Evaluating performance of production scheduling from an economic perspective

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Abstract: Production scheduling which is a part of the planning and control of production units lies at the heart of the performance of manufacturing organizations. Production scheduling determines organizational performance. The need for efficient scheduling has greatly increased in recent decades owing to market demands for product quality, flexibility and order flow times, and other measures. However, although scheduling research activities have in the same period moved from purely academic exercises to serious attempts to solve practical problems in companies, successful implementations of scheduling techniques in practice are still scarce [1-6] and less attempt on solving the same from an economic perspective. In many companies, scheduling is still a typically human domain. However, the task of scheduling production units can become very complex. Humans are not very well equipped to barely control or optimize large and complex systems without computational tools, and the relations between actions and effects are difficult to assess. This paper will focus on problems that are related to the complexity of scheduling in practice. Scheduling based on this techniques often changed by the scheduler due to random disruptions or are not carried out exactly as preplanned on the shop floor. Because of the complex production processes, schedules are often difficult to assess mainly in terms of production cost. This paper takes a leap approach by assessing production performance in terms of cost. A new criterion of optimality is also proposed and used. This criterion is termed “total opportunity cost” and takes into account the different single criterion in a weighed term.

1. INTRODUCTION

Businesses that succeed and make money constantly assess themselves and improve in all dimensions of their business; ‘metrics’ are the cornerstone of their assessment, and the foundation for any business improvement. “… measuring the success of a reengineering effort from monetary viewpoint is a challenging phenomenon. More than half of the executives said they have no consistent, reliable way of measuring reengineering benefits”. Successfully planned metrics should help companies resolve this problem. A metric is nothing more than a standard measure to assess companies’ performance in a particular area. Metrics are at the heart of a good, customer-focused process management system and program directed at continuous improvement. The focus on customers and performance standards show up in the form of metrics that assess companies’ ability to meet their customers’ needs and set business objectives. A key with metrics is to measure the right things. Companies need to measure their financial performance from the first operations downstream. This is to ensure that processes are all optimized in terms of financial performance. Production scheduling has been employed by companies to measure their performance in terms of different metrics, i.e., productivity with an emphasis on output quantity. This paper proposes a new strategy of optimizing companies’ performance from economic perspective.

In this paper, production scheduling performance is measured in terms of “opportunity cost”. Opportunity cost is defined as a cost of any activity measured in terms of the value of the next best alternative forgone (that which is not chosen): some opportunity costs are either explicit while others are implicit. It is the sacrifice related to the second best choice available to companies, which picked among several mutually exclusive choices. Opportunity cost is a key concept in economics, and has been described as expressing the basic relationship between scarcity and choice. The notion of opportunity cost plays a crucial role in ensuring that scarce resources are used efficiently. Thus, it shall be seen on later sections that opportunity costs are not restricted or limited to monetary or financial costs: the real cost of output forgone, lost time, pleasure or any other benefit that provides utility should also be considered as opportunity costs.

Cost of Production
The market forces of demand and supply. Supply and demand are the two words that Economists use most often. These are forces that make market economies work.

What are Costs?
According to the law of supply; firms are willing to produce and sell a greater quantity of a good when the price of the good is high; this result in a supply curve that slopes upward. The firm’s objective is to maximize profit.
**Profit** is defined as the differences between **Total Revenue** and **Total Cost**

where:

- total revenue is the amount a company receives for the sale of its output
- total cost is the market value of the inputs a company uses in production

### 1.1. Opportunity Costs

A company’s cost of production includes all the opportunity costs of making its output of goods and services. These costs include both implicit and explicit costs. Implicit costs are input costs that require a direct outlay of money by a company; and explicit costs are those that do not require an outlay of money by a company (illustrated in Figure 1 below). This may be regarded to as a view by an Economist. An Accountant may view an economic profit and implicit as an accounting profit.

![Fig. 1. Relationship between revenue and opportunity costs](image)

**1.2. Classification of Scheduling Problem**

The first step to well solve any problem is to understand the dynamics associated with it. Using general framework; production scheduling problem is wholly defined by the following criteria:

- Production Requirements;
- Process Structure; and
- Scheduling Objectives.

**1.3. Production Requirements**

A company can be classified as “closed or an open shop” configuration. In a closed shop, all customer orders are satisfied by products in inventory and a cyclic operation mode is employed. With this operandi modus, production lines produce batches in the same sequence over a time period of fixed length. Production is initiated by the need to replenish inventory; scheduling decision involves determining the size of batches and a sequence of operation in order to optimize a certain objective function.

