The Effect of Hamming Distances on Permutation Codes for Multiuser Communication in the Power Line Communications Channel

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***Abstract*—We partition permutation sequences into groups to form permutation codes for multiuser communication. Each group becomes a codebook for each user in a multiuser communication system. We present simulation results for the performance of different partitions of permutation codes for multiuser communication, where the codes are to be used in channels with background noise and jamming, such as the Power Line Communications (PLC) channel. With the help of the simulation results, we show that by partitioning codebooks according to distance properties we can affect the performance of the codes. The permutation codes have codewords of length M with symbols taken from an alphabet whose cardinality is M, where M is any integer. Each symbol may be seen as representing one out of the M frequencies in an M-ary Frequency Shift keying modulation scheme, for example. Each user has a codebook of cardinality greater or equal to M and there can be a maximum of $M-1$ users communicating at the same time through a multiple access OR channel.**

I. Introduction

Balakirsky and Vinck [1] showed that it is possible to construct *uniquely decodable* permutation codes for a multiple access OR channel where the inputs originate from a Fast Frequency Hopping / Multiple Frequency Shift Keying modulation (MFSK). In [1], T independent senders are permitted to transmit M-ary codewords of length M and T receivers receive a vector of sets of length M. The t-th receiver tries to determine the message of the t-th sender. The receiver consists of M frequency detectors to identify which symbol is present or absent.

The construction in [1] is termed an M, T, L, d scheme, where M is the length of the permutation codeword, T is the number of pairs (sender receiver), L is the number of codewords per sender, and $(d-1)$, the maximum number of jamming signals the code can handle and still allow for unique decodability. The uniquely decodable permutation codes are constructed based on the so-called, *individual entry* which allowed for unique decodability. The individual entry allows a codeword, for the case $L=1$, to be uniquely identified by having a symbol that is unique to that particular user in a specific time slot. The idea of code construction using individual entries was extended to other constructions with different parameters (M, T, L, d). It was noted that for the case where $L>1$ (more than one codeword per sender) the presence of individual entries was not the necessary condition for unique decodability.

In this work we show that a set of permutation sequences, of given cardinality and Hamming distance, can be partitioned to form (M, T, L, d) uniquely decodable permutation codes for multiuser communication with varying performances. The code is uniquely decodable if, per codeword transmission, each user receiver is able to identify the codeword that was sent by its corresponding sender in the presence of other users transmitting at the same time. Each codeword is a set of permuted integers from an alphabet $\{1,\ldots,M\}$ such that each integer appears only once in a codeword. The integers can be seen as representing different frequencies in the case of Frequency Hopping / MFSK. Our focus in this work is on adjusting the values of the parameters T and L such that we affect the distance properties of the code. The error handling capabilities of the codes we present will be probabilistically evaluated, hence we do not specify the parameter d. We define jamming as in [1]: sending of an interfering signal containing all M frequencies in a time slot of a codeword transmission (impulse noise) or containing a fixed frequency spanning all the time slots of a codeword transmission (frequency disturbance).

II. System Model

The communications system model is that of data transmission over a multiple access channel as shown in Fig. 1, see [1] and [2]. As described in [1] and [3], a permutation codeword may be represented as a binary $M\times M$ matrix, X. Each row

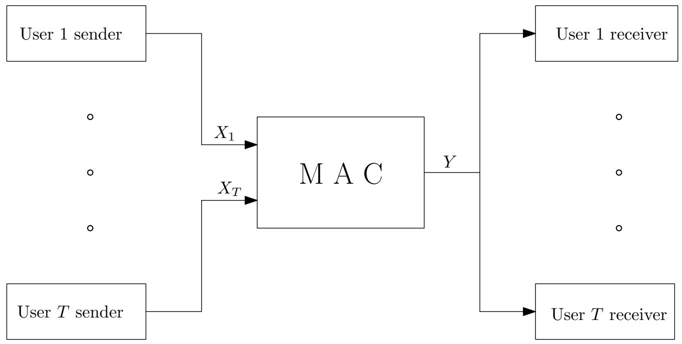

Fig. 1. Multiple access communications system model

and each column of X has exactly one bit equal to 1. A row of X represents the integers in the codeword and a column represents the time slot when each integer appears. For our multiuser case, each user can transmit one of M codewords as a matrix X_i, where X_i is a representation of a codeword from user i, and $1 < i \leq M - 1$. Below is an example of the binary matrix representations X_1 and X_2 of permutation codewords (12453) and(12534), respectively where $M = 5$:

