Narrow-band Interference Model for OFDM Systems for Powerline Communications

Thokozani Shongwe†
Department of Electrical and Electronic Engineering Science,
University of Johannesburg,
P.O. Box 524, Auckland Park, 2006,
Johannesburg, South Africa
Email: tshongwe@uj.ac.za

Victor N. Papilaya†
Electronic and Computer Engineering Faculty,
Satya Wacana Christian University
Jl. Diponegoro 52-60,
Salatiga 50711 Jawa Tengah
Indonesia
Email: victor.papilaya@uni-due.de

A. J. Han Vinck
†University of Duisburg-Essen,
Institute for Experimental Mathematics,
Ellernstr. 29, 45326 Essen, Germany
E-mail: vinck@iem.uni-due.de

***Abstract*—A narrow-band interference (NBI) model for the powerline communications channel is presented. We give frequency domain details and analysis of the NBI model specifically for OFDM systems; it can easily be adapted to model NBI for other communications systems. We also show that by making the same assumptions as in the Middleton class A model, our NBI model becomes the Middleton Class A noise model.**

***Index Terms*—Narrow-band interference, noise model, Powerline communications, OFDM**

I. Introduction

It is widely accepted that the power line communications channel is characterised by three main noise types namely, coloured background, impulse and narrow-band interference (NBI). Of these three noise types, the narrow-band interference has not been given much attention and as a result it lacks a well defined model. We find some good efforts in describing or modelling narrow-band interference in [1]–[5]. Galda and Rohling [1] gave a brief NBI model description, where the bandwidth of the interfering signal was considered to be small compared to the OFDM subcarrier spacing, hence the interfering signal could be modelled as a single-tone. The frequency and phase of the single-tone interfering signal were assumed to be stochastic with a uniform distribution. The power of the interfering signal was a fixed parameter. In [2], the interfering signal was considered to be a stochastic process in the time domain. The authors in [3] and [4] gave descriptions of interference models together with expressions for the interfering signal. With the aid of the expressions, they explained the conditions under which an interfering signal affects only one subcarrier or several adjacent subcarriers. The work [1]–[4] mostly described the signal processing of the NBI which was sufficient for the purposes of that work. Unfortunately, not much statistical detail of the models was given. The contribution of this article is therefore to give a detailed and well-defined narrow-band interference model applicable to an OFDM system, which is an extension of our previous NBI model in [5]. We will focus to the OFDM system when used in the powerline communications (PLC) channel.

Any communications system, with a given finite bandwidth, is susceptible to frequency interference. The frequency interference can originate from several sources and appear on the frequency spectrum of the system as interfering signals. We will call an interfering signal, an *interferer*. In the powerline communications, the sources of frequency interference can be classified into two main classes: 1. interference due to electrical devices connected in the same PLC network as the transmitter of the desired signal, and 2. interference due to radio broadcasters. The devices connected to the PLC network cause interference at their switching frequency and they usually affect PLC transmission taking place in frequencies up hundreds of kilohertz. Radio broadcasters commonly operate in the megahertz region of the frequency spectrum and will interfere with PLC transmission occurring in the megahertz region. In this work, we shall focus on interferers with narrow bandwidth and occurring from independent sources. This view is somewhat generic to the two classes of sources of frequency interference in PLC already mentioned.

Now, we assume that interferers originate from different independent sources, and we also assume that a very large number of interferers rarely occurs within a given frequency band. We therefore model the probability of a certain number k of interferers on the system's spectrum as a Poisson distribution

$$P_k = \frac{\eta^k e^{-\eta}}{k!}, \tag{1}$$

where $k = 0, 1, \ldots \infty$.

In general, η in (1) is a quantity indicating the average number of occurrences of certain events, defined over a specified observation period. In our model, we define the average fraction of bandwidth occupied by NBI in a system bandwidth W as

$$\lambda = \frac{\eta \bar{\Omega}}{W},$$

where η the average number of interferers with average bandwidth $\bar{\Omega}$. P_k then is the probability that there are k such interferers on the frequency band W. The next task is then to find the power of the interferer(s) that affects the system as

NBI, and to also approximate the probability distribution of the power of this interference (noise) in the system.

We specify our system of interest in this paper as the OFDM (Orthogonal Frequency Division Multiplexing) system, and we shall present our NBI model for this system.

