A Group Genetic Algorithm for the Fleet Size and Mix Vehicle Routing Problem

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Abstract - **In logistics management, the use of vehicles to distribute products from suppliers to customers is a major operational activity. Optimizing the routing of vehicles is crucial for providing cost-effective services to customers. This research addresses the fleet size and mix vehicle routing problem (FSMVRP), where the heterogeneous fleet and its size are to be determined. A group genetic algorithm (GGA) approach, with unique genetic operators, is designed and implemented on a number of existing benchmark problems. GGA demonstrates competitive performance in terms of cost and computation time when compared to other heuristics.**

Keywords - **Logistics, vehicle routing, genetic algorithms**

I. INTRODUCTION

In most supply chains, the management of distribution activities is a major operational task because distribution costs contribute a significant portion of the total operational costs. The need for effective distribution systems continues to come up in the supply chain industry due to escalating fuel costs. Distribution is a major part of logistics and a substantial cost to many companies. An effective distribution system can save millions of dollars each year. It can assist the decision maker in long-range planning, contract negotiations, and operations improvement. Hence, the development of effective and efficient distribution management systems is imperative.

In logistics industry, firms often require their vehicles to serve networks of hundreds of customers at various locations. As such, planning and scheduling can consume much time and effort, yet with little or no realizable cost efficiency. Several questions naturally arise: How many vehicles are needed to accommodate customer demand? What are the required vehicle capacities? What are the best routes? How best can customer demands be satisfied at the least possible cost? Due to multiple potential combinations of vehicle types and routing patterns, solutions to these questions are complex. This problem is a variant of the vehicle routing problem (VRP) [1].

The rest of the paper is as follows: Section II gives a brief outline of the VRP. Section III describes the fleet size and mix VRP (FSMVRP). A group genetic algorithm is proposed in Section IV. Computational tests and results are given in Section V. Section VI concludes the paper.

II. THE VEHICLE ROUTING PROBLEM

The VRP, first studied by Dantzig and Ramser [1], mainly seeks to minimize transportation costs, number of vehicles used, and customer waiting times. Since its inception in the 1950s, other VRP variants followed. The capacitated VRP (CVRP) is concerned with optimizing the dispatch of goods required by customers, using a fleet of capacitated homogenous vehicles [2]. Another variant is VRP with time window constraints (VRPTW), where arrival after the latest time window is penalized [3]. VRP problems with heterogeneous vehicles are frequently encountered in logistics industry. The heterogeneous fixed fleet VRP (HFFVRP) is a CVRP variant with a fixed number of available vehicles. The decision involves how best to utilize the existing vehicle fleet [4]. On the other hand, the FSMVRP is a CVRP variant where the fleet size and its composition are to be determined [5].

III. THE FSMVRP PROBLEM DISCRIPTION

Formally, the FSMVRP can be described as follows. There are n customer locations, $\{1, 2,..., n\}$. A fleet of T vehicle types are available at the depot, represented by 0. The number of vehicles for each type is unlimited, and one of the decisions is to determine the number of vehicles of each type. Each vehicle type t has a capacity Q_t, a fixed cost f_t and a variable cost per unit distance v_t. Assume that between two vehicle types a and b, we have $f_a < f_b$ if $Q_a < Q_b$. Two cost structures exist: (i) different fixed costs with uniform variable costs [4-9], and (ii) different variable costs with no fixed costs [10-12]. For the FSMVRP with fixed costs and uniform variable costs;

$$v_t = 1; \quad f_t > 0 \quad \forall t \tag{1}$$

For the FSMVRP with variable costs and no fixed costs;

$$v_t > 0; \quad f_t = 0 \quad \forall t \tag{2}$$

Each customer node $i > 0$ has a non-negative demand d_i. Let the travelling distance between location i and j be non-negative $\tau_{i,j}$. These distances are symmetric and satisfy the inequality, $\tau_{i,j} = \tau_{j,i}$ and $\tau_{i,j} + \tau_{j,k} \geq \tau_{i,k}$. Thus, the total variable cost of travelling from location i to location j is $v_t\tau_{i,j}$. The FSMVRP consists in determining the vehicle fleet composition and the route of each vehicle, so that the total cost of delivering goods to all customers is minimized; each route starts and ends at the depot; each customer is visited exactly once; customer demands are

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satisfied; and vehicle capacity is not violated. Owing to FSMVRP complexity, the use of exact methods on large-scale instances is not viable. Not surprisingly, most approaches rely on heuristics that obtain good solutions, including tabu search [6] [7], memetic algorithm [11], genetic algorithm [11], particle swarm optimization [14], and evolutionary algorithm [12]. We propose a group genetic algorithm (GGA) to address the FSMVRP.

