FUZZY SYSTEM DYNAMICS SIMULATION FOR MANUFACTURING SUPPLY CHAIN SYSTEMS WITH UNCERTAIN DEMAND

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ABSTRACT

Real-world manufacturing supply chain systems are characterised by imprecise and dynamic factors. As a result, decision-making takes place in a complex, dynamic and fuzzy environment in which managerial goals and the impacts of possible actions are not precisely known. In a demand driven manufacturing supply chain system, the presence of a fuzzy demand is a serious cause for concern. The present study integrates fuzzy theory and system dynamics simulation to address the fuzzy and dynamic nature of demand-supply factors, from a systems perspective. A set of performance indices were defined to evaluate the system performance. Based on typical demand scenarios, comparative simulation experiments were conducted using the base scenario as a benchmark. The simulation results show the utility of the fuzzy system dynamics approach: (a) the approach represents the real-world picture of a supply chain with fuzzy demand, (b) the supply chain system performs better under dynamic fuzzy policies, and (c) computational "what-if analysis" showed that dynamic fuzzy-based policies are more robust than conventional crisp rules, even in turbulent demand situations. Further managerial insights and practical evaluations are provided in this study.

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1 INTRODUCTION

In the real world, supply chain systems are characterised by a number of dynamic, imprecise, and humanistic factors that play a significant role in their overall behaviours [1-3]. Most of the managers have to make decisions in such dynamic fuzzy environments in which the goals, the constraints and the impacts of possible actions are not precisely known. In a demand-driven supply chain system, with imprecise demand information and ambiguous variable descriptions, designing robust supply chain strategies and policies is a difficult task that calls for reliable decision support tools.

For many manufacturing supply chains characterised by high demand uncertainty, robust demand-supply policies are critical. However, due to external uncertainties in the market, it is difficult to precisely determine future demand, which creates a challenge for many supply chain decision makers. As a result, the development of effective dynamic policies for supply chain strategies is extremely difficult in a dynamic environment characterised by uncertain demand. To obtain reasonable results in uncertain environments, it is necessary to include management judgement, with a systems view.

The major difficulty in designing a supply chain structure is in deciding strategy for capacity augmentation. Clearly, under uncertainty of market demand, manufacturers tend to adopt a cautious approach to capacity build up. The most common idea is to run the plant at maximum utilization with the aim of reducing per unit production cost and invest in capacity as necessary. However, due to variation in demand there exists a trade-off between capacity investment and inventory holding. In no uncertain terms, the variation in demand increases as one moves up the supply chain. Uncertain fluctuations and amplifications of orders and inventory are often experienced mainly due to lack of timely information sharing on production caused by delays and feedback in the decision rules among the enterprises of the supply chain. This phenomenon, known as the bullwhip effect has been extensively studied by Lee et al. [4]. The same phenomenon is popularly explained in terms of Forrester effect [5], as demonstrated in Figure 1. System dynamics concepts and control theoretic approaches have been used to explain the demand implication effects.

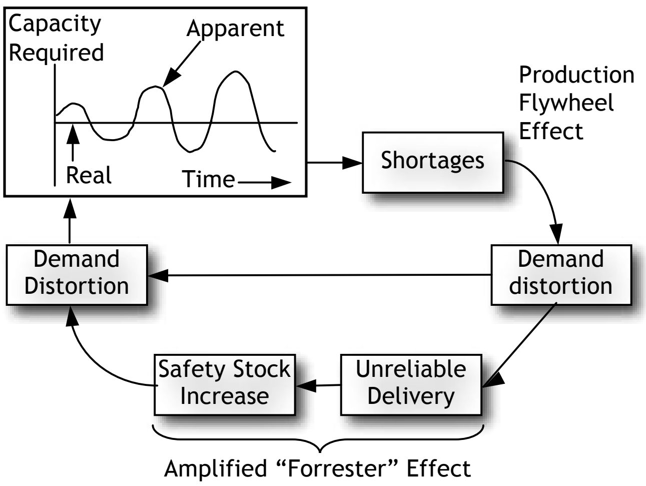

Figure 1: The Forrester Effect (Houlihan, 1987) [2]