Our NBI model defines the interferers in the frequency domain, with the assumption that they correspond to real signals in the time domain. The effect of the interferers on the system is also completely described in the frequency domain.

OFDM Signal Generation Overview:

OFDM is a multicarrier transmission scheme, where data is carried on several subcarriers which are orthogonal to each other to avoid mutual interference. In the OFDM system of interest, an IDFT (inverse discrete Fourier transform) takes in as input, data symbols carried in vector D_s, from a phase-shift-keying (PSK) modulation scheme and produces a discrete sequence in the time domain, d_n. The relationship between D_s and d_n is represented by

$$d_n = \frac{1}{\sqrt{N}} \sum_{s=0}^{N-1} D_s e^{j2\pi ns/N}, \tag{2}$$

where N is the number of subcarriers used to carry data. d_n is the complex baseband transmit signal from the output of the IDFT normalized by the factor $\frac{1}{\sqrt{N}}$.

II. NBI Power

Let us define an arbitrary interferer $x(n)$, of bandwidth Ω, as a discrete-time signal which is a sum of arbitrary single-tone signals as

$$x(n) = \sum_{i=0}^{\Omega} A_i e^{j(2\pi f_i n + \phi_i)}, \tag{3}$$

where A_i, f_i and ϕ_i are the corresponding amplitudes, frequencies and phases of the different arbitrary signals, respectively. At the receiver the interferer goes through the N-point DFT and appears on the OFDM spectrum. The result of this operation is described by

$$X(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}, \tag{4}$$

where $X(\omega)$ is the amplitude spectrum of $x(n)$ after the DFT. We represent the continuous amplitude spectrum of $X(\omega)$, in frequency f, as $X(f)$.

If the interferer is a single-tone then it can be described by the i^{th} signal as

$$x_i(n) = A_i e^{j(2\pi f_i n + \phi_i)} \tag{5}$$

and its amplitude spectrum, after applying a rectangular window and DFT, is given by

$$X_i(f) = A_i e^{j\phi_i} e^{j(N-1)(\pi f_i - \pi f)} \frac{\sin N(\pi f_i - \pi f)}{\sin(\pi f_i - \pi f)} \,^{1}. \tag{6}$$

[1]The derivation of the equation can be found in the Appendix

From here onwards, we will refer to the power or amplitude spectrum of a time domain signal (or interferer) after windowing and DFT, simply as power or amplitude spectrum of the signal (interferer).

Now, returning to the amplitude spectrum in (4) which has a bandwidth Ω. We are interested in the effect of the power spectral density (PSD) of the interferer $x(n)$ on the OFDM spectrum and hence, how the interferer affects the OFDM signal as noise in the frequency domain.

In Fig. 1 we show an arbitrary power spectrum of the interferer $x(n)$ denoted by $|X(f)|^2$; together with that of two consecutive subcarriers SC$_m$ and SC$m+1$, centred at f_m and f_{m+1}, respectively.

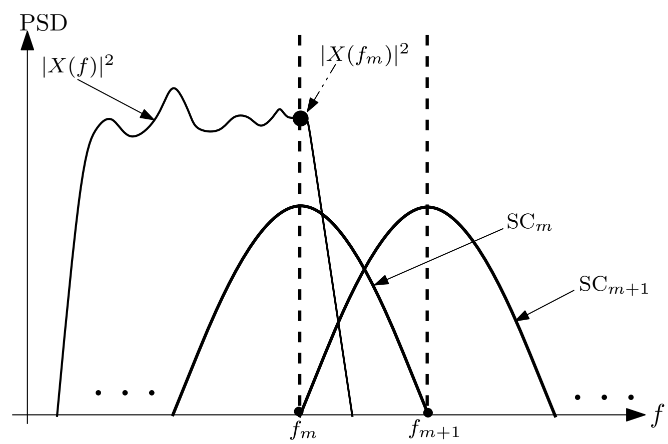

Fig. 1. The effect of the power spectrum $|X(f)|^2$ of an arbitrary interferer $x(n)$, on the OFDM spectrum showing two neighbouring subcarriers SC$_m$ and SC$_{m+1}$. The Centre frequencies of SC$_m$ and SC$m+1$ are f_m and f_{m+1}, respectively. $|X(f_m)|^2$ is the power contribution of the spectrum $|X(f)|^2$, of the interferer, on SC$_m$.