IV. GROUP GENETIC ALGORITHM APPROACH

We describe GGA and its elements, including chromosome coding, initialization, and genetic operators.

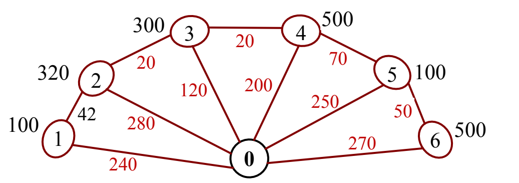
A. GGA Coding Scheme

The GGA performance strongly depends on the type of the coding scheme used. While most authors use depot(s) as trip delimiters [11] [12], a few do not use delimiters [15]. We develop our coding scheme from the later. The evaluation of a chromosome $k = [1, 2, 3, \ldots, n]$ involves partitioning customer orders along k into groups so that the cumulative load for each group does not exceed the vehicle capacity, and the cumulative delivery cost incurred is minimized. This is represented by a graph, $G(X)$ with vertex $V(G) = \{i \mid 0 \le i \le n\}$. Let $E(G)$ be the set of directed arcs on $G(X)$, where $(i,j) \in E(G)$ iff $\sum_{m=i+1}^{j} d_m \le Q_t$, and t be the vehicle type chosen. Each arc (i,j) represents a feasible trip, where the vehicle departs from node 0 (depot) and visits nodes i+1, i+2,…, j-1, and j, consecutively. The total load for trip (i,j) is given by $\sum_{m=i+1}^{j} d_m$. The objective is to select a vehicle type t with the least cost and capacity not less than the trip load. Then, for the FSMVRP with fixed cost, trip cost $c_{i,j}$ is equivalent to fixed cost plus variable costs [5];

$$c_{i,j} = f_t + \tau_{0,i+1} + \sum_{h=i+1}^{j-1} \tau_{h,h+1} + \tau_{j,0} \tag{3}$$

On the other hand, for the FSMVRP with no fixed cost;

$$c_{i,j} = \left(\tau_{0,i+1} + \sum_{h=i+1}^{j-1} \tau_{h,h+1} + \tau_{j,0}\right) \cdot v_t \tag{4}$$



Vehicle $t1$: $Q_1 = 500, f_1 = 300$; Vehicle $t2$: $Q_2 = 550, f_2 = 400$

Fig. 1. Typical data for chromosome representation

Consider a typical distribution center with $T = 2$ unlimited vehicle types to serve $n = 6$ retailers (see Fig. 1). The numbers on arc(i,j) and node j represent $\tau_{i,j}$ and d_i, respectively. The capacities of vehicle types $t1$ and $t2$ are $Q_1 = 500$ and $Q_2 = 550$, respectively. Their fixed costs are $f_1 = 300$ and $f_2 = 400$, respectively. The proposed GGA uses a group structure for each feasible solution based on three codes (see Fig. 2). Code 1, of size n, is a group structure upon which the genetic operators act. Code 2 shows the vehicle type assigned to each trip, while code 3 represents the position of the last node of each trip.

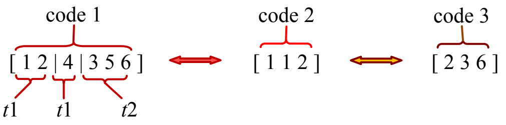


Fig.2. Chromosome representation

The chromosome [1 2 4 3 5 6 · 1 1 2], comprises codes 1 and 2 ("·" demarcates codes 1 and 2). According to code 1, vehicle type 1, with $f_1 = 300$ and $v_t = 1$, is assigned trip (0-1-2-0). From (3) the total cost for this trip is 300 + 240 + 42 + 280 = 862. Other trips, (0-4-0) and (0-3-5-6-0), are evaluated in a similar manner, as shown in Table I.