In an open loop, customer orders trigger production operation to start and finished products are not stocked – this is regarded as a make-to-order (MTO). This type often use short term scheduling, and is usually used by companies that manufacturer a large number of low volume, high-value added products.

### 1.4. Various Measures of Cost

Costs of production may be divided into two: fixed and variable costs. Fixed Costs (FMC) are those costs that do not vary with the quantity of products produced, whereas, Variable Costs (TVC) are those that do vary with the quantity of products produced. Therefore, Total Costs given the above parameters can be calculated as the sum of FC and VC (i.e., TC = FMC + TVC). Average costs can be determined by dividing the company’s costs by the quantity of output it produces. Variable costs makes calculations of total opportunity costs a bit more challenging.

### 1.5. Process Structure

This criterion addresses the complexity of the process, specifically the number of processing steps, and most importantly the configuration of the process units for each task. Processes can be generally categorized into four configurations: single-unit process, parallel, serial unit, and generalized serial and parallel unit as shown in the figure 1A below.

To ensure that all configurations are considered in this study; all types of company-layouts (namely; fixed-position layout, production-line layout, process layout and product layout) are examined. The above configurations one way or the other falls under one the company-layouts considered herein.
2. SCHEDULING OBJECTIVES

Morton and Pentico [7] defined scheduling as “a scheduling system that dynamically makes decisions about matching activities and resources in order to finish jobs and projects needing these activities in a timely and high-quality fashion while simultaneously maximizing throughput and minimizing direct operating cost.”

Simply stated, scheduling is about allocating available production resources to complete a certain set of tasks in a given time period while satisfying certain sets of objectives, e.g., productivity increment, minimize production costs, machine idle times, etc. These objectives can be categorized as either performance-based (e.g., tardiness; flow times; makespan; etc.) or economic-based (inventory holding costs; shortage costs; labour costs; etc.).

- Objective of this paper deals with providing insight into the economic aspects of the scheduling process under consideration. The scheduling problem is analyzed and relations for its various cost components are developed. A total opportunity cost function is developed for use as the comprehensive criterion of optimality in scheduling problems. Sensitivity analysis conducted thus far (Gupta, [8]) shows that none of the individual criteria (minimum operational costs, raw material costs, penalty costs, or utilities costs) gives optimal or near optimal results when compared to the total opportunity costs.

- Another attempt to break-through the use of conventional objective functions commonly aiming to minimize makespan, tardiness, or total cost of production is presented by Gupta [8] in his PhD thesis entitled Economic Aspects of Scheduling Theory. Gupta’s work shows that the measure of performance that should be optimized is the total opportunity cost function. In addition, the use of minimization of maximum flow time as the optimality criterion may increase production cost for the company concerned rather than optimizing it. Using sensitivity analysis, Gupta concluded that none of the individual criteria (minimum operational costs, raw material costs, penalty costs, or utilities costs) gives optimal results when compared to the total opportunity cost. Thus, a modified version of the total opportunity cost function presented by Gupta is used in this paper as the criterion of optimality for the scheduling model developed in later sections.
In this paper total opportunity cost as first suggested by Gupta [8] is used as the optimization criterion for the scheduling problem. It is defined mathematically as the sum of operation cost, job waiting cost, machine idle cost, and penalty cost of jobs. This serve as the primary basis of the objective function in the proposed model. The individual cost components presented below are modified versions of the ones found in Gupta [8]'s model. However, the objective functions in both models aim to achieve the same goal — to minimize total opportunity cost. The only difference between the two is the use of variable processing rate in the proposed model, whereas in Gupta's model, the output rate of the process is assumed to be fixed. This makes the processing model to be more usable, superior and to represent if not, more closely the real phenomenon than the one first presented by Gupta [8]. In real world, processing rates are always subject to change due to different reasons, e.g., it can be due to machine worn out, or any other reason.

3. METHODOLOGY

As given in previous sections, mathematically; Total Opportunity Cost (TOC) is given as the sum of Operation Cost (OC), Job Waiting Cost (JWC), Machine Idle Cost (MIC), Penalty Jobs Cost (PJC) and Defective Products (DP)*. 

* However, some researchers excludes the last component: defective product. By excluding this term in the calculation of the total opportunity cost, the model is obliged to somewhat enforce the quality constraints without any exception. Companies that exclude DP aims for zero quality defects tolerance; which is what companies are aiming for but practically unattainable.