In Fig. 1 we also show some *power contribution* of the interferer on subcarrier SC$_m$ as $|X(f_m)|^2$. This power contribution on a subcarrier(s) is the interferer's power that affects the subcarrier as noise or effective noise power. The power contribution $|X(f_m)|^2$ of the interferer on a subcarrier(s) can vary depending on the position of the interferer's PSD on the OFDM spectrum. We will shortly explain this power contribution in relation to the position of a given interferer on a subcarrier.

Focusing on one subcarrier SC$_m$, we are interested in finding the power contribution of the interferer $x(n)$ on the subcarrier, in the frequency domain. The DFT (of the OFDM receiver) samples the received signal at the centre frequency of each subcarrier to get the transmitted data. Therefore, the interferer contributes its power to the subcarrier centred at f_m when its power spectrum is evaluated at f_m. This evaluation of the interferer's PSD at f_m results in the power contributed being $|X(f_m)|^2$, as already stated.

It is sufficient to show the power contribution of the interferer(s) on one subcarrier, because the analysis of the power contribution on every other subcarrier follows the same manner. As such, the analysis of the power contribution on one subcarrier can be applied to all subcarrier.

It should be noted that when the bandwidth of the interferer, Ω is larger than the subcarrier spacing, the interferer may contribute power to several adjacent subcarriers. The contribution of the interferer's power to one subcarrier can be analysed without affecting the contribution to another subcarrier. This is in agreement with our argument that analysing the interferer's power contribution on one subcarrier is enough to give us an understanding of the frequency interference on every subcarrier.

Now, let us have an interferer of fixed average amplitude A and bandwidth Ω ($0 < \Omega < W$). This interferer can be located anywhere along the OFDM frequency spectrum W, its position is therefore unknown (variable y). We are interested in finding the noise power contribution of this interferer on a particular subcarrier with a centre frequency f_m. The power contributions of the interferer at f_m are the instantaneous points on the interferer's PSD when evaluated at f_m. Let the position of the interferer, around the subcarrier, have a uniform probability distribution. Since the interferer has bandwidth Ω, then the probability distribution of its power contributions on the subcarrier at f_m, is $P = 2\Omega/W$. The factor of 2 is due to the fact that the interferer's PSD can be on either side of f_m, in position, and still contribute power at f_m. So, each power contribution of the interferer on a subcarrier has equal probability P, where each power contribution corresponds to the instantaneous location/position of the interferer. This enables us to calculate the average power contribution of the interferer in question on a subcarrier, which will be the sum of all the possible power contributions weighted by the their probabilities P. We denote this average power contribution of a single interferer on subcarrier centred at f_m by $\bar{\chi}$, and is defined as

$$\begin{aligned}\bar{\chi} &= \int_0^{\Omega} P|X_y(f_m)|^2 dy \\ &= \frac{2\Omega}{W}\int_0^{\Omega} |X_y(f_m)|^2 dy, \qquad (7)\end{aligned}$$

where y are the different frequency values (positions on the spectrum) the interferer can assume, which is a variable over the bandwidth of the interferer. The term $\int_0^{\Omega} |X_y(f_m)|^2 dy$ in (7) gives the total power of the interferer, and $\bar{\chi}$ gives the average effective power, which is the power contributed by the interferer on the subcarrier.

To make the analysis simpler for any given number of interferers k, we assume they have an average bandwidth $\Omega = \bar{\Omega}$ and the same A, and hence the same average effective power $\bar{\chi}$ as calculated in (7). Therefore the power contribution, due to the k interferers, is the sum of the average effective power of the individual interferers,

$$\sigma_k^2 = k\bar{\chi}. \qquad (8)$$

We call σ_k^2 the effective narrow-band interference (NBI) power, given k interferers. The total average effective NBI power in the system is

$$\begin{aligned}\sigma^2 &= \sum_{k=1}^{\infty} \sigma_k^2 P_k \\ &= \bar{\chi}\sum_{k=1}^{\infty} kP_k \\ &= \bar{\chi}\eta, \qquad (9)\end{aligned}$$

where P_k is as defined in (1) and $\bar{\chi}$ as defined in (7).