TABLE I.
GGA CODING SOLUTION EXAMPLE

Trip	Vehicle type	Cost
0-1-2-0	1	862
0-4-0	1	700
0-3-5-6-0	2	930
Total Cost		2492

B. Initialization

An initial population of the desired size, *popsize*, is produced by (i) savings [16] and sweep heuristics [17], and (ii) random generation. The savings algorithm is applied using one vehicle type at a time. The sweep algorithm is also used to generate initial solutions. These initial solutions are then concatenated into chromosomes. More chromosomes are generated as follows;

```
Repeat
1. Assign a location to each vehicle $t$, (t = 1,2,…, $m$)
2. Randomly assign the remaining locations,
3. Encode the string and add to initial population,
Until (population size = popsize).
```

The GGA approach minimizes some cost function f which is mapped to a score function, as suggested in [23];

$$f_k(\tau) = \max[0, f^{\max} - g_k(\tau)] \tag{5}$$

where, $g_k(\tau)$ is the objective function of chromosome k at time τ and $f^{\max}$ is the largest objective function.

C. Selection Operator

Several selection strategies have been suggested by Goldberg [18], including deterministic sampling, remainder stochastic sampling with/without replacement, and stochastic tournament. The remainder stochastic sampling without replacement is applied in this work; each chromosome k is selected and stored in the mating pool according to its expected count e_k calculated thus;

$$e_k = \frac{f_k}{\left(1/popsize\right)\sum_{k=1}^{popsie} f_k} \tag{6}$$

Here, f_k is the score function of the k^{th} chromosome. Each chromosome receives copies equal to the integer part of e_k, that is, $[e_k]$, while the fractional part frac(e_k) is treated as a success probability of obtaining additional copies of chromosome k into the mating pool.

D. Crossover Operator

Crossover is an evolutionary mechanism by which selected chromosomes mate to produce new offspring, called *selection pool*. This enhances exploration of unvisited regions in the solution space. The proposed group crossover operator exchanges groups of genes of selected chromosomes (see Fig.3), with probability *prcoss*, until the desired pool size, *poolsize*, is obtained:

Repeat
1. Generate the crossover point in (1, g-1), g = trips.
2. Swap the groups to the right of the crossover point.
3. Repair the offspring, if necessary.

Until (selection *poolsize* is achieved).

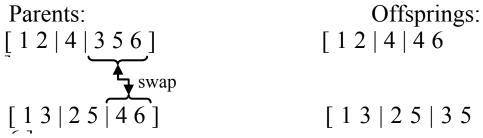


Fig. 3: Crossover operator

After crossover, some customers may appear in more than one trip, while others may be missing. Such offspring should be repaired by eliminating duplicated customers to the left of the crossover point (see Fig.4) and inserting missing ones into the trip with the least loading. Thus, group coding takes advantage of the group structure. The basic single-point crossover is applied on code 2.

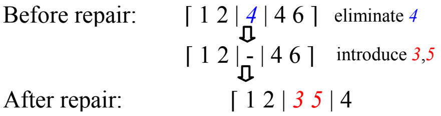


Fig. 4: Chromosome repair mechanism

E. Mutation Operator

Mutation is applied to every new chromosome using two mutation operators; *swap mutation* and *shift mutation*. The swap mutation operates by exchanging genes between two groups in a chromosome according to the following procedure:

1. Randomly select two numbers from set {1, 2,…,g};
2. Randomly choose a gene from each group;
3. Swap the selected genes.

Fig. 5 illustrates the swap mutation mechanism.

Offspring chromosome	:	[1 2 | 4 | 3 5 6]
Select group or trip	:	2 and 3
Select genes or nodes	:	**4** and **6**
Mutated offspring	:	[1 2 | 6 | 3 5 4

Fig. 5. Swap mutation

The shift mutation works by shifting the frontier between two adjacent groups by one step, either to the right or to the left (see Fig. 6) as follows;

1. Randomly generate the frontier in (1, g-1).
2. Randomly choose the shift direction: *right* or *left*.
3. Shift the frontier in the selected direction.

offspring chromosome	:	[1 2 | 4 | 3 5 6]
select frontier, rand (1,2)	:	1 ↖ shift frontier
select direction	:	ieit
mutated offspring	:	[1 | 2 4 | 3 5

Figure 6: Shift mutation operator

As for the genes that correspond to code 2, a basic mutation operator is applied by replacing a randomly selected gene with a randomly generated integer in {0, 1,…,T}. Mutation essentially provides GGA with a *local search* capability, called *intensification*. However, shift mutation is a more localized search than swap mutation.

F. Inversion Operator

To curb premature convergence, inversion is applied at a low probability on selected chromosomes. Inversion rearranges chromosome groups in a reverse order (Fig. 7).