System dynamics, originated by Forrester in the 1960s [5], was developed as a methodology to study and model complex industrial and social systems using principles from control engineering. It is based on a theory of system structure and a set of tools for representing complex systems and analysing their dynamic behaviour. As demonstrated by Angerhofer [6] and Sterman [7], the tool has been widely applied to supply chain management problems in the literature. However, as observed in a number of studies [1-8], the use of basic system dynamics and control theoretic approaches leads to unwanted fluctuations (bull-whip effects) in the supply chain. Therefore, inherent fuzzy behaviour has to be included in modelling fuzzy systems in order to include more realism into the model [9-11]. In this study, we move a step forward in determining how the manufacturer can use limited information from the retailer to make decisions about capacity adjustment, in the presence of high imprecision in the available information. We base this view on the fact that demand information is in most cases imprecise due to lack of past data, lack of similar demand information, or vague information. The end goal is to ensure that variations between demand and supply are minimized and unwanted fluctuations are minimized as much as possible. Therefore, the major purpose of this study is to develop a fuzzy system dynamics model for a typical manufacturing supply chain with uncertain demand. To achieve this aim, the objectives of this research are:

- to develop a causal loop diagram that represents the causal linkages between the main variables of the system;
- to develop a fuzzy system dynamics model that captures expert judgement using Simulink on a Matlab platform; and,
- to perform illustrative simulation study based on typical sample demand input patterns.

The remainder of this paper is structured as follows: In section 2, we provide a brief background description of a manufacturing supply chain system with a fuzzy demand. In section 3, a fuzzy system dynamics model is proposed for the supply chain system. Section 4 presents the experimental design used to illustrate the application of the proposed model, based on common sample demand inputs. Results and the relevant discussion are presented in Section 5, together with managerial insights into the application of the fuzzy concepts and system dynamics in typical manufacturing supply chains. Finally, Section 6 concludes the paper, summarising the main contributions of this study.

2 PROBLEM BACKGROUND

In this study, we simulate the activities of a manufacturing supply chain in which a manufacturing firm operates a make-to-order system. This implies that customer orders arriving at a rate *OrderRate* accumulate as backlog till they could be produced and shipped. The actual average delay *DeliveryDelay* in delivering orders, that is, the mean residence time of orders in the backlog, is dependent on the relative ratio between the backlog *Backlog* and the current shipment rate *ShipRate*. Meanwhile, the firm perceives delay *PerceivedDelay*, and this builds pressure on capacity adjustments. The desired capacity is augmented according to the perceived pressure obtained by comparison of perceived delay and the company target for delivery target *DeliveryTarget*. As such, the capacity, *Capacity*, is augmented with a delay *cap_AT*. In turn the actual capacity build-up influences the actual shipment rate, depending on the allowable capacity utilisation rate *capacity utilization*.

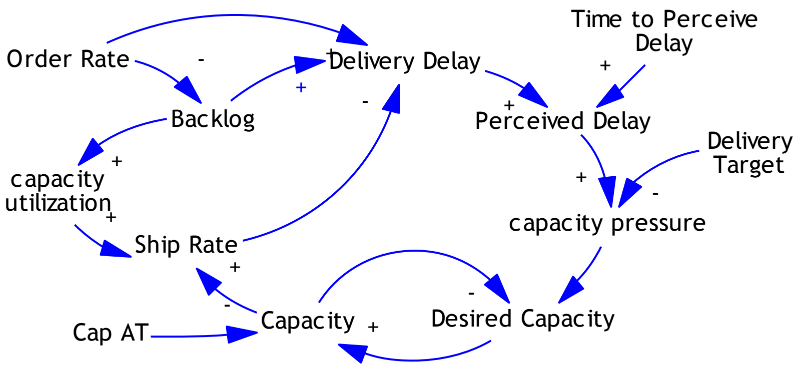

Figure 2: Supply Chain Causal Loop Diagram

The main aim is to build the capacity in accordance with the perceived information in regards to backlog and delivery delays. This, however, is subject to the minimum delay set up by the manufacturing firm. Figure 2 represents a causal loop diagram for the manufacturing supply chain used in this paper.

