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Validation of International Code-Type Concrete Elastic Modulus Estimation Methods

ABSTRACT
The elastic modulus of concrete is utilized in the design of reinforced concrete structures, including in predicting creep deformation. This elastic modulus can be estimated, using models contained in national design codes, by considering one or more properties (usually compressive strength).

The proposed paper assesses the accuracy of eleven empirical elastic modulus estimation models, when compared with the actual values measured on a range of concretes under laboratory controlled conditions. The equations considered are those contained in BS 8110 (1985), SANS 10100 (2000), SANS 10100 (2000) Modified, ACI 209 (1992/2008), AS 3600 (1988, 2001 and 2009), CEB-FIP (1970, 1978 and 1990), EC 2 (2004), GL 2000 and 2004), GZ (1993) and RILEM Model B3 (1995).

The test results indicated that the discrepancies between the measured and estimated values were only significant in the case of the SANS 10100 (2000) Modified method (P = 3,1 %) and the CEB-FIP (1970) method (P = 2 %). The most accurate methods were the SANS 10100 (2000) and AS (2009) which both yielded a coefficient of variation (ω_j) of 9,3 %. The least accurate method was the CEB-FIP (1970) which yielded a coefficient of variation (ω_j) of 22,7 %.

Furthermore, the test results of this research were used to establish which factors influence the elastic modulus of concrete. It was found that the concrete density, the density of the included aggregate and the coarse aggregate content separately correlated significantly with the elastic modulus ($P \leq 3$ %).

INTRODUCTION

Elastic modulus (E) is defined as the ratio of stress to resultant strain. The elastic modulus value of concrete (E) is an important design consideration for the durability, long-term serviceability and the load carrying capacity of structures.

The importance of the elastic modulus of a concrete structure increases with increasing deformation sensitivity of the structure. The more critical the deflections and unacceptable secondary cracking and distress of a structure, the stiffer the concrete required and the more important the accurate determination of the E value. On the other hand, concrete with a low E may be preferred where cracking due to restrain is to be avoided[1].

The value of E is also important in the prediction of creep of concrete. With the exception of the RILEM Model B3 (1995), design code type creep prediction models utilise an estimated E value in the calculation of creep strain.

The E may be determined by laboratory testing using methods such as BS1881[2] and ASTM 469-02[3] or estimated by means of empirically based expressions of various complexity, such as those typically incorporated in design codes. Such models typically use parameters such as compressive strength as input. In general, the more deformation sensitive the structure, the more justifiable the cost and time of laboratory testing.

The objectives of the investigation reported in this paper were to:

- Assesses the accuracy of empirical equations that are used for estimating E which are incorporated in the following creep prediction models, deriving mainly from national structural design codes, for six different concretes, incorporating combinations of three aggregate types and two w/c ratios.

- Assess the influence of various factors on the measured estimated elastic modulus.

The models considered were the following.

- British Standards Institution - Structural Use of Concrete, BS 8110 - Part 2 - (1985)[4].
- SANS 10100 (2000)[5] (formerly SABS 0100, 1992).
- SANS 10100 (2000) Modified model.
- American Concrete Institute (ACI) Committee 209 (1992), reapproved by ACI Committee 209 in 2008[6].
- Standards Association of Australia - Australian Standard for Concrete Structures - AS 3600 (1988)[7].
- AS 3600 (2001)[8].
- AS 3600 (2009)[9].
- Comité Euro-International Du Béton - Federation Internationale De La Précontrainte (CEB-FIP) Model Code (1970)[10].
- CEB-FIP Model Code (1978)[11].
- CEB-FIP Model Code (1990)[12].
- EUROCODE (EC 2) – BS EN 1992-1-1:2004[13], which will be referred to as EC 2 (2004). This model, which supersedes the BS 8110 (1985) model, is the same as the CEB-FIP (1999) model.
- Gardner and Lockman 2000[14] and 2004[15] versions which will be referred to as GL (2000) and GL (2004), respectively. The GL (2000) model was published in 2001.
- Gardner and Zhao (GZ, 1993)[16].
- International Union of Testing and Research Laboratories for Materials and Structures (RILEM) Model B3 (1995), after Bazant and Baweja (1995)[17].

MODELS CONSIDERED

Basis of Models

The equations considered are all empirical. Some of the equations considered derive from superseded version of national design codes. Nevertheless, it is appropriate to assess their accuracy.

The SANS 10100 (2000) equation is based on the superseded British Standard method, BS 8110 (1985), with a small modification where aggregate specific values (based on stiffness) are used in K_0 instead of the constant value of 20 (for normal concrete).