Note: If the positions of the interferers are mostly static on the frequency band of interest, as might be the case with the PLC channel, the model will still apply, but with the following changes on the average effective power $\bar{\chi}$ in (7). The position of an interferer will no longer have a probability distribution because it is fixed, and its power that affects a subcarrier centred at f_m will be one of the $|X_y(f_m)|^2$, for $y = 0 \ldots \Omega$. We can define the average effective power, $\bar{\chi}$, as the average of all the values $|X_y(f_m)|^2$, for $y = 0 \ldots \Omega$.

$$\bar{\chi} = E\left\{|X_y(f_m)|^2\right\}, \qquad (10)$$

where $E\{.\}$ means the expectation. It should be noted that whether the positions of the interferers are static or changing, that has no bearing on the probability of having interferers in the system, P_k.

A second issue to be noted is that when the average bandwidth of the interferers $\bar{\Omega}$ gets larger, there is likely going to be more adjacent subcarriers affected by an interferer as NBI. Remembering from Section I that $\lambda \propto \bar{\Omega}$, then as $\bar{\Omega}$ gets larger so does λ. This means that the probability of having NBI in the system increases in proportion to an increased $\bar{\Omega}$. As stated, interferers with larger bandwidth may affect adjacent subcarriers and this will likely result in a burst of errors. Whether adjacent or random subcarriers are affected, the average number of errors that will occur in the system does not change. This burst error phenomenon can easily be taken into account by using models that consider channels with memory for example, a Gilbert-Elliot model.

Similarities to Middleton's Class A Noise Model:

Just as in the Class A noise model by Middleton [6], the NBI model, in the frequency domain, can be seen as an infinite number of parallel channels each with effective NBI power σ_k^2 ($k = 0 \ldots \infty$) and additive white Gaussian noise of variance σ_g^2, where each channel is selected with probability P_k prior to transmission. This channel model is shown in Fig. 2, where a symbol D is affected by AWGN of variance σ_g^2 and NBI of variance σ_k^2.

Now, let us make the assumption that the NBI amplitude due to k interferers is a Gaussian random variable, and can take any value z, with mean μ and variance σ_k^2. Then this interference (noise) due to k interferers has a Gaussian distribution with a PDF (probability density function) defined as $\mathcal{P}(z|k) = \mathcal{N}(z; \mu, \sigma_k^2)$.

$$\mathcal{P}(z|k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\frac{-(z-\mu)^2}{2\sigma_k^2}. \qquad (11)$$

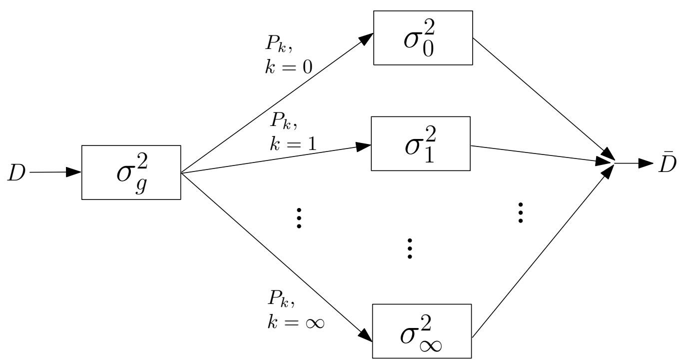

Fig. 2. Narrow-band interference model including additive white Gaussian noise (AWGN). A symbol D affected by AWGN of variance σ_g^2. Then it enters one of infinite parallel channels with probability P_k, and in that channel the symbol is affected by NBI of power σ_k^2 such that the output data symbol $\bar{D}$ is the original D plus noise of variance $\sigma_g^2 + \sigma_k^2$.

Then the probability distribution of the NBI z is

$$\mathcal{P}(z) = \sum_{k=1}^{\infty} \mathcal{P}(z|k) P_k. \quad (12)$$

With the results of Equation (11) and Equation (12), we have arrived at the same result of Middleton Class A noise model [6], where it was assumed that the noise has a Gaussian distribution.

III. How the NBI Model is Applied

To demonstrate the application of our NBI model, we shall use an example. Firstly, we modify the infinite-parallel-channel model in Fig. 2 into an easy to use two-parallel-channel model as shown in Fig. 3.