3 FUZZY SYSTEMS DYNAMICS MODELLING

In this section, we present a brief background to fuzzy theory, introducing the concepts fuzzy logic relevant to fuzzy and dynamic manufacturing supply chains.

3.1 Background to fuzzy logic concepts

Fuzzy logic is a logical system that utilises the theory of fuzzy sets, a theory which relates to classes of objects with un-sharp boundaries in which membership is a matter of degree. Fuzzy logic is built on top of the experience of experts who already understand the system under study. It is built on the structures of qualitative description used in everyday natural language, which makes it easy to use. Oftentimes, systems do not have enough precise data to allow statistical analysis which normally demand data collection over a long time. Fuzzy logic, being tolerant of imprecise data, builds this understanding into the process rather than tacking it onto the end. Moreover, fuzzy logic can model nonlinear functions of arbitrary complexity.

From a mathematical point of view, fuzzy sets are a generalization of classical notion of sets. If X is the universe of discourse and its elements are denoted by x, then a fuzzy set A in X is defined as a set of ordered pairs;

$$A = \{x, \mu_A(x) \mid x \in X\} \tag{1}$$

where, $\mu_A(x)$ is called the membership function of x in A, which maps each element of X to a membership value in the interval [0,1].

In this study, we use the Matlab fuzzy logic toolbox which has built-in membership function types, which are, in turn, built from several basic functions such as piece-wise linear functions, Gaussian distribution function, and sigmoid curve. Fuzzy sets and fuzzy operators "and", "or", and "not" are the subjects and verbs of fuzzy logic. These if-then rule statements are used to formulate the conditional statements that comprise fuzzy logic. A single fuzzy if-then rule assumes the form: if x is A then y is B, where A and B are linguistic values defined by fuzzy sets on the ranges (universes of discourse) X and Y, respectively. Her, "x is A" is called the *antecedent*, while "y is B" is known as the *consequent*. This provides strong constructs for *fuzzy inference*.

Fuzzy inference is the process of formulating the mapping from a given input to an output based on some fuzzy logic *set of rules* [12, 13]. The mapping provides a basis from which decisions can be made based on a set of linguistic control rules obtained from experienced decision makers. The process of fuzzy inference involves the following constructs: membership functions, logical operations, as well as if-then rules. The fuzzy inference process involves crisp (non-fuzzy) inputs, linguistic (fuzzy) rules, and defuzzifier and the crisp output. More specifically, the actual fuzzy inference process consists of five parts, namely [13]:

(i) fuzzify input variables: determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions;

(ii) apply fuzzy operator: fuzzy operator (AND or OR) in the antecedent to determine the degree to which each part of the antecedent is satisfied for each rule;

(iii) apply implication method: apply implication on a single number given by the antecedent, to obtain the output as a fuzzy set;

(iv) aggregation of all outputs: fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set through aggregation; and,

(v) defuzzification: conversion of a fuzzy set (the aggregate output fuzzy set) into a single output number upon which a decision will be based.

In the next section, we implement the concepts of fuzzy theory, fuzzy inference, and system dynamics to develop our fuzzy system dynamics model for the manufacturing supply chain problem with a fuzzy customer demand.

3.2 Fuzzy system dynamic model

In system dynamics modelling, the simplest possible model is always used to capture the mental models of policy makers who are experienced in the field of interest. In this regard, fuzzy system dynamics goes a step further by representing fuzzy input and decision rules of expert top management executives, allowing for human judgement to be included into the fuzzy system dynamics model. The concepts of fuzzy system dynamics modelling are applied to capture dynamic and fuzzy variables in a manufacturing supply chain.

Figure 3 shows the overall block diagram for the manufacturing supply chain system. The main variables are the *OrderRate* which is an exogenous demand input to the model. Various demand patterns can be represented. Another important variable is the delivery delay *DelDelay* which is influenced by the backlog *Backlog* and the shipment rate *ShipRate* in accordance with the following expression [8];

$$DelDelay = Backlog / ShipRate \quad (2)$$

In practice, supply chain managers have a pre-set minimum delay, called, *MinDelay*, which acts as the minimum acceptable delay below which action has to be taken to augment the capacity of the supply chain.