The equation used in the SANS 10100 (2000) Modified model is essentially the SANS 10100 (2000) equation with a further aggregate specific modification (α) to the calculation of the elastic modulus. Values for both K_0 and α arose from research conducted by Davis and Alexander (1992) on concretes with 23 South African aggregate types.

The ACI 209 (1992) and the AS 3600 (1988 & 2001) methods adopt the same equation. The AS 3600 (2009) method uses a different equation in the case of cylinders strengths exceeding 40 MPa.

The GL (2000), GL (2004) and GZ (1993) use a common equation for the estimation of elastic modulus.

Factors Considered by Each Model

The factors considered in the calculation of the E by each of the models are summarised in Table 1.

Tab. 1 Summary of factors considered in the calculation of E by the different models

FACTORS		Concrete Density	Aggregate Stiffness	Normal/ Light Weight Concrete	Cube Strength at 28 Days	Cube Strength at age t	Cylinder Strength at 28 Days	Cylinder Strength at age t	Age of Concrete	Cement Type	Method of Curing
MODEL	BS 8110 (1985)		X	X	X	X			X		
	SANS 10100 (2000)		X	X	X	X			X		
	SANS 10100 (2000) Modified		X	X	X	X			X		
	ACI 209 (1992/ 2008) and AS 3600 (1988/ 2001)	X					X	X	X	X	X
	AS 3600 (2009)	X					X	X	X	X	X
	CEB-FIP (1970)			X				X			
	CEB-FIP (1978)						X				
	CEB-FIP (1990)		X				X				
	EC (2004)		X				X	X	X	X	
	GL (2000/ 2004) and GZ (1993)						X	X	X	X	
	RILEM Model B3 (1995)						X		X		

The following is evident from Table 1.

- Most of these equations estimate the E on the sole basis of concrete compressive strength.
- The ACI and the AS 3600 methods are only methods that consider concrete density.
- The BS 8110 (1985), SANS 10100 methods, CEB-FIP (1990) and EC (2004) are the only methods that consider aggregate stiffness.
- The factors listed in the three most extreme columns are considered, by the relevant models, in the estimation of concrete strengths for ages other than 28 days, which are in turn used to estimate E values at these ages.

EXPERIMENTAL DETAILS

Materials

A single batch of CEM I 42,5 cement from the Dudfield factory of Alpha Cement was used for all the tests carried out in this investigation. Quartzite (Q) from the Ferro quarry in Pretoria, granite (G) from the Jukskei quarry in Midrand and andesite (A) from the Eikenhof quarry in Johannesburg were used as both the stone and sand aggregates for the concrete. The stone was 19 mm nominal size and the fine aggregate was crusher sand.

Representative boulders were collected from each of the quarries for the determination of the elastic modulus of the aggregates.

Laboratory Procedures

Determination of elastic moduli of the aggregates

Measurements of aggregate elastic modulus or stiffness were carried out on samples obtained from the representative boulders collected. The stiffness of each rock type as determined on the boulder samples was taken to be representative of the stiffness of the corresponding aggregates used in the concrete specimens.

Three cores measuring 42 mm in diameter and 82 mm long were cut from each set of two boulders and these were tested according to the procedure described in BS 1881[2] to determine the elastic modulus of the aggregates used in this investigation.

The cores were loaded to a maximum stress equal to approximately 25 per cent of the average unconfined compression strength values respectively determined by Davis and Alexander[18] as 250 MPa, 190 MPa and 527 MPa for the quartzite, granite and andesite from the same sources.

Concrete mixture proportions

A total of six mixtures were prepared, using water/cement (w/c) ratios of 0.56 and 0.4, for each of the three aggregate types included in the investigation. For each mix, a constant water content of 195 l/m^3 was used. This approach ensured that, for the different aggregate types used, concretes with the same w/c ratio had the same volume of cement paste. Table 2 shows the mix proportions and test results of the six concrete mixes.