Below, mathematical expressions of TOC components with respective discussions:

3.1. Operation Cost (OC)

The operation cost incurred in the span of a scheduling horizon is composed of two parts: 1) setup/changeover cost, and 2) processing cost. Costs associated with changeovers are assigned to the preceding job in a pair of consecutive jobs, e.g., the changeover cost incurred when switching from job i to job j will be assigned to job i.

\[ SC_i = \sum_{j \in S_{i_j}} (CC \cdot C_{ij} \cdot X_{ij}) \]  

where:

\[ SC_i \] setup or changeover cost of job i (ZAR)
\[ CC \] changeover cost per unit time (ZAR)
\[ C_{ij} \] changeover time from job i to j

\[ X_{ij} \] binary variable indicating assignment of job j after job i

The processing cost of job i can be defined as the cost incurred directly by the actual compounding process, such as utilities and manpower but not including cost of raw materials. This is dependent on the output rate of the extruder and can be calculated as follows:

\[ PC_i = PRC \cdot \frac{Q_i}{PR_{iu}} \cdot W_{iu} \]  

where:

\[ PC_i \] processing cost of job i
\[ PRC \] processing cost per unit
\[ Q_i \] size of job i in kg
\[ PR_{iu} \] processing or output rate of job i on unit u in kg/hr
\[ W_{iu} \] binary variable indicating assignment of job i to unit u

Taking the sum of the set up cost and the processing cost of each individual job result in the total operation cost (OC) of all jobs in the scheduling horizon; thus; a schedule can also be measured based on economic aspects.

\[ OC = \sum_i (SC_i + PC_i) \]  

3.2. Job Waiting Time (JWC)

It is a common sense that company loses their well-deserved share if raw materials and unfinished goods sit on the plant floor, waiting to be processed due to machine unavailability. Capital that is otherwise available to generate revenue is tied up in the in-process goods. Gupta [8] called this cost "job waiting cost", or otherwise known as in-process inventory cost. Gupta [8] defined job waiting cost as the sum of value of raw materials required for the job and the subsequent value that has been added to it by the preceding production steps prior to the step that require waiting due to machine availability.

In flow-line environment, as opposed to fixed-position layout, the entire plant can be treated as one lone process, since there is no disruption between each of the stages. Therefore, when calculating the job waiting cost, only the cost of raw materials has to be take into account and the additional value added on the previous processing stations can be omitted:

\[ JWC = \sum_i (WT_i \cdot R_i \cdot MC_i) \]
where:

\[ JWC \text{ total waiting cost of all jobs in the scheduling horizon} \]
\[ WT_i \text{ waiting time for job } i \text{ in unit time} \]
\[ R_i \text{ expected return of raw materials used in job } i \]
\[ MC_i \text{ raw material cost of job } i \]

3.3. **Machine Idle Cost (MIC)**

When a machine is idle and not processing, manufacturing (valued) opportunity is lost due to tied up occupied capital for in-process inventory – an opportunity cost also exist. This machine idling cost is directly proportional to the length of time that the machine is not utilized and represents the average revenue that the machine could have otherwise been generating by producing goods during that period of idling time.

\[ MIC = \sum_u R_u \cdot IT_u \]  
(5)

where:

\[ MIC \text{ total idling cost of all units } u \text{ during scheduling horizon} \]
\[ R_u \text{ expected rate of return on unit } u \text{ per unit time} \]
\[ IT_u \text{ idling time of unit } u \]

3.4. **Job Penalty Cost (JPC)**

Job penalty cost is somewhat difficult to quantify, as not all the consequences of completion of job past the due date can be directly measured. Gere [9] suggests that the penalty costs incurred by late deliveries should include contractual penalty clauses, additional expenses incurred for dealing with customer, cost of expediting tardy jobs, and loss of goodwill cost.

What makes this difficult to compute is the determination of loss due to goodwill. When a customer is dissatisfied with the services provided by the company, it results in customer dissatisfaction, and consequently, my result in lost sales due in the future and/or loss of other potential customers due to damaged reputation. This cannot be accurately measured or predicted, as it is solely based on the behavioral tendencies of the customer in question and can only be estimated with a high degree of error.

It will be assumed that the penalty cost is directly proportional and increasing with tardiness of the job. This can be mathematically expressed as follows:

\[ \text{JPC} = \sum_i D_i \cdot Pn_i \]  
(6)

where:

\[ \text{JPC} \text{ total penalty cost of all jobs in scheduling horizon} \]
\[ D_i \text{ tardiness of job } i \text{ in unit time} \]
\[ Pn_i \text{ penalty cost for late job } i \]

3.5. **Total Opportunity Cost (TOC)**

Reader should not confuse this for theory of constraints. In this paper, TOC is used to refer to the total opportunity cost and not theory of constraints. This is calculated by summing up expression (3) to expression (6); total opportunity cost of a schedule can be expressed as:

\[ \text{TOC} = \text{OC} + \text{JWC} + \text{MIC} + \text{JPC} \]  
(7)

From RHS of the above main expression; expression (3) represent OC; expression (4) represent JWC; expression (5) represent MIC; expression (6) represent JPC.