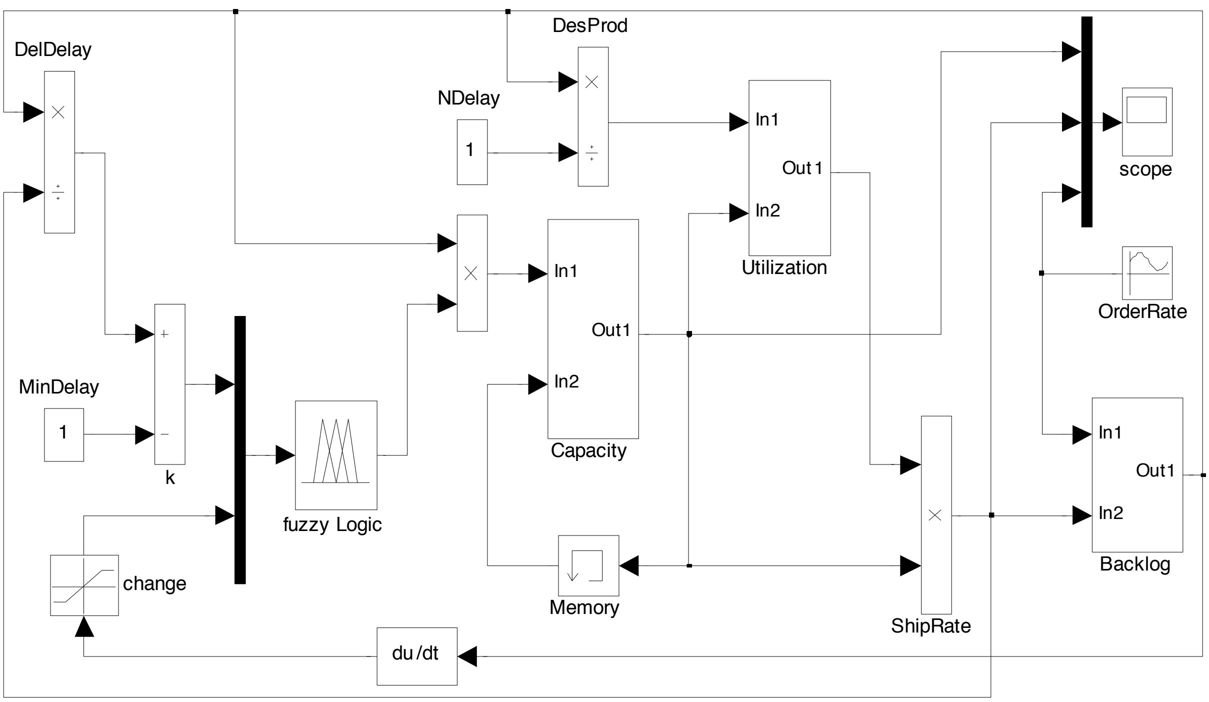

Figure 3: The Block Diagram For The Fuzzy System Dynamics Model

In order to capture the mental model of the fuzzy control of the capacity of the supply chain, two important input variables are defined. The first variable is defined by the magnitude of error between *OrderRate* and *ShipRate*. Ideally, the order rate should equal the shipping rate. Therefore, the preferred error should be as close to zero as possible. The second variable is defined by the current rate of change of *Backlog*. We define a control variable k to represent the normalised level of deviation from zero. We assume that the decision maker wishes to have a minimum delivery delay *MinDelay* of 1 time unit. In this respect, we define preferred error k as follows,

$$k = \frac{Backlog}{ShipRate} - MinDelay \tag{3}$$

Setting the *MinDelay* to 1, the expression can be simplified as follows:

$$k = DelDelay - 1 \tag{4}$$

where, *DelDelay* is as defined in (1).