Tab. 2 Details of the mixes and laboratory test results

Aggregate Type	Quartzite		Granite		Andesite	
Mix Number	Q1	Q2	G1	G2	A1	A2
Water (l/m^3)	195	195	195	195	195	195
CEM I 42,5N (kg/m^3)	348	488	348	488	348	488
19 mm Stone (kg/m^3)	1015	1015	965	965	1135	1135
Crusher Sand (kg/m^3)	810	695	880	765	860	732
w/c Ratio	0,56	0,4	0,56	0,4	0,56	0,4
a/c Ratio	5,24	3,50	5,30	3,55	5,73	3,83
Slump (mm)	90	50	115	70	95	55
Cube Compressive Strength (MPa)	37	65	38	65	48	74
Cylinder Compressive Strength (MPa)[a]	30	53,5	30,7	53,5	38	59
Characteristic Cube Strength (MPa)	30	50	30	50	30	50
Characteristic Cylinder Strength (MPa)[a]	25	40	25	40	25	40
Concrete Density (kg/m^3)	2371	2410	2385	2432	2596	2585
Average Elastic Modulus of included Aggregate (GPa)	73		70		89	
[a] Inferred from cube strength using the conversions from EC 2 (2004)						

Preparation of concrete specimens

Six 100 mm cubes were cast for each of the six mixes. In the case of each mix, three cubes were tested at seven days and three at 28 days after casting. The 28 day strength of each concrete, which is shown in Table 2, was taken as the average of the three compressive strength tests at that age. For each concrete type, three prisms, measuring 101.6 x 101.6 x 200 mm, were prepared for the determining measuring the elastic modulus. All the concrete samples were cured in a water bath, at a temperature maintained at 22 ± 1 °C.

Determination of elastic moduli of the concrete

The prisms were removed from the curing bath at an age of 28 days after casting and placed in a compression frame. Thereafter, the secant elastic modulus was measured for each mix, as follows.

- The load corresponding to a stress strength ratio of 25 per cent (σ_a) was applied to the samples, maintained for 60 seconds, and then unloaded;

- Thirty seconds later, a pre-load of approximately 1Mpa (σ_b) was applied and maintained for 60 seconds;
- The load was increased to σ_a, maintained for 60 seconds and unloaded to σ_b;
- Thirty seconds later a set of readings was taken (at σ_b) and regarded as the zero-strain readings;
- The load was increased to σ_a and a set of readings was taken within ten minutes. These readings were taken as being the immediate elastic deflections (ASTM C512-76)[19].

RESULTS AND DISCUSSION

Accuracy of Models

Table 3 shows the estimated elastic moduli for each of the concretes according to the different models with the average elastic moduli measured, at 28 days after casting. The most and least accurate elastic modulus estimations are indicated in green and red, respectively.

Tab. 3 Measured and predicted elastic moduli and corresponding statistics

MEASURED		ELASTIC MODULUS OF CONCRETE (GPa)						CoV (ω_j %)	P (%)
		Q1	Q2	G1	G2	A1	A2		
		25.8	34.0	27.8	28.9	36.7	40.9		
MODEL	BS 8110 (1985)	27.4	33.0	27.6	33.0	29.6	34.8	15.0	45.0
	SANS 10100 (2000)	24.4	30.0	27.6	33.0	38.6	43.8	9.3	67.7
	SANS 10100 (2000) Modified	31.8	43.0	27.6	33.0	38.6	43.8	14.9	3.1
	ACI 209 (1992/ 2008) and AS 3600 (1988/ 2001)	27.2	37.2	27.8	37.7	35.1	43.4	12.8	16.5
	AS 3600 (2009)	27.2	35.0	27.8	35.4	35.1	40.0	9.3	40.7
	CEB-FIP (1970)	32.5	43.4	32.9	43.4	36.6	45.6	22.7	2.0
	CEB-FIP (1978)	29.5	35.8	29.7	35.8	31.9	37.0	13.9	63.3
	CEB-FIP (1990)	31.1	37.7	31.3	37.7	33.6	38.9	15.4	20.2
	EC (2004)	32.1	38.2	32.3	38.2	34.5	39.3	16.4	12.2
	GL (2000/ 2004) and GZ (1993)	27.1	35.0	27.3	35.0	30.0	36.5	14.4	78.5
	RILEM Model B3 (1995)	25.9	34.6	26.2	34.6	29.2	36.4	15.2	54.6

Table 3 includes the following statistics.

- The coefficients of variation of errors (ω_j) after Bazant and Panula[20]. The lower the ω_j , the more accurate the method of estimation.
- The t-Test results, which relate to a comparison between the measured and the predicted values for each of the concretes, determined by a particular model. T-test probabilities exceeding five per cent indicate that the discrepancies in the paired values are not significant.

It is evident from Table 3 that, in the case of the concretes containing the andesite aggregate (A1 and A2), which had an average density of 2591 kg/m^3, the models generally underestimated the elastic modulus. In the remaining mixes, which had an average density of 2400 kg/m^3, the models generally overestimated the elastic modulus. No trend was established regarding the variance exhibited in the values predicted for the lower w/c ratio mixes (Q2, G2 and A2) in comparison with the higher w/c ratio mixes.

