIX

Static in-sample evaluations

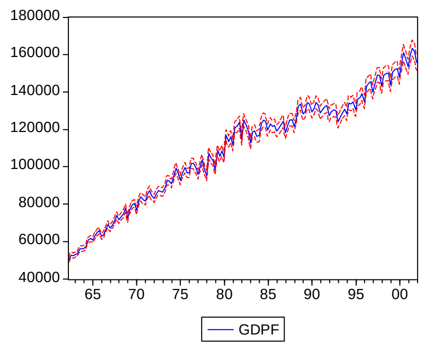
a. Linear model



Forecast: GDPF
Actual: GDP
Forecast sample: 1961:1 2002:1
Adjusted sample: 1963:2 2002:1
Included observations: 156

Root Mean Squared Error	1837.121
Mean Absolute Error	1407.418
Mean Abs. Percent Error	1.342663
Theil Inequality Coefficient	0.007996
Bias Proportion	0.002162
Variance Proportion	0.018594
Covariance Proportion	0.979245

b. TAR model

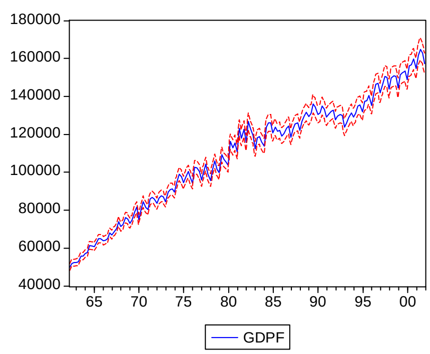


Forecast: GDPF
Actual: GDP
Forecast sample: 1961:1 2002:1
Adjusted sample: 1962:2 2002:1
Included observations: 160

Root Mean Squared Error	1387.470
Mean Absolute Error	1070.050
Mean Abs. Percent Error	0.988999
Theil Inequality Coefficient	0.006105
Bias Proportion	0.000157
Variance Proportion	0.007790
Covariance Proportion	0.992053

Static in-sample evaluations

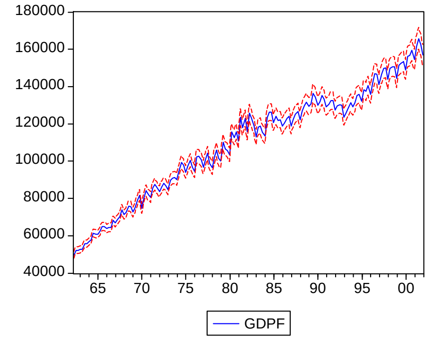
a. LSTAR model



Forecast: GDPF
Actual: GDP
Forecast sample: 1961:1 2002:1
Adjusted sample: 1962:2 2002:1
Included observations: 160

Root Mean Squared Error	1814.023
Mean Absolute Error	1368.725
Mean Abs. Percent Error	1.308576
Theil Inequality Coefficient	0.007975
Bias Proportion	0.002147
Variance Proportion	0.017382
Covariance Proportion	0.980471

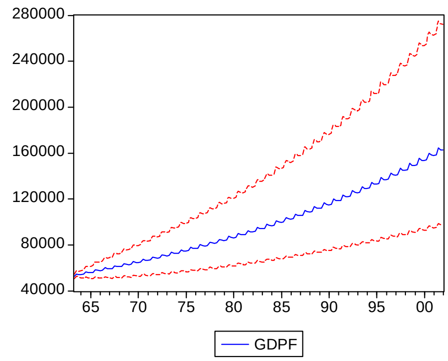
b. **ESTAR model**



Forecast: GDPF	
Actual: GDP	
Forecast sample: 1961:1 2002:1	
Adjusted sample: 1962:2 2002:1	
Included observations: 160	
Root Mean Squared Error	1819.393
Mean Absolute Error	1377.394
Mean Abs. Percent Error	1.316483
Theil Inequality Coefficient	0.007999
Bias Proportion	0.002064
Variance Proportion	0.015063
Covariance Proportion	0.982873

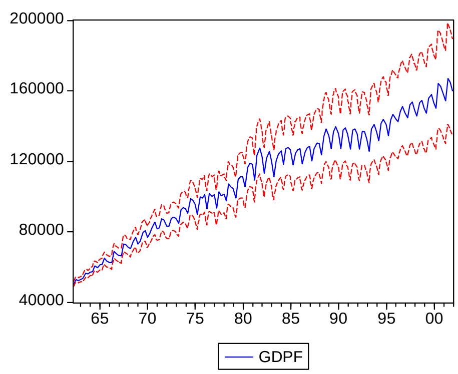
Dynamic in-sample evaluations

a. Linear model



Forecast: GDPF	
Actual: GDP	
Forecast sample: 1961:1 2002:1	
Adjusted sample: 1963:2 2002:1	
Included observations: 156	
Root Mean Squared Error	15318.23
Mean Absolute Error	12801.06
Mean Abs. Percent Error	12.41136
Theil Inequality Coefficient	0.070076
Bias Proportion	0.657916
Variance Proportion	0.047893
Covariance Proportion	0.294191

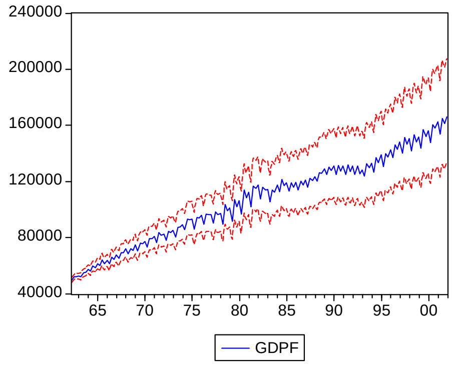
b. **TAR model**



Forecast: GDPF	
Actual: GDP	
Forecast sample: 1961:1 2002:1	
Adjusted sample: 1962:2 2002:1	
Included observations: 160	
Root Mean Squared Error	4789.766
Mean Absolute Error	3799.555
Mean Abs. Percent Error	3.289760
Theil Inequality Coefficient	0.020895
Bias Proportion	0.105799
Variance Proportion	0.106259
Covariance Proportion	0.787942

Dynamic in-sample evaluations

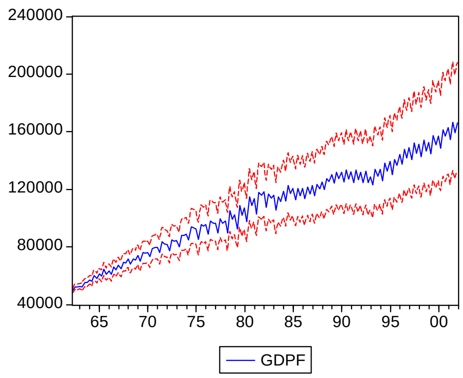
a. LSTAR model



Forecast: GDPF
Actual: GDP
Forecast sample: 1961:1 2002:1
Adjusted sample: 1962:2 2002:1
Included observations: 160

Root Mean Squared Error	5676.144
Mean Absolute Error	4372.276
Mean Abs. Percent Error	4.015632
Theil Inequality Coefficient	0.025240
Bias Proportion	0.232649
Variance Proportion	0.019318
Covariance Proportion	0.748033

b. ESTAR model



Forecast: GDPF
Actual: GDP
Forecast sample: 1961:1 2002:1
Adjusted sample: 1962:2 2002:1
Included observations: 160

Root Mean Squared Error	5832.481
Mean Absolute Error	4626.631
Mean Abs. Percent Error	4.225243
Theil Inequality Coefficient	0.025910
Bias Proportion	0.194139
Variance Proportion	0.029162
Covariance Proportion	0.776699