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ANALYSIS OF A SELF-STARTING SYNCHRONOUS RELUCTANCE MACHINE WITH ROTOR CAGE BARS IN THE AIR BARRIERS

By

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In the

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UNIVERSITY OF JOHANNESBURG

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Co-Supervisor: Prof D. Nicolae

November 2018
DECLARATION

I, ANAYOCHUKWU VITALIS AGUBA hereby declare that the thesis submitted for the degree MPhil in Electrical and Electronic Engineering Science at the University of Johannesburg is my own original work and has not been previously submitted to any other institution of higher learning. I also declare that all sources cited or quoted are indicated and acknowledged by means of comprehensive list references.

........................................

Anayochukwu Vitalis Aguba
DEDICATION

This dissertation is dedicated to my parents, siblings, uncles, and my Nephews. Your prayers, immense love, sacrifices and moral support has took me all this far and made all this come together, I will forever remain blessed and grateful. To the Faculty of Engineering, the department of Electrical and Electronic Engineering and all the staff members, the love and supports have anchored me throughout the period of this study. This work is also dedicated to all my fellow postgraduate students, my colleagues at work and to all my friends out there. I am very delighted to be one of you.
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ABSTRACT

This work presents the modelling and analysis of a self-starting synchronous reluctance machine (SSSynRM) having distributed cage bars in the flux barriers of the rotor structure. The frame of a conventional 1.5 kW, four-pole, 50Hz, three-phase Induction machine (IM) is used to conceptualize and model the SSSynRM. Due to microscopic machine design variables involved in the modelling of the SSSynRM, two-Dimensional (2D) Finite Element Method (FEM) is used to design and model the machine in both magnetic-static and AC magnetic transient solvers.

The numerical computation of electromagnetic parameters and performance indexes of the SSSynRM are obtained through Finite Element Analysis (FEA). The flux-density distribution and the machine inductances are obtained from the magnetic-static solver, while the performance characteristics under transient and steady-state operations are obtained from AC magnetic transient solver.

In addition to the FEA, the dynamic model of the SSSynRM has been studied and established in both machine variables and arbitrary reference frame. The validation of the dynamic model has been carried out in Matlab/Simulink platform to analyse transient characteristics of the self-starting SynRM. Results obtained from FEA have been compared to those obtained from Matlab/Simulink environment, hence have shown a clear correlation in between the results.

In view of appreciating the performance behaviours of the SSSynRMs, a different model with cage bars outside the rotor flux barriers (SynRM-A) and inside rotor flux barriers (SynRM-B) have been studied. Results obtained from both of the models have proven that SSSynRM with rotor cage bars produces a sufficient starting torque with much minimised copper losses in the rotor, hence with reduced torque ripple contents. It also shown the reduction of torque harmonic contents in the variation of $dq$-axis inductances.
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LIST OF SYMBOLS AND ABBREVIATIONS

Roman alphabet

\( B_{qq} \) \( q \)-axis air-gap flux density
\( B_{gd} \) \( d \)-axis air-gap flux density
\( Coe \) Coenergy
\( f \) Field
\( IE4 \) Premium efficiency class
\( i_{qs} \) \( q \)-axis stator current
\( i_{qr} \) \( q \)-axis rotor current
\( i_{ds} \) \( d \)-axis stator current
\( i_{dr} \) \( d \)-axis rotor current
\( i_m \) Magnetizing current
\( i_s \) stator current
\( i_r \) Rotor current
\( j \) inertia
\( L_{is} \) stator leakage inductance
\( L_m \) Magnetizing inductance
\( L_{ms} \) Magnetizing inductance of stator
\( L_{mr} \) Magnetizing inductance of rotor
\( L_{mA} \) Average magnetizing inductance
\( L_{mB} \) Half-average magnetizing inductance
\( L_q \) \( q \)-axis inductance
\( L_d \) \( d \)-axis inductance
\( L_{ir} \) rotor leakage inductance
$L_{mq}$  
$q$-axis magnetizing inductance