$$12453 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$12534 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The multiple access communication channel described in Fig. 1 accepts the binary matrices, X_i, from the T users and performs an OR function on corresponding elements to produce a single $M \times M$ binary matrix, Y per codeword transmission.

$$Y = \vee X_i, i = 1, \ldots, T$$

The matrix Y feeds into every user receiver and for unique decoding to be possible, the i-th user receiver should be able to determine the codeword sent by the i-th user sender from Y. As an example, taking $T = 2$ and using the codewords above, (12453) and (12534) for the first user and second user, respectively, then the output of the OR channel, Y is:

$$Y = X_1 \vee X_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad (1)$$

To perform decoding at the receivers, each receiver performs an AND operation between Y and each of its given binary matrices. The binary matrix with the largest number of agreements with Y (smallest Hamming distance) is taken as the transmitted matrix, and hence its corresponding codeword is recovered.

III. Problem Statement

It has already been shown in [3] – [6] that permutation codes are good candidates for coding for channels with jamming (frequency disturbances and impulse noise) and background noise because they provide frequency and time diversity. It is known in coding that the minimum Hamming distance is a measure of the error correction capability of a code. In this work we investigate the effect of the Hamming distance on the error handling capabilities of M-ary permutation codes for multiuser communication, by looking at the number of codewords contributing to particular Hamming distances.

In a codebook, the codewords may contribute to different Hamming distances towards each other, we investigate this effect on the performance of permutation codes for multiuser communication that are uniquely decodable in the presence of jamming and background noise. We use examples of codes to show that by partitioning permutation sequences according to Hamming distances properties, it is possible to design codes with varying performances.

Before we present our different partitions of permutation codes for multiuser communication, we first show how the different types of noise mentioned above (frequency disturbances, impulse noise and background noise) affect coded information in the channel model described in Section II. We model the effects of frequency disturbance, impulse noise, and background noise on the binary Y matrix as done in [3] as follows:

For the purposes of demonstrating the effects of the noise, we shall use the error-free binary matrix Y in (1) for each type of noise as an example.

- *Frequency disturbance*: an interfering third-party signal on one or more of the signaling frequencies across one or more time slots. The effect of a continuous single frequency disturbance is represented by an all-ones row in the binary matrix Y, where the row corresponds to the frequency in question. As an example, let frequency 2 be the frequency disturbance on Y. Then we receive the corrupted matrix as:

$$Y_{freq.disturbance} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- *Impulse noise*: we consider an impulse duration equal to the symbol time slot, where a single impulse occurs in a symbol time slot. The effect of an impulse is represented by an all-ones column in the binary matrix Y. As an example, let an impulse occur in symbol time slot 1 on Y. Then we receive the corrupted matrix as:

$$Y_{impulse} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- *Background noise*: can cause the appearance or disappearance of a symbol when a 0 is changed into 1 or vice versa in the binary matrix Y. As an example, let row 1, column 2 in Y encounter a reversal error (background noise). Then we receive the corrupted matrix as:

$$Y_{background} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

For a detailed explanation about the PLC channel characterization and how noise affects data, we refer the reader to [6].