Since *OrderRate* is supposed to be as close as possible to *ShipRate*, the range of values of the preferred *error* k is expected to vary between -1 and 1. However, error values close to zero are most preferable. The level of preference diminishes as the values of k approach 1 or -1. In this development, we represent this situation as a fuzzy relationship as demonstrated in Figure 4;

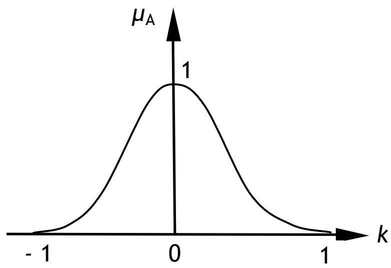

Figure 4: Fuzzy set (preferred error)

In addition to the error k explained above, we define the rate of change of backlog, *rate*, as follows;

$$rate = \frac{d}{dt}(Backlog) \tag{5}$$

The rate variable defines whether *Backlog* is increasing or decreasing. It follows that if the rate is increasing, then the corresponding capacity adjustment is amplified. On the other hand, if the rate is decreasing, then the desired capacity adjustment should be minimal. Based on the fuzzy relationship in Figure 4 and the concept of rate of change of *backlog*, *rate*, a fuzzy rule base is built to represent the fuzzy control of the capacity build-up for the manufacturing supply chain. Let *cap_change* denote the desired capacity adjustment. The fuzzy rule base is outlined as follows;

Rule 1: IF (*error* is ok) THEN (*cap_change* is zero);

Rule 2: IF (*error* is low) THEN (*cap_change* is reduce fast);

Rule 3: IF (*error* is high) THEN (*cap_change* is increase fast);

Rule 4: IF (*error* is ok) and (rate is positive) THEN (*cap_change* is reduce slowly);

Rule 5: IF (*error* is ok) and (rate is negative) THEN (*cap_change* is increase slowly);

In accordance with rule1, the desired capacity adjustment is zero since *cap_change* is zero. As for rule 2, the preferred *error* is low (or negative). This implies that the desired capacity adjustment is to reduce (ideally) the current capacity fast since backlog is very low. Conversely, when the prerred *error* is high (or positive), the implication is that the current capcity should be increased fast. Furthermore, if the error is “ok,” that is, in the neighbourhood of zero, then the consequent decision depends on whether the current trend (*rate*) of *Backlog* is increasing (positive) or decreasing (negative). If *rate* is positive then capacity should ideally be reduced slowly. On the other hand, if rate is negative, then capacity should be increased slowly since the trend shows that the backlog is somewhat on the increase. All these linguistic variables are coded into the Fuzzy tool box and Simulink on a matlab platform.

In modelling the fuzzy supply chain system, the above set of fuzzy rules were implemented in Simulink on a matlab platform. As explained by Zadeh [10], fuzzy theory concepts are introduced to handle uncertain, fuzzy, or linguistic variables. A linguistic or fuzzy variable represents the ranges of values that the variable can take, for instance, "high", "low" and "medium" value ranges. The set of rules maps the input variable (or a combination of input variables) to one output response variable. The membership function is a graphical representation of the magnitude of participation of each input that associates a weighting with each of the inputs that are processed and defines the functional overlap between inputs [3].The advantage of using this approach is that the process of model building is fast, using fuzzy logic tools to represent practical scenarios.

The actual shipment rate is determined by current capacity and capacity utilisation, which is a function of the ratio of the desired production to the capacity. These can be modelled by the following expressions;

$$ShipRate = capacity * utilization \tag{6}$$

Here, the variable *utilization* is defined by a heuristic function *f* as follows:

$$utilization = f(DesProd / capacity) \tag{7}$$

The heuristic function *f* is explained graphically as in Figure 5. Supply chain and operations managers tend to accommodate variations in demand through changes in the level of capacity utilisation due to the fact that investment in capacity has to be done cautiously in order to avoid over-investment. The rule of the thumb is that the higher the backlog, the higher the utilisation rate. This relationship is limited to some utilisation saturation point when the firm's plants operate at the maximum possible rate.