$L_{md}$  
$d$-axis magnetizing inductance

$L_s$  
Stator inductance

$L_r$  
Rotor inductance

$N_s$  
Number of turns

$P$  
Number of poles

$PF_{\text{Max}}$  
Maximum power factor

$r_q$  
$q$-axis resistance

$r_d$  
$d$-axis resistance

$R_r$  
Rotor resistance

$R_s$  
Stator resistance

$T_{\text{ave}}$  
Average torque

$T_{em}$  
Electromagnetic torque

$T_L$  
Load torque

$T_{\text{ripple}}$  
Percentage torque ripple

$v$  
Voltage

$v_{\text{abc}}$  
Voltage Variables associated with stator

$v_q$  
$q$-axis voltage

$v_d$  
$d$-axis voltage

$w$  
Angular velocity

$w_r$  
Rotor angular velocity
Greek Letters

$\lambda$  Flux linkage

$\lambda_{mq}$  $q$-axis magnetizing flux density

$\lambda_{md}$  $d$-axis magnetizing flux linkage

$\lambda_q$  $q$-axis stator flux linkage

$\lambda_r$  $q$-axis rotor flux linkage

$\theta_r$  Electrical angular displacement

$\theta_m$  Rotor angular displacement

$\theta$  Vector angle

$\Delta L$  Change in inductances

Abbreviations

ANN  Artificial Neutral Network

DOLSynRM  Direct-online synchronous reluctance machine

EVs  Electrical vehicles

FEM  Finite Element Method

FEM  Finite Element Analysis

FUF  Fully filled flux barrier

HEVs  Hybrid electrical vehicles

IM  Induction Machine

IPM  Interior Permanent magnet

LSPMaSynRM  Line-start Permanent Magnet Assisted Synchronous Reluctance Motor

LSSynRM  Line-Start Synchronous Reluctance Motor

MTPA  Maximum Torque per Ampere
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>MRT</td>
<td>Maximum rate of change of torque</td>
</tr>
<tr>
<td>MPF</td>
<td>Maximum power factor</td>
</tr>
<tr>
<td>ME</td>
<td>Maximum Efficiency</td>
</tr>
<tr>
<td>MMF</td>
<td>Magneto motive force</td>
</tr>
<tr>
<td>PM</td>
<td>Permanent magnet</td>
</tr>
<tr>
<td>PAF</td>
<td>Partial filled flux barrier</td>
</tr>
<tr>
<td>SSSynRM</td>
<td>Self-Starting Synchronous reluctance machine</td>
</tr>
<tr>
<td>SynRM</td>
<td>Synchronous Reluctance Machine</td>
</tr>
<tr>
<td>TLSynRM</td>
<td>Transverse-Laminated Synchronous reluctance machine</td>
</tr>
<tr>
<td>VSD</td>
<td>Variable speed drives</td>
</tr>
<tr>
<td>2D</td>
<td>Two dimensional</td>
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CHAPTER 1

INTRODUCTION

1.1 Background Information

Synchronous reluctance machines (SynRMs) have a reach history and they have been in existence since the origins of electrical machinery (Muteba, 2017). In the 1900s, Danielson introduced the Synchronous reluctance machine in an attempt to improve the power factor of an induction motor (Griffin, 1954, Hortman, 2004). The partial review of the SynRMs development started with Kostko in 1923 and ended in the late 1950s with successful designs in the United States of America (Armando et al., 2009, Kamper et al., 1996, Muteba, 2017). During that period of time, attention was given to the development of SynRMs because it was considered that the motor can start directly from the mains and get locked into synchronous speed (Hortman, 2004). As from the 1980s, several research publications validated the development of modern power electronic application technology and compared SynRMs with high performance industrial drives for variable speed applications (Vagati, 1994, Lipo, 1991, Boldea, 1996). Hence the introduction of power electronics assured machine designers more promising insight with reluctance machine rotor designs, since the stator of a three phase SynRM is similar to a three-phase induction Motor (IM). Power electronic devices and other control techniques were used to validate if SynRMs have high performance at low speed, produce a constant power operation over a long period of time and with a low acoustic noise (Kamper and Volsdhenk, 1994). It was reported by Lipo, Kamper, Bianchi and Nagrial that SynRMs have high efficiency as compared to IMs because they have no copper losses in the rotor (Lipo, 1991, Kamper and Volsdhenk, 1994, Bianchi et al., 2009, Bianchi et al., 2006, Nagrial et al., 2011). Despite the above mentioned advantages, SynRMs cannot start directly on line, and several methods have been suggested to provide the SynRMs with starting torque,
including the introduction of some electrical conducting materials in the rotor slots (Ugale et al., 2014, Rodger et al., 2006, Nagrial et al., 2011, Miljavec et al., 2009, Marcic et al., 2008, Marčič, 2011). The aims were to start the SynRM directly from line without the application of power electronics. The introduction of buried rare earth permanent magnet in flux-barriers was suggested as a means to provide the SynRM with a starting torque, and the method also assists with the enhancement of torque density and efficiency (Wang et al., 2013, Staton et al., 1993). However, due to high cost and limited deposit of rare-earth permanent magnet materials, SynRMs without magnets are becoming more appropriate alternative solution to rare-earth permanent magnet synchronous machines (Wang et al., 2013, Staton et al., 1993, Rahman et al., 2000, Kamper and Volsdhenk, 1994, Bianchi et al., 2006).