IV. Partitions

Let us define an M-ary permutation codebook to be C_{M-k} with minimum Hamming distance, $d_{M-k} = M-k$, where $k \in \{0, 1, 2, \dots, M-2\}$. For all the values of k, the distances are related as follows, $d_{M-0} > d_{M-1} > d_{M-2} > \dots > d_2$. The maximum cardinality a codebook can have for a corresponding distance is given by,

$$|C_{M-k}| = \frac{M!}{(d_{M-k}-1)!} \tag{2}$$

Equation (2) is adapted from [4], where it is used for single user transmission case. It should be noted that for any C_{M-k}, the distance d_{M-k} is the minimum Hamming distance of the code which means it includes all other codewords contributing to Hamming distances larger than d_{M-k}. For example, a codebook of distance d_{M-2} will also include codewords of distances d_{M-1} and d_{M-0}. The codebook C_{M-k}, where $k \neq 0$, can be partitioned into sub-codebooks such that a permutation code for multiuser communication is formed. Each sub-codebook will be assigned to a particular user, hence the number of sub-codebooks equals the number of users communicating at the same time. To form codebooks for multiuser communication, with good error handling capabilities, the minimum Hamming distance within each sub-codebook should be as large as possible. Since the codewords within a sub-codebook may have different Hamming distances, we strive to create codes that have the largest number of codewords contributing to the largest distance per sub-codebook. This is achieved by introducing codewords within a user such that they optimize the distances and still allowing the code to be uniquely decodable. The following example illustrates the contribution of codewords to the different distances within a user codebook.

TABLE I
Uniquely decodable code for M=4, T=3, L=4

	USER 1	USER 2	USER 3
A	1234	1342	1423
B	3124	2143	3412
C	3241	4132	4213
D	4321	2431	2314

TABLE II
Uniquely decodable code for M=4, T=3, L=4

	USER 1	USER 2	USER 3
A	2143	1342	1423
B	3124	4132	1234
C	3412	4213	2314
D	4321	2431	3241

Example 1: Below are two uniquely decodable permutation codes for multiuser communication with $M = 4$, a minimum Hamming distance $d_3 = 3$, $T = 3$, and $L = 4$.

Since $d_3 = 3$, there are also other codewords with higher distances, $d_4 = 4$, in the codes above. For the codes in Tables I and II, the distance relationships between codewords of user 1, are represented in Fig. 2 (a) and 2 (b) respectively. The distance relationships for codewords of users 2 and 3 give the same diagrams.

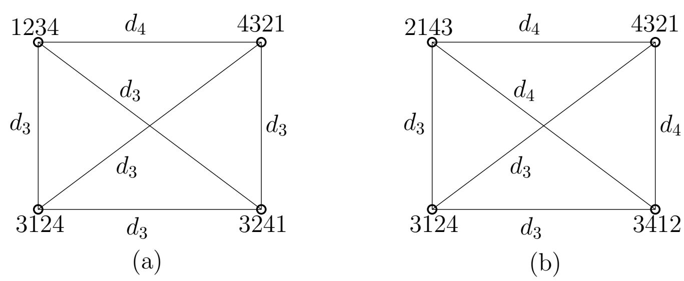

Fig. 2. Hamming distances relationship between codewords for (a) user 1 in Table I and (b) user 1 in Table II

In Fig. 2 (a) above, it can be seen that there are 5 d_3s and 1 d_4, while in Fig. 2 (b) there are 3 d_3s and 3 d_4s. The code represented by Fig. 2 (b) will perform better because it has more larger distances compared to that represented by Fig. 2 (a).

The best known codes which give maximum Hamming distance per user are Linear Congruence codes (LCCs), first introduced by Titlebaum [7]. LCCs are constructed such that, for a given prime M, all the codewords in a user codebook contribute to $d_M = M$, where d_M is the maximum possible Hamming distance. The only partitions we could form that performed better than the LCCs are those with reduced number of users and/or codewords per user compared to the LCCs. An example of such a code is presented in section V.

We shall show with simulation results the effect of the distance within each sub-codebook in Section V. The codes' performances will be evaluated for the probability that a codeword that was not sent appears at the receiver due to errors in the channel. Each codeword has a particular probability of appearing at the receiver when it was not sent. To simplify