The AS 3600 (2900) method and SANS 10100 (2000), which yielded a coefficient of variation (ω_j) of 9,3 %, were the most accurate methods. The CEB-FIP (1970) was the least accurate method ($\omega_j = 22{,}7$ %). This method was also the least accurate in the case of five of the six mixes. Furthermore, this method yielded a t-Test probability less than five percent (2 %), hence indicating that the differences between the measured and predicted values are significant.

The SANS 10101 (2000) Modified model, which makes allowance for the stiffness of 23 aggregates, did not yield more accurate results that the other models which make a general stiffness based aggregate type allowance.

The methods that consider concrete density (ACI and AS 3600) yielded relatively low ω_j values.

No correlation was found to exist between the accuracy of a model and the number of factors considered by that model.

Specific Factors Influencing E

As is evident from Table 1, most models estimate E solely on the basis of compressive strength.

The results of this investigation were used to ascertain the effect of other factors on the E, by correlating a number of properties with the measured E values. Table 4 shows the statistics pertaining to the relationships established.

Tab. 4 Relationships between measured E values and other properties

Property	Correlation Coefficient (r)	Level of Significance (P %)
Cylinder compressive strength	0.637	17.4
Aggregate / Cement (by mass)	0.247	63.7
Fine aggregate content (by mass)	0.378	46.0
Coarse aggregate content (by mass)	0.854	3.0
Total aggregate content (by mass)	0.381	45.6
E of aggregate	0.864	2.6
Concrete density	0.883	2.0

Referring to Table 4, the coarse aggregate content, E of the aggregate and concrete density correlated significantly ($P \leq 3$ %) with the measured E of the concrete. These relationships, which are depicted in green in the table, are shown in Figures 1 to 3.