Consequently, a self-starting SynRM should have the capability to start on its own and synchronise at the required synchronous speed when supplied from a steady state voltage without an external control. While considering the environmental awareness and global market, the self-starting mode of synchronous reluctance machine promises the highest possible efficiency that makes it a defensible substitute to induction motors.

Three-phase IMs have been used for several applications because of its numerous advantages such as low cost, easy to construct and they do not require external starting sources (Boldea and Nasar, 2009). Although IMs are the most used types of line-start electrical motors, Self-starting SynRMs have also been popular in recent years due to their constant power usability at synchronous speed and better efficiency in comparison with IMs (Negahdari and Toliyat, 2016). Better efficiency arises from zero slip at steady state that induces no current through rotor cage bars. But due to IMs poor efficiency and coper loss issues, the induction machine seems unsuitable and does not meet the international requirements for high efficiency IE4 and
the ongoing research classes of electric motors (Dorrell, 2014). Hence, SynRM$s$ appear more desirable alternative over IM$s$ for industrial applications.

Similarly, SynRM$s$ can be an alternative to other alternating current (AC) machines in terms of high torque density and high efficiency. In terms of torque density and efficiency, the performance of SynRM is similar to an induction machine (Vagati, 1994, Hortman, 2004, Kamper and Volsdhenk, 1994, Boldea, 1996).

Moreover, with a robust rotor configuration, synchronous reluctance machine operates at a very high speed over a long period of time (Fukao et al., 1989). The performance of synchronous reluctance machines is based mostly on two performance indexes, “the inductance difference ($L_d - L_q$), and the saliency ratio ($L_d / L_q$). The higher the inductance difference, the better the torque density. Similarly, the higher the saliency ratio, the better the power factor of the SynRM (Staton et al., 1993, Lipo, 1991, Boldea et al., 1994). A copper or aluminium cage is introduced in the rotor structure to provide the SynRM with capable starting and accelerating torques that will speed up the motor to synchronism so that it could enter into the steady-state mode of operation. The design of a self-starting SynRM$s$ should be such to ensure proper performance in all three modes of operation: -starting, accelerating and steady-state.

To ensure that the self-starting SynRM rises up to its expectations, it should operates properly in all its three-modes. Consequently, several work presented in recent years focussed more on the design guidelines to ensure proper operation of the self-staring SynRM. In 2013, Gamba has presented a lumped parameters model of the self-staring SynRM to investigate the second mode of operation, providing guidelines to overcome dilemma between synchronization capability and steady-state performance (Gamba, et al., 2015). In the later, the transient pull-in capability of the self-starting SynRM was studied using FEA and successfully validated through experimental measurements. Though the effect of filling conducting material in flux
rotor barriers of SynRM has been evidenced through simulation and experimental results in Gamba’s work, the influence that the number of flux barriers and their positioning has on the starting capability and other electromagnetic parameters of interests, has still yet to be reported. A novel self-starting SynRM was reported by Yeswanth in 2015. In the report, four different SynRM rotor structures filled with copper and aluminium in alternative flux barriers was proposed to improve the operational performance in all three modes (Yeswanth, et al., 2015). On the other hand, Negahdari studied intensively the effect of crawling of the self-starting SynRM, utilizing a sample cage configuration. The self-starting SynRM model and characteristics have been presented to give better perception of successful and unsuccessful synchronization consequences (Negahdari and Toliyat, 2016).