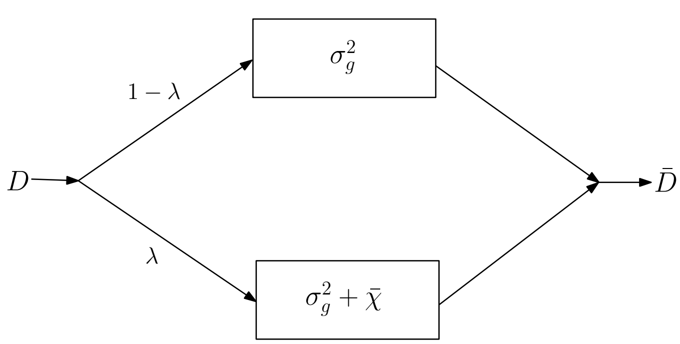

Fig. 3. Narrow-band interference model including additive white Gaussian noise (AWGN). A symbol D either enters a channel with AWGN (variance σ_g^2), with probability $1-\lambda$, or enters a channel with noise $\sigma_g^2 + \bar{\chi}$, with probability λ.

A transmitted symbol from any modulation is affected by AWGN of variance σ_g^2, with probability $1-\lambda$. The symbol is affected by AWGN and NBI of average power $\sigma_g^2 + \bar{\chi}$, with probability λ. That is, a symbol "chooses" one of the channels in Fig. 3 according to the entrance probabilities λ or $1-\lambda$.

Now, let $\lambda = 10^{-2}$ and $\bar{\chi} = 10$. For AWGN, $\sigma_g^2 = 1$. Having specified the variances of the AWGN and the NBI, we now want to specify the noise samples of the AWGN and that of the NBI, which we call n_g and n_N, respectively. We focus on n_N because n_g is known, it is AWGN.

Let us look at two distributions for the NBI sample n_N, which are the *uniform distribution* and *Gaussian distribution*.

Case A: NBI has a uniform distribution.
For a uniform distribution with limits a and b:

- $n_N = a + (b-a)R_u$.
- R_u, a function that generates a random number from a standard uniform distribution on the open interval $(0,1)$.
- the variance $\bar{\chi}$ and given by $(b-a)^2/12$.
- since we know the variance we need to specify a and b, let $a = -b$, then we have $b = \sqrt{\bar{\chi}}\sqrt{3} = \sqrt{10}\sqrt{3}$.
- $n_N = -\sqrt{10}\sqrt{3} + 2\sqrt{10}\sqrt{3}R_u$.

Case B: NBI has a Gaussian distribution with mean $\mu = 0$.

- $n_N = \sqrt{\mu} + \sqrt{\bar{\chi}}R_g$.
- R_g, a function that generates a random number from a standard normal distribution.
- $n_N = \sqrt{10}R_g$.

The symbol is affected by noise as follows:

- with probability λ: $\bar{D} = n_g + n_N + D$
- with probability $1-\lambda$: $\bar{D} = n_g + D$,

where $n_g = R_g$ because $\sigma_g^2 = 1$.

If D is complex valued, then n_N and n_g have to be complex too. For example, in [5] we generated each NBI sample n_N as a complex random value using the function R_g, such that the sample was $\sqrt{\bar{\chi}}R_g + j\sqrt{\bar{\chi}}R_g$. As such, the NBI generated in [5] can be viewed as a random phasor that can rotate in any direction, with the real and imaginary components each having a magnitude determined by R_g and $\sqrt{\bar{\chi}}$.

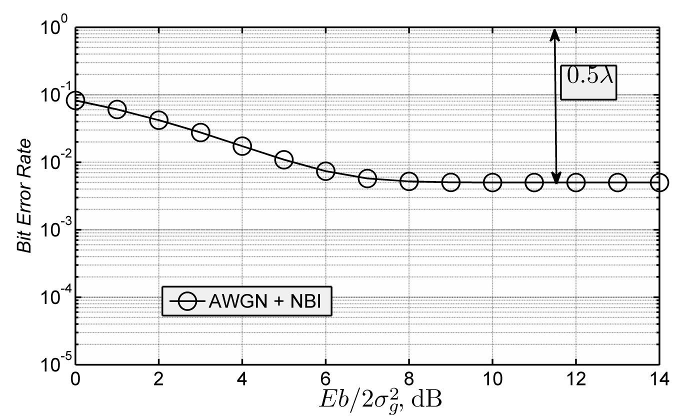

Fig. 4. BPSK–256OFDM modulation bit error rate performance in the presence of NBI with $\bar{\chi} = 10$ and $\lambda = 10^{-2}$, and AWGN with $\sigma_g^2 = 1$.