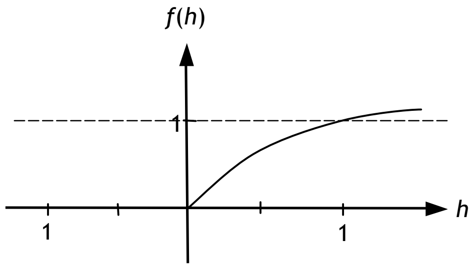

Figure 5: utilization vs normalised demand *h*

The desired production *DesProd* rate is influenced by the backlog and the normal delivery delay *NDelay* which is the normal time required to process and ship an order. It follows that the desired production is the ratio of backlog to normal delay as follows;

$$DesProd = Backlog / NDelay \tag{8}$$

The final capacity adjustment is computed according to an exponential smoothing function with a capacity build-up delay.

4 ILLUSTRATIVE EXPERIMENTS

The supply chain system was simulated under different demand patterns, namely, s-shaped, sudden increase (step), steady increasing, and fluctuating (sinusoidal). Customer demand rate which is the order rate is very influential to the behaviour of the system. The s-shaped demand gives insight into a scenario where demand changes due to introduction of a new product that gains market share slowly at first, followed by rapid increase up to an anticipated maximum. The simulate demand ranges from 500 to 600 units per unit time over a simulation time horizon of 200 time units (months). In the presence of high uncertainty due to lack of past data, the demand input is modelled as a fuzzy input variable.

When a sudden rise in demand is anticipated, the sudden increase can be modelled using a step function, with a low initial depend and a high final demand. In this instance, demand is expected to increase from 500 to 700 at the 100th time value. This scenario is of common occurrence with new products that gain sudden acceptance into the market, and the manufacturing supply chain is expected to be agile enough to respond swiftly to the changes in the market with introducing losses due to over- or under-investment in production capacity augmentation.

A steady increasing demand is when demand is expected to increase at a fairly constant rate, as opposed to sudden or s-shaped increase. In this scenario, it is anticipated that demand will be constant at 500 units for a period of 100 time units and then rises to 600. A ramp function is used to model this scenario. This scenario is slightly similar to the s-shaped demand in that both cases are characterised by increasing demand.

Finally, a fluctuation demand is a common occurrence where high uncertainty is anticipated and the variation is expected to be around an approximated average value. This demand type is modelled by a sinusoidal function, with a bias of 500 units and amplitude of 25 units. These demand scenarios are summarised in Figure 6.

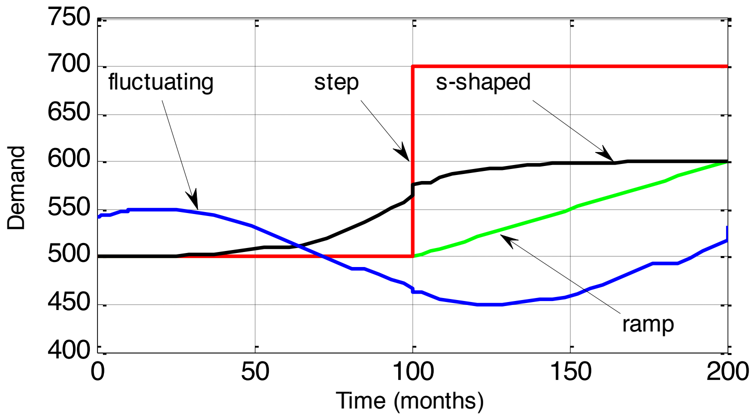

Figure 6: Demand Scenarios For Illustrative Experiments

In all these demand scenarios, the basic measure of performance used was the *gap = OrderRate - ShipRate*, which measures the variation of order rate and shipment rate. Ideally, the order rate and the shipment rate should be equal in order avoid excess inventory and backlog. The next section provides the results of the model simulation and the relevant discussions, providing practical managerial implications.

5 RESULTS AND DISCUSSION

The effects of fluctuating demand, illustrated by the sinusoidal demand pattern are represented by the variation of demand about an average of 500 units. As shown in Figure 7, the shipment rate quickly follows the order rate within a negligible time period. Using a capacity building delay of 50 time units (weeks), the capacity slowly builds up over the period to the desired level of 525 units per period. These results demonstrate the utility of fuzzy system dynamics simulation in the presence of uncertain demand or order rate. With a fuzzy control approach, unwanted fluctuations can be avoided.