1.2 Problem Statement

The research background in previous section indicates that recent work done to ensure proper performance of the self-starting SynRM in all three modes of operations focused mostly on design guidelines and on issues related to successful and unsuccessful synchronization. It also provides details of effect of filling as much space as possible with conducting materials in flux rotor flux barriers on the starting capability of SynRMs. There is not a comprehensive study that illustrates the effect that the positioning of conducting material in rotor structure has on performance parameters in all three modes of operations: -starting, accelerating and steady-state.

Therefore this dissertation investigates and analyses the effect that the positioning of conducting material in rotor structure has on transient characteristics of a self-starting SynRM in all three modes of operations. The study is accomplished by means of numerical method: - ac transient magnetic solutions, dynamic modelling: - in both machines variables and arbitrary
reference frame, and then validation of dynamic model through Matlab/Simulink computation.

In this context the following are the sub-problems that are addressed in this dissertation:

- The utilization of proper design guidelines to ensure that the steady-state performance indexes of the SynRM are not compromised for the sole purpose to provide the motor with excellent transient performance.
- The use numerical method to develop and implement a 2D finite element model of the machine to study the electromagnetic parameters for transient AC magnetic solutions.
- The study and establishment of the dynamic model of the self-starting SynRM in both machines variables and arbitrary reference frame.
- The validation of dynamic model in Matlab/Simulink environment in order to analyse transient characteristics of the self-starting SynRM in all three modes of operations.

1.3 Research Objectives

The objectives of this study are to:

- Use proper design guidelines during the process of inserting conducting materials in the rotor structure without compromising the steady-state performance indexes of the SynRM.
- Develop and implement a 2D finite element model of the self-starting SynRM in order to successfully study the electromagnetic parameters for transient AC magnetic solutions.
- Study and establish the dynamic model of the self-starting SynRM in both machines variables and arbitrary reference frame.
- Validate the dynamic model in Matlab/Simulink platform.
1.4 Hypothesis

The interaction between electrical and magnetic field leads to skin effect. The latter is due to the increase in current density from the centre of a conductor to its peripheral point. The skin effect leads to an uneven distribution of current density, as a result its affects the starting characteristics of the machine. Moreover, an increase in resistance of a conductor leads to skin effect which outcomes to unnecessary losses in the conductor of the motor. Also, inserting cage bars in the air flux barriers of the rotor structure near the air-gap minimises the conductor losses. This leads to the smooth distribution of current density and provide desirable transient behaviours of a synchronous reluctance machine.

1.5 Delimitation of Study

This dissertation deals only with the modelling and analysis of a self-starting transverse-laminated SynRM ((TLSynRM) having rotor cage bars in the air barriers. The work focused mostly on the analysis of the transient characteristics of the self-starting SynRM. Issues related to crawling during acceleration, successful and unsuccessful start of the self-starting SynRM do not form part of this work. Although the FEA takes into account saturation, skin effect and eddy current, they have been left out in the dynamic model and in the overall analysis.

1.6 Contributions

This work has presented the transient performance behaviour of a self-starting synchronous reluctance machine (SSSynRM) with uniformly distributed rotor bars. The various findings of the analytical and simulation results are reported. The contributions of this work are found in chapters four, five and six of this dissertation. In chapter four, the self-starting SynRM is modelled using a 2D finite element method for magnetostatic and ac magnetic transient solutions. In chapter five, the dynamic model is studied and established, then validated in
chapter six through Matlab/Simulink. The research outputs of this work are reported in three
different technical papers that have been published in conference proceedings as detailed
below:

  Reluctance Motors with Uniformly Distributed Cage Bars in the Rotor Structure”.  
  *International Conference on Optimization of Electrical and Electronic Equipment*
  *(OPTIM) & Intl Aegean Conference on Electrical Machines and Power Electronics*
  *(ACEMP), 2017 International Conference. Brasov, Romania.*

- A. V. Aguba, M. Muteba, and D.V. Nicolae, “Transient Analysis of a Start-up
  Synchronous Reluctance Motor with Symmetrical Distributed Rotor Cage Bars”.  