In Figs. 4 and 5 we give the simulation results of **Case B**, where the system is OFDM with $N = 256$ subcarriers, and BPSK is used as the modulation. In Figs. 4, we set $\lambda = 10^{-2}$ and $\bar{\chi} = 10$. The probability of error caused by NBI for a BPSK modulation will be $0.5 \times \lambda = 0.5 \times 10^{-2}$. The error floor in Fig. 4 confirms this estimate of the probability of error. The Signal to Noise Ratio (SNR) is $Eb/2\sigma_g^2$ in Fig. 4, hence the persistent error floor. The role of Fig. 4 is to indicate the probability of error caused by NBI without paying special attention to the power of the NBI. We address the issue of the effect of the NBI power in the SNR in Fig. 5.

Fig. 5 shows the simulation result with similar parameters to that of Fig. 4, except that now the SNR includes the NBI power and is $Eb/2(\sigma_g^2 + \bar{\chi})$. Also in Fig. 5, we have set $\lambda = 10^{-2}$ and $\bar{\chi} = 100$. The SNR gap between the graph of AWGN only and that of AWGN + NBI confirms the NBI power, $\bar{\chi} = 100$.

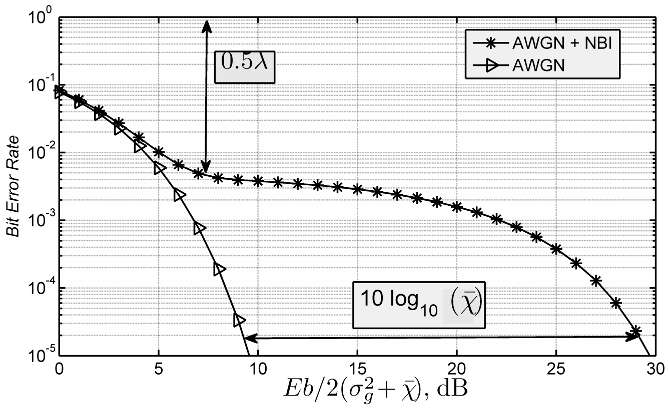

Fig. 5. BPSK–256OFDM modulation bit error rate performance in the presence of NBI with $\bar{\chi} = 100$ and $\lambda = 10^{-2}$, and AWGN with $\sigma_g^2 = 1$.

IV. Conclusion

We have given a narrow-band interference model which is applicable to the PLC channel when an OFDM system is used. In the model we gave the probability with which this NBI power affects data. We also showed how to calculate the average effective power of the narrow-band interference, from a number of interferers, that affects the OFDM system. The average effective NBI power can be modelled with an appropriate distribution; in this paper we demonstrated the use of two distributions which were the uniform and Gaussian distribution, and gave numerical results for the Gaussian distribution case.

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* All the authors in this paper contributed equally.

Appendix

Let $x_i(n)$ denote a discrete time signal,

$$x_i(n) = A_i e^{j(\omega_i n + \phi_i)}, \tag{13}$$

where $\omega_i = 2\pi f_i$, A_i is the amplitude and ϕ_i phase of the signal. To simplify the expression we set $A_i = 1$ since it is a constant that does not play any role in the finding of the Fourier transform of $x_i(n)$.

The Fourier transform of $x_i(n)$, $X_i(\omega)$ from an N-point DFT, is

$$\begin{aligned} X_i(\omega) &= \sum_{n=0}^{N-1} e^{j(\omega_i n + \phi_i)} e^{-j\omega n} \\ &= e^{j\phi_i} \sum_{n=0}^{N-1} e^{jn(\omega_i - \omega)} \\ &= e^{j\phi_i} \frac{1 - e^{jN(\omega_i - \omega)}}{1 - e^{j(\omega_i - \omega)}} \\ &= e^{j((\frac{N-1}{2})\omega_i + \phi_i)} e^{-j(\frac{N-1}{2})\omega} \frac{\sin \frac{N}{2}(\omega_i - \omega)}{\sin \frac{1}{2}(\omega_i - \omega)} \\ &= e^{j\phi_i} e^{j\frac{N-1}{2}(\omega_i - \omega)} \frac{\sin \frac{N}{2}(\omega_i - \omega)}{\sin \frac{1}{2}(\omega_i - \omega)}, \end{aligned}$$

where $\omega = r\frac{2\pi}{N}$, for $r = 0 \ldots N$.

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