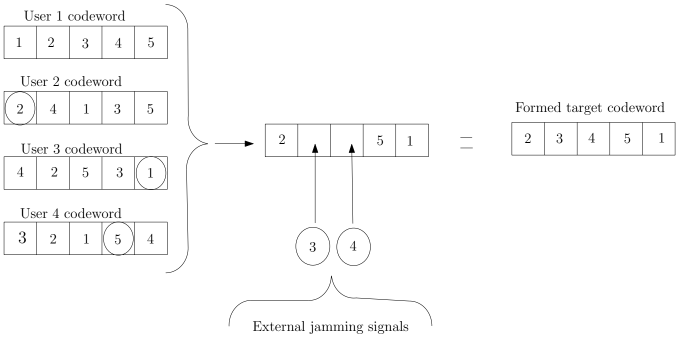

Fig. 3. Codeword construction from other users' codewords and external jamming signals

the simulations we shall look at cases where the distance properties of each user in the code are the same, otherwise it is possible to have asymmetric distance properties.

Another approach of forming the partitions would be to start with empty user codebooks and start populating the codebooks according to the desired distance properties.

A. Error Handling Capabilities of LCCs

We can easily show that the LCCs by [7] can correct at least an impulse or frequency disturbance. Assume we want to try and match all M symbols of a particular codeword j in a particular user i which was not sent. Note that user i cannot contribute any matching symbol to codeword j because codewords in a user have the maximum Hamming distance, M. In the LCC there is a maximum of $M-1$ users, assuming a worse case scenario where each of the $M-2$ users contributes to the matching of the symbols of codeword j in user i, we have $d = M-(M-2) = 2$ symbols unmatched every time. It will therefore take $d = 2$ impulses or frequency disturbances to match all symbols in the worse case scenario. In the worst case scenario the d jammers must contribute precisely to the d unmatched symbols for decoding to fail.

TABLE III
UNIQUELY DECODABLE CODE FOR M=5, T=4, L=5

	USER 1	USER 2	USER 3	USER 4
A	12345	13524	14253	15432
B	23451	24135	25314	21543
C	34512	35241	31425	32154
D	45123	41352	42531	43215
E	51234	52413	53142	54321

The following example illustrates how a codeword that was not sent during a transmission may appear at the receiver of one user and cause decoding failure.

Example 2: Consider the LCC for $M = 5$ in Table III. Assume users 1–4 send the codewords shown in Fig. 3, and that the target codeword to be constructed (codeword that was not sent but appear at one of the receivers) is 23451 in user 1.

In Fig. 3 it is shown that for the target codeword (23451) to be constructed, users 2, 3 and 4 each contributes symbols 2, 1 and 5 respectively, to the matching of 23451, and user 1 cannot contribute a matching symbol. Since the three users 2, 3 and 4 can only contribute one symbol each, the target codeword is short of two symbols to be fully constructed and those two symbols can only come from external jamming signals (assuming no other types of errors are present apart from jamming). As shown in Fig. 3, the jamming signals contribute the missing symbols 3 and 4 at positions two and three, respectively. Note that if a jamming signal contributes a symbol in a position that already has a contribution, either by one of the users or another jamming signal, that jamming signal is considered ineffective in the contribution of symbols to the target codeword.

V. Results

We present simulation results for $M = 4$, $T = 3$, $L = 4$, and $T = 2$, $L = 7$ codes with various error handling capabilities. The simulations were carried out using the channel model presented in Section II. The performance measure for each code was the probability that codewords from a particular user that were not sent were detected at the output of the channel. This becomes an error probability distribution of codewords per user, and we take the average of the error probabilities of the codewords of a user and get the average error probability per user, Pe_U. The system was tested for Pe_U in the presence of background noise, impulse noise and frequency disturbances. We focused on a single user's Pe_U because the codes used gave similar error probability distribution for all users. In Figs. 4–6, we shall simply refer to Pe_U as the probability of error. In this section, we shall refer to the codes in Tables I and II, as Code2 and Code3, respectively. In addition to Code2 and Code3, we present two more codes namely, Code1 (in Table IV) and Code4 (in Table V).