Before inversion	:	[1 2 | 4 | 3 5 6]
After inversion	:	[3 5 6 | 4 | 1 2]

Fig. 7: Inversion operator

G. Diversification

As iterations proceed, the population converges to a particular solution. Premature convergence may occur before an optimal solution is obtained. To check diversity, define an entropic measure H_i for each location i;

$$H_i = \sum_{j=1}^{m} \frac{(n_{ij}/popsize)\cdot\log(n_{ij}/p\,opsize)}{\log(m)} \tag{7}$$

where n_{ij} is the number of chromosomes in which location i is assigned position j in the current population; m is the number of locations. Then, diversity H is defined as,

$$H = \sum_{i=1}^{m} H_i/m \tag{8}$$

Therefore, inversion is applied to improve diversity to a desired value. The best candidates from diversified and undiversified populations are always preserved.

TABLE II
SPECIFICATIONS FOR THE BENCHMARK PROBLEMS

No.	Q_1	f_1	v_1	Q_2	f_2	v_2	Q_3	f_3	v_3	Q_4	f_4	v_4	Q_5	f_5	v_5	Q_6	f_6	v_6
3	20	20	1.0	30	35	1.0	40	50	1.0	70	120	1.0	120	225	1.0			
4	60	1000	1.0	80	1500	1.0	150	3000	1.0									
5	20	20	1.0	30	35	1.0	40	50	1.0	70	120	1.0	120	225	1.0			
6	60	1000	1.0	80	1500	1.0	150	3000	1.0									
13	20	20	1.0	30	35	1.1	40	50	1.2	70	120	1.7	120	225	2.5	200	400	3.2
14	120	100	1.0	160	1500	1.1	300	3500	1.4									
15	50	100	1.0	100	250	1.6	160	450	2.0									
16	40	100	1.0	80	200	1.6	140	400	2.1									
17	50	25	1.0	120	80	1.2	200	150	1.5	350	320	1.8						
18	20	10	1.0	50	35	1.3	100	100	1.9	150	180	2.4	250	400	2.9	400	800	3.2
19	100	500	1.0	200	1200	1.4	300	2100	1.7									
20	60	100	1.0	140	300	1.7	200	500	2.0									

G. The GGA Implementation

The overall GGA (Fig. 8) incorporates the operators described in previous sections, using carefully chosen genetic probabilities: crossover (0.4), mutation (0.01), and inversion (0.05) as in [21].

```
GGA Algorithm:
BEGIN
1:  Input: initial data input: select GGA parameters;
2:  Initial population, oldpop: create chromosomes;
      (i)   savings and sweep algorithms; then, (ii) random generation;
Repeat
3:  Selection/recombination:
      (i)   evaluate strings by fitness function;
      (ii)  create temporal population, temppop: use int [$e_k$], then frac($e_k$)
4:  Group crossover/recombination:
      (i)   select 2 strings by selection strategy from temppop.
      (ii)  apply crossover operator to the 2 strings.
      (iii) if successful, apply inversion, else go to 5.
      (iv)  apply repair mechanism if necessary.
5:  Mutation: mutate and  move offspring to new population, newpop;
6:  Replacement strategy:
      (i)   compare selection pool spool and oldpop strings, successively;
      (ii)  take the one that fares better in each comparison;
      (iii) select the rest of the strings with probability 0.55;
7:  Diversification:
      (i)   calculate population diversity $H$;
      (ii)  if ($H < H_{\min}$) then diversify till $H \geq H_{\min}$;
      (iii) re-evaluate strings by fitness function;
8:  New population:
      (i)   oldpop = newpop
      (ii)  advance population, gen = gen + 1
Until (gen ≥ maxgen)
END
```

Fig. 8: Proposed GGA Implementation

V. COMPUTATIONAL TESTS AND DISCUSSIONS

A. Problem Sets

The proposed GGA was implemented in Java and executed on a Pentium 4 at 3GHz based on 12 benchmark problems in [20]. Table II specifies the costs, vehicle capacity Q_t, (t = 1, 2,…, 6); fixed cost f_t; and variable cost v_t. Using the notation in [5], problem sets 3 to 6 have 20 customers, 13 to 16 have 50, 17 to 18 have 75, and 19 to 20 have 100. Computational results were compared with those from best-performing heuristics in the literature, including tabu search [21] [7] [9], column generation-based heuristics [8], and evolutionary algorithm [12].

B. Computations, Results and Discussions

Table III presents the computational results of the FSMVRP with fixed costs. A count of the best-known solutions obtained by the heuristics is provided. Out of the 12 benchmark problems, our GGA approach produced 11 best known solutions, compared to only 6 found in [21], 6 in [7], 5 in [15], 8 in [8], 9 in [9], and 10 in [5].