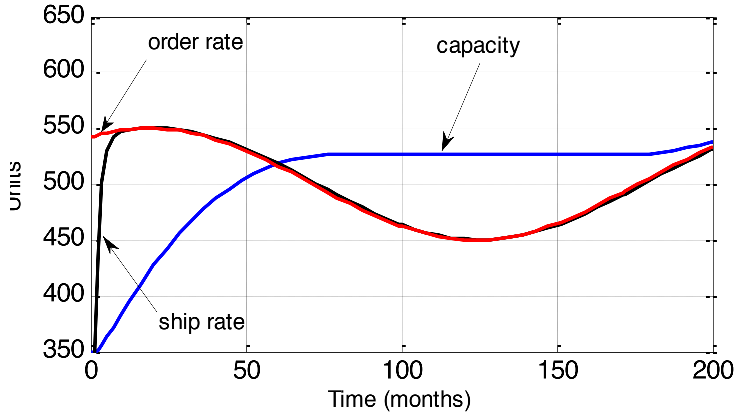

Figure 7: Fluctuating Order Rate And Shipment Rate

Figure 8 illustrates the variation of the shipment rate, the order rate, and the capacity build up when the order rate is s-shaped. As can be seen from the graph, the shipment rate follows the order rate closely, with negligible gap between the two rates. As the order rate varies from 500 to 600, the shipment rate follows the demand closely. Similarly, capacity is built up over the period from 350 to about 650 over a period of 200 time units. Again, this illustrates the effectiveness of the fuzzy system dynamics approach in modelling practical scenarios in manufacturing supply chains.

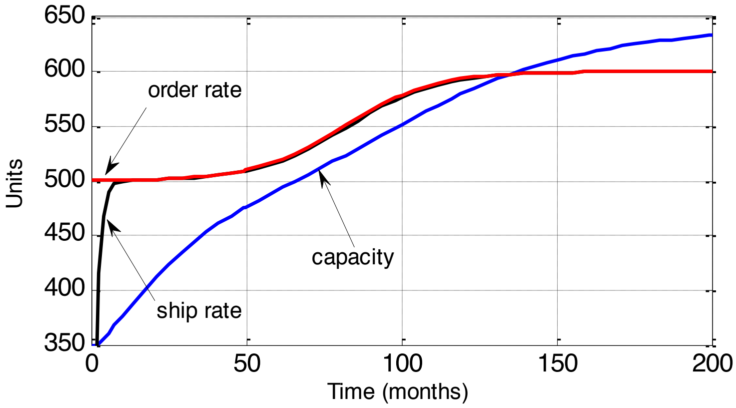

Figure 8: S-Shaped Order Rate And Shipment Rate

A step demand is known to cause unwanted shocks in the system. However, as shown in Figure 9, the shipment rate follows the order rate closely, even when the sudden demand is introduced at 100 time period. In the same manner, capacity build-up is smoothed over the period, avoiding costly sudden changes in capacity building. Therefore, fuzzy-based control can effectively minimize unwanted fluctuations by avoiding sharp responses to changes in the demand patters. Therefore, the bull-whip effects can be minimized by wisely incorporating human judgement in form of fuzzy rules.