Theoretically, another term representing the opportunity loss from defective products, defined as the products that do not meet quality specifications, can be included in the total opportunity cost (equation 7). Some companies would argue that they aim for zero tolerance on defective products. However, considering the practical situation it becomes also necessary to introduce and/or include the same in the final expression. If a term denoting loss due to quality issues were to be included in the objective function, it can potentially allow the mathematical model to generate a solution that result in a lower total opportunity cost, but compromising on quality while increasing production costs due to discarding and/or recycling of defective products. By including this term, the expression above then changes to the following:

\[ \text{TOC} = \text{OC} + \text{JWC} + \text{MIC} + \text{JPC} + \text{DP} \]  
(8)

where, \( \text{DP} = \text{defective products due to quality issues, which can be give mathematically as:} \)

\[ \text{DP} = I - O \text{, where:} \]
\[ I = \text{input, and } O = \text{output} \]

**Defective product (DP)** is understood as the difference between input and output; measured per scheduling horizon. For this kind of calculations, we use expected return of the product (price charged contractually to the client and not manufacturing costs) – this guarantee accurate results.
Objective function: 
\[ TOC = Z = OC + JWC + MIC + JPC + DP \] 
the optimization criterion for this scheduling problem is the total opportunity cost, \( Z \). An optimal schedule will be a schedule that minimizes the total opportunity cost, while taking into account all relative constraints.

3.6. Method Comparison

The method of measurement that Gupta [8] used for comparison of the total cost of a schedule obtained by minimizing one of the optimization criteria and one obtained by minimizing the total opportunity cost is called the presciational efficiency. Let \( C_{na} \) be the total cost of the schedule as defined by Equation 7 and 8, obtained by optimizing the \( n^{th} \) criterion, and let \( C_{jo} \) represent the total cost of the schedule obtained by minimizing the total opportunity cost, then the presciational efficiency of the \( n^{th} \) criterion of optimality is defined by the equation 9 below:

\[ \Lambda_n = \frac{C_{na} - C_{jo}}{C_{na}} \times 100 \]  
(9)

The presciational efficiency evaluates the percentage deviation of the total cost of a schedule from the total cost of the optimal schedule obtained using minimization of the total opportunity cost as the optimality criterion.

From expression 9 above, related measure called “effectiveness” can also be calculated. If the total cost of a schedule obtained by optimizing the \( n^{th} \) criterion is the same as the one obtained by optimizing the total opportunity cost, then \( C_{na} - C_{jo} \). This makes the presciational efficiency to be zero (0) and the effectiveness of such a schedule would be 100%, yielding the same results cost-wise as a schedule generated by using the total opportunity cost as the criterion of optimality. The lower the presciational efficiency, the more effective the \( n^{th} \) criterion is. Relationship is given below:

\[ \xi_n = 100 - \Lambda_n \]  
(10)

3.7. Problem Constraints.

Below, definitions and respective formulations of production schedule optimization criterion. Information required for the determination of the production schedules are categorized as parameters. These are fixed values that are dependent on the job.

Let:

\[ D_i \] tardiness of job \( i \)

\[ IT_u \] idle time of unit \( u \)

\[ LTF \] latest completion time of set of jobs

\[ PR_i \] output rate of job \( i \) on unit \( u \)

\[ TF_i \] completion time of job \( i \)

\[ TS_i \] starting time of job \( i \)

\[ TT \] total tardiness in days

\[ WT_i \] waiting time of job \( i \)

\[ W_{iu} \] assignment of job \( i \) to be processed on unit \( u \)

\[ C_{ij} \] changeover time from job \( i \) to job \( j \)

\[ Q_i \] size of job \( i \)

\[ R_i \] expected rate of return on raw materials

\[ R_u \] expected rate of return on machine

\[ RTO_i \] release time of job \( i \)

\[ RTU_u \] release time of line \( u \)

3.8. Definition of Tardiness

Tardiness is also one measure that relates more closely with organizational profitability. It is the positive difference between actual completion time of an order and its target due date. If for example, an order is completed prior to the due date, then it would have been “earliness”, and completion exactly as expected it would have a tardiness of zero (0). Whereas, if the target due date of an order \( s \), and the actual completion time of the order was \( s+3 \), the tardiness of the job would be 3.

\[ D_i = TS_i + \sum_{w \in L_i} \left( W_{iu} \frac{Q_i}{PR_{iu}} \right) \]  
(11)