Table IV presents the percentage deviation of each solution from the best-known and the computation times for each problem. All algorithms showed remarkable accuracy, with average percent deviation less than 1%. Our GGA performed competitively in terms of percentage deviation and computation times. The results demonstrate the utility of the GGA developed in this research.

VI. CONCLUSIONS AND FURTHER RESEARCH

TABLE III
COMPUTATIONAL RESULTS FOR THE BENCHMARK PROBLEMS

No.	Best known	Gendreau et al. (1999)	Wassan and Osman (2002)	Lima et al. (2004)	Choi and Tcha (2007)	Brandao (2008)	Liu et al. (2009)	GGA (2012)
3	961.03	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**
4	6437.33	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**
5	1007.05	**1007.05**	**1007.05**	**1007.05**	**1007.05**	**1007.05**	**1007.05**	**1007.05**
6	6516.47	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**
13	2406.36	2408.41	2422.10	2408.60	**2406.36**	**2406.36**	**2406.36**	**2406.36**
14	9119.03	**9119.03**	9119.86	**9119.03**	**9119.03**	**9119.03**	**9119.03**	**9119.03**
15	2586.37	**2586.37**	**2586.37**	2586.88	**2586.37**	**2586.37**	**2586.37**	**2586.37**
16	2720.43	2741.5	2730.08	2721.76	**2720.43**	2728.14	2724.22	**2720.43**
17	1734.53	1749.5	1755.1	1758.53	1758.53	**1734.53**	**1734.53**	**1734.53**
18	2369.65	2381.43	2385.52	2396.47	2371.49	**2369.65**	**2369.65**	**2369.65**
19	8659.74	8675.16	**8659.74**	8691	8664.29	8661.81	8662.95	8661.81
20	4038.46	4086.76	4061.64	4093.29	4039.49	4042.59	**4038.46**	**4038.46**
Bests:	12	6	6	5	8	9	10	11

TABLE IV
PPERCENT DEVIATIONS AND CPU TIMES FOR FSMVRP ALGORITHMS

No.	Best	Gendreau et al. (1999)		Lima et al. (2004)		Choi and Tcha (2007)		Liu et al. (2009)		GGA (2012)	
	known	Deviation	Time (s)	Deviation	Time (s)	Deviation	Time (s)	Deviation	Time (s)	Deviation	Time (s)
3	961.03	0.000	164	0.000	89	0.000	0	0.000	0	0.000	0
4	6437.33	0.000	253	0.000	85	0.000	1	0.000	0	0.000	1
5	1007.05	0.000	164	0.000	85	0.000	1	0.000	2	0.000	2
6	6516.47	0.000	309	0.000	85	0.000	0	0.000	0	0.000	0
13	2406.36	0.085	724	0.093	559	0.000	10	0.000	91	0.000	89
14	9119.03	0.000	1033	0.000	669	0.000	51	0.000	42	0.000	65
15	2586.37	0.000	901	0.020	554	0.000	10	0.000	48	0.000	55
16	2720.43	0.775	815	0.049	507	0.000	11	0.139	107	0.000	10
17	1734.53	0.863	1022	1.384	1517	1.384	207	0.000	109	0.000	113
18	2369.65	0.497	691	1.132	1613	0.078	70	0.000	197	0.000	211
19	8659.74	0.178	1687	0.361	2900	0.053	1179	0.037	778	0.024	804
20	4038.46	1.196	1421	1.358	2383	0.026	264	0.000	1004	0.000	1047
Averages:		0.3012%	776.9	0.3171%	887.4	0.1462%	172.4	0.0176%	209.0	0.0024%	209.7

FSMVRP involves the determination the fleet size and the mix of heterogeneous vehicles, assuming that the number of vehicles of each type is unlimited. This paper presents a GGA for solving the FSMVRP with fixed and variable costs. The approach obtained best-known solutions based on the comparative analysis tests on benchmark problems. Moreover, the GGA approach performed competitively within a reasonable computation time. In terms of the average solution cost, GGA demonstrated competitive performance.

This research contributes logistics and transportation. The current GGA uses unique group genetic operators, demonstrating its competitive performance when compared to related approaches in the literature. Possible further research directions include the design of more efficient algorithms for solving the FSMVRP problem in which the customer demand is uncertain or fuzzy.

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