TABLE IV
UNIQUELY DECODABLE CODE FOR M=4, T=3, L=4

	USER 1	USER 2	USER 3
A	1234	1342	1423
B	2143	3124	4132
C	3412	4213	2314
D	4321	2431	3241

TABLE V
UNIQUELY DECODABLE CODE FOR M=4, T=2, L=7

	USER 1	USER 2
A	1234	1342
B	2143	3124
C	3412	4213
D	4321	2431
E	1423	2314
F	4132	3241
G	4123	3214

Figs. 4 and 5 show results for $M = 4$, $T = 3$, and $L = 4$ uniquely decodable permutation codes for multiuser communication with the following distance properties: Code1

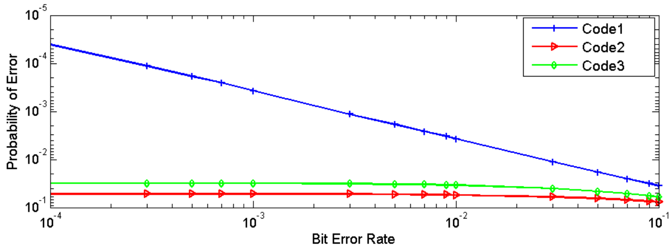

Fig. 4. Comparison of the probability of error for Code1, Code2 and Code3 in the presence of background noise and frequency disturbance at frequency 2

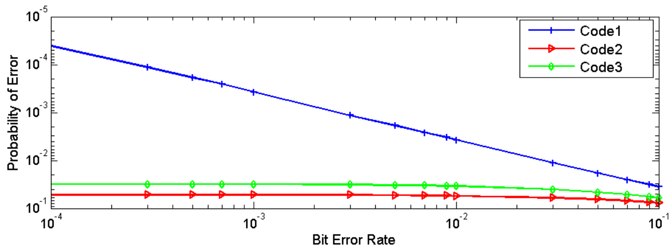

Fig. 5. Comparison of the probability of error for Code1, Code2 and Code3 in the presence of background noise and impulse in time slot 1

has 6 d_4s and no other lower distances, Code2 has 3 d_4s and 3 d_3s, and Code3 has 1 d_4 and 5 d_3s. Fig. 4 shows the performance of the three codes (Code1, Code2 and Code3) in the presence of a frequency disturbance, at frequency 2, and background noise. Fig. 5 shows the performance of the three codes is the presence of an impulse, in time slot 1, and background noise. From the results of Figs. 4 and 5 we see that a code with more higher distances performs better than other codes with less higher distances for both impulse noise and background noise, and frequency disturbance and background noise.

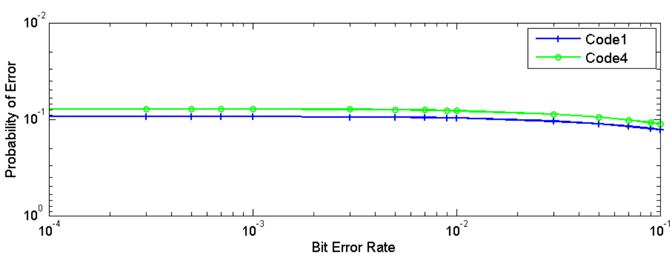

Fig. 6. Comparison of the probability of error for Code1 and Code4 in the presence of background noise, impulse in time slot 1 and frequency disturbance at frequency 2

In Fig. 6 we compare the performance of Code1 against Code4 ($M = 4$, $T = 2$, $L = 7$) in the presence all three types of noise, background noise, impulse in time slot 1 and frequency disturbance at frequency 2. Code4 has the following distance properties, 9 d_4s, 8 d_3s and 4 d_2s.

Code4 performs slightly better than Code1 due to the sacrifice of reducing the values of T and L.

VI. Conclusion

We presented examples of uniquely decodable permutation codes for multiuser communication, and simulation results to show the effect of the Hamming distance on the performance of the codes. The results show that it is possible to design good performing permutation codes for multiuser communication, in the presence of background noise and jamming, by carefully partitioning (or selecting) codewords into groups such that the Hamming distances are optimized.

The codes presented here can find use in cases where several devices are communicating at the same time and each device sends one of several messages through a channel characterized by background noise, frequency disturbances and impulse noise, such as the PLC channel.

The codes presented in this paper are not the only codes one could create, there are many other codes that can be created. An algorithm for generating permutation codes for multiuser communication based on distance properties would be a desirable contribution.

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