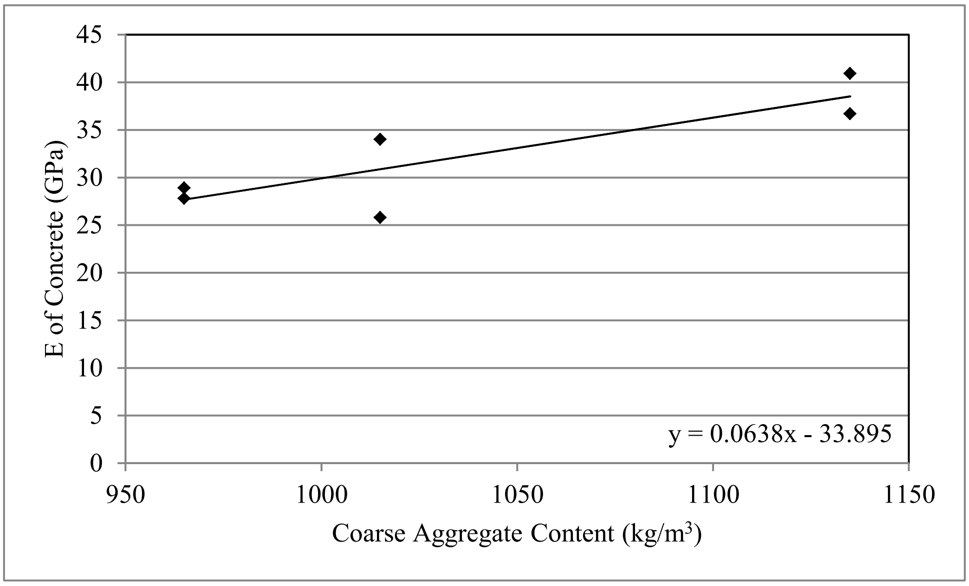

Fig.1 Relationship between E of concrete and coarse aggregate content

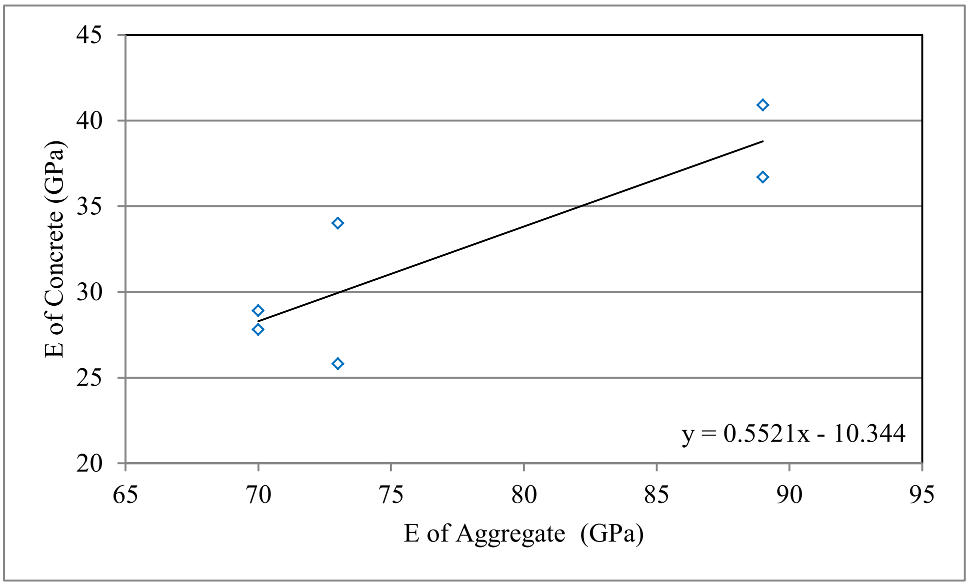

Fig. 2 Relationship between E of concrete and E of aggregate

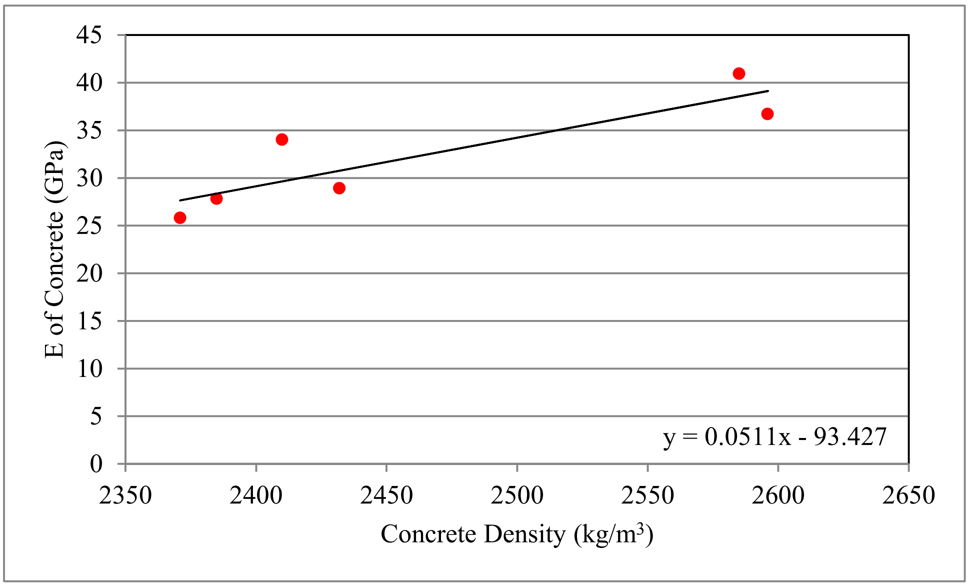

Fig. 3 Relationship between E of concrete and concrete density

The significant correlations established indicate that compressive strength should not be solely used to estimate the E of concrete. Furthermore, the following properties should also be considered in the estimation of E.

- Aggregate properties, in particular aggregate stiffness and coarse aggregate content.
- Concrete density – as is currently done by the ACI 209 and AS3600 models.

CONCLUSIONS

The eleven concrete elastic modulus estimation models considered in this research include concrete strength as a factor. However, some models consider up to six factors. No correlation was found between the accuracy of a model and the number of factors considered by that model.

The AS 3600 (2900) method and SANS 10100 (2000) which yielded a coefficient of variation (ω_j) of 9,3 % were the most accurate methods. The CEB-FIP (1970) was the least accurate method ($\omega_j = 22,7$ %).

The SANS 10101 (2000) Modified model, which makes allowance for the stiffness of 23 aggregate types, did not yield more accurate results that the other models which make a general stiffness based aggregate type allowance.

The methods that consider concrete density (ACI and all the AS 3600) yielded relatively accurate results, compared to the other models.

The influence of various properties on the E of concrete was assessed by correlating the E values measured in this research with these properties. It was found that the E of the concrete correlated significantly with the coarse aggregate content, E of the included aggregate and concrete density. These correlations allude to the inclusion of concrete density and aggregate properties as criteria in equations used to estimate the E of concrete.

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