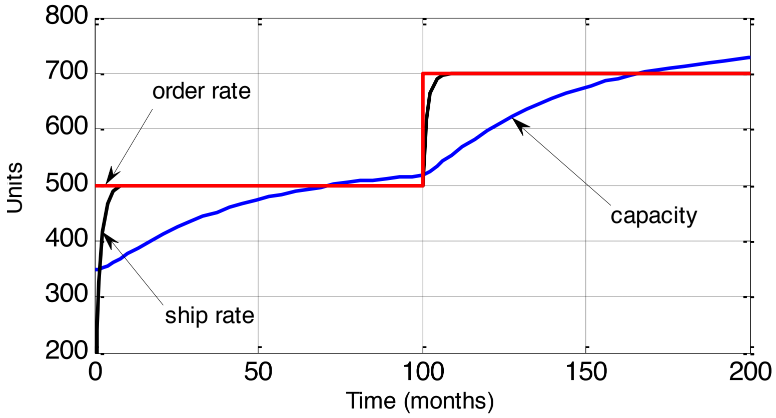

Figure 9: Step- Increase Order Rate And Shipment Rate

Finally, Figure 10 shows the case when input demand is expected to be steady over a period of 100 time units and then steadily rising to about 550 units. As the order rate rises at time =100, the capacity build up responds smoothly based on the fuzzy rule set, without causing costly fluctuations and imbalances. Likewise, the shipment rate follows the order rate closely without introducing any fluctuations in the system, thereby avoiding the bull-whip effects. The set of experiments demonstrated the effectiveness of fuzzy system dynamics in modelling supply chain systems with uncertain or fuzzy inputs. Therefore, the approach provides a platform for developing a useful decision support tool for fast informed decisions even when data is not precisely known.

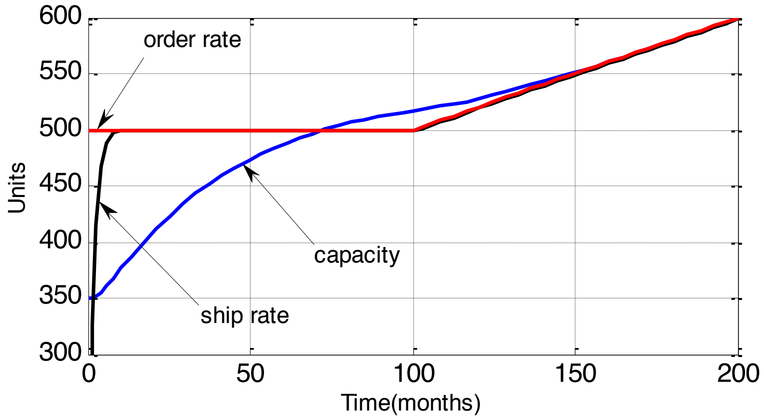

Figure 10: Steady Order Rate With Ramp-Increase Vs Shipment Rate

6 CONCLUSIONS

In this study, we implemented an artificial intelligence construct using fuzzy logic concepts to model the dynamics of a manufacturing supply chain with uncertain demand. Simulink and fuzzy logic tools in Matlab platform allow complex nonlinear dynamic models to be specified rapidly, simplifying model building to simple steps. This is especially suitable in real-world applications characterised by fuzzy demand patterns and complex dynamic interactions among different demand and supply factors in the manufacturing supply chain.

The fuzzy system dynamics model was tested using various demand input patterns in order to test the robustness of the approach. By coding the fuzzy input into a set of fuzzy rules, and the dynamic interactions into a system dynamics structure, the fuzzy interactions of uncertain demand were modelled effectively from a systems and fuzzy logic point of view. It was shown from the simulation results that the approach is effective and robust with all demand input patterns: s-shaped, steady increasing, sudden increase, and fluctuating. Compared to basic system dynamics or control-theoretic approaches based on simple proportional control, fuzzy system dynamics is capable of minimizing unwanted fluctuations considerably. The implication is that, the use of expert opinion is quite effective especially in the presence of fuzzy input and complex nonlinear relationships.

This research is of value to both researchers and practitioners. First, the research contributes to the body of knowledge of system dynamics, artificial intelligence, as well as fuzzy logic applications. In addition, the study is crucial to the manufacturing supply chain practitioners, such as supply chain managers who often encounter challenges in decision-making when demand is uncertain or demand data is scarce. It is also important when factor interactions are characterised by complex dynamics. Fuzzy system dynamics then offers a quick and intelligent approach that can assist the decision maker in building simulated practical scenarios in a reasonable time. Fuzzy rules can be constructed based on mental models and modelled and tested by simulation before actual implementation, thereby assisting the policy maker to make informed decisions on time. Therefore, the study is useful both to the academics and to practitioners in the manufacturing supply chain industry.

Acknowledgement

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