3.8.1. Definition of Order Completion Time

The completion time of an order is simply given as sum of the starting time and its processing time, as given below:

\[ TF_i = TS_i + W_{iu} \frac{Q_i}{PR_{iu}} \]  
(12)

3.8.2. Definition of Job Waiting Time

This measure is important for the determination of the opportunity cost of jobs waiting processing. From the moment a job is released for processing until the actual completion starting time of the job, materials are tied-up as in-process inventory costs instead of being transformed into much more valuable finished product.

\[ WT_i = TS_i - RTO_i \]  
(13)

3.8.3. Completion Time of Last Job Processed

This is also an important measure for determining total opportunity cost for a certain scheduling horizon. The latest completion time of all jobs \( i \) can be derived from the following equation:
\[ LTF_i \geq TF_i \]  \hspace{1cm} (14)

3.8.4. Definition of Unit Idle Time

Simply stated, the idle time of a unit is the actual time that the unit is not processing any jobs. Therefore, the idle time of unit \( u \) is the latest completion time of the last job in the production schedule less processing times of all jobs allotted to the same unit while considering respective jobs released times. The justification in using the latest completion time of the last job in the entire schedule instead of the last job in the entire schedule instead of the last job assigned to that particular unit is that, even though one particular unit may have complete processing all jobs assigned to it, it is still sitting idle while other units are still running.

\[ IT_u \geq LTF - \sum_{i \in I_u} \left( W_{in} \cdot \frac{Q_i}{PR_{in}} \right) \]  \hspace{1cm} (15)

Considering all expressions, from (1) to (15); scheduling model is then developed considering all factors as discussed above. It is assumed that the schedule constructed following above criterions and respective constraints shall represent if not more closely a real world phenomenon under stochastic environment.

4. THE SCHEDULING MODEL

In this section, scheduling problem is given and defined.

4.1. Problem Consideration and Definition

The objective of this model as given in previous sections is also to generate a production schedule populated from economic perspective. A plant that operates 24 hours a day, 7 days a week and run on a three shift schedule. For scheduling purposes, is assumed to have smooth transition between shifts, with no break in production. Pre-emption of jobs among different machines are not allowed. This is to ensure quality consistency within batches and to eliminate any errors that may occur. It is further assumed that there is no resource constraints, manpower or material-wise, unless otherwise stated in the form of delayed job or unit released time. It is also assumed that any planned maintenances should take place outside scheduling period. Batch sizes (kg) are given as the sum of size of job (kg) and safety factor (kg) divided by size of mixer (kg). The general purpose of safety factor is to ensure that sufficient space in the mixer for achieving an even distribution of materials and prevent overloading of the mixer.

Batch Size (kg) = \( \frac{\text{Size of Job (kg)} + \text{Safety Factor (kg)}}{\text{Size of Mixer (kg)}} \)

A company’s total costs are divided into fixed and variable costs; fixed costs do not change when company alters the quantity produced; whereas variable costs do change as the company alters quantity of output produced.

4.2. Production Function (Prod. Fn)

In terms of cost, production function relates to quantity of inputs (value) used to make goods and the quantity of output (value) of the very goods.

Prod Fn: Marginal Product & Diminishing Marginal Product
- Marginal Product of any input in the production process is the increase in output that arises from an additional unit of that input.
- Diminishing marginal product is the property whereby marginal product of an input declines as the quantity of the input increases.

Mini Case Study: as more and more workers are hired at a company, each additional worker contributes less and less to the production because of limited company equipments. Relationship between output and number of workers hired, output and relative costs are illustrated graphically below.

Table 1. A Production Function & Total Cost

<table>
<thead>
<tr>
<th>No. of Workers</th>
<th>Output</th>
<th>Marginal Product of Labour</th>
<th>Cost of Factory</th>
<th>Cost of Workers</th>
<th>Total Cost of Inputs</th>
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</tbody>
</table>

Fig. 2. Production Function
Fig. 3. Total Cost Curve

5. CONCLUSION

The goal of companies is to maximize profit, which equals to the difference between total revenue and total costs. This makes more important when analyzing companies’ behavior to include all the opportunity costs of production. The focus was on the criterion of optimization. Various criteria were discussed and a mathematical relationship for the same was given, sensitivity analysis showed that the use of a single criterion alone (e.g., minimize inventory) is not enough for optimizing operation. In fact, such use might even increase the overall production cost rather than optimizing it. A new criterion of optimality is proposed and used. This criterion is termed the total opportunity cost and takes into account the different single criterion in a weighed term.

One major novelty of the present work is the inclusion of quality constraints which makes the problem more practical than models that excluded defective criterion.

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