  Analysis of Synchronous Reluctance Motor with Cage Bars in the Rotor Structure”.  
  *In proceeding of SPEEDAM 2018. 20-22 June 2018, Amalfi Coast –Italy.*

1.7 Dissertation Outline

Chapter 2

This chapter presents the literature review of a standard synchronous reluctance machine
developments and its performances. The theoretical analysis and analytical achieved
performance indexes were reported and presented. Different rotor geometries using electrical
machine design approaches were reviewed, analysed and presented.
Chapter 3

Chapter three discusses on the mathematical modelling of a standard three-phase four-pole SynRM to set up the hypothetical background for the modification of the proposed machine and as a guide for the introduction of this dissertation realisation.

Chapter 4

It focused on finite element method (FEM), a technique used to develop and build the geometrical structure for the proposed self-starting SynRM machine. The characteristics of the proposed SynRM machine were evaluated using 2D finite element analysis (FEA) and the performance indexes with emphasis on the benefits of the machine when fitted with cage bars in the flux barriers of the rotor structure.

Chapter 5

This chapter presents the mathematical modelling and analysis of synchronous reluctance machine fitted with uniform distributed cage bars in the rotor structure. The $dq$ model of the proposed SynRM machine is established and its equivalent circuit variables derivations are presented in this chapter. The derived machine variable set up the benchmark for the validation of the proposed machine in MATLAB software.

Chapter 6

In this chapter, the Matlab/Simulink model of the proposed SynRM is presented. The model is built using the mathematical equations that were obtained in chapter five. The models were splits into different sections for simplicity and better understanding. This chapter actually validate the performance results obtained from theoretical analysis, finite element simulations and as well the derived mathematical modelling of the proposed machine. The obtained
Simulink simulation results were compared with the finite element simulation results obtained in terms of starting, synchronising and step in load changes.

**Chapter 7**

The overall findings using the proposed machine characteristics were concisely elaborated and presented. Conclusions and recommendations for future work were also presented in this chapter.
CHAPTER 2
REVIEW OF A SynRM DEVELOPMENT AND ITS PERFORMANCES

2.1 Introduction

With respect to the research background in previous chapter, it shows that previous work done with the starting of SynRMs were mostly focused on the use of external device sources and utilises reluctance and rotating sinusoidal MMF for torque production. This type of motor consists of a salient rotor without a conducting material attached but an arranged laminated steel based on the reluctance principle, thus utilising the principle of magnetisation. It also elaborates work done on SynRMs using permanent magnets in the rotor slots. Different starting techniques of the synchronous reluctance machine (both internal and external) are also numerically studied and published with reputable publishers.

This chapter briefly presents the history of synchronous reluctance machines and the used methods by different authors. The basic principles of synchronous reluctance machine using magnetic field interactions between rotor and stator in the machine airgap is reviewed and analysed in this chapter. The Synchronous Reluctance Motor time-to-time rotor configurations and its control techniques were reviewed, the applications that deals with self-starting of synchronous reluctance machines and the overall theoretical review that draw this chapter into conclusion were also elaborated.

2.2 Basic principle operations

Fundamentally, synchronous reluctance machines make practical and effective use of reluctance concept and rotation of magnetic field for torque production. The electro-mechanical device converts electrical energy to mechanical energy. The synchronous
reluctance machine is primarily designed using first principle to generate magnetic field that produces the magnetic reluctance differences between the direct axis \((d\text{-}axis)\) and quadrature axis \((q\text{-}axis)\). Basically, two-pole salient rotor is placed in a certain position between \(d\text{-}\) and \(q\text{-}\) axis in a magnetic field and aligns with the field producing field distortion in the main field when aligned with \(q\text{-}\) axis. In this condition, there is production of torque aiming to minimise the field distortion experienced in the \(q\text{-}\) axis. This concept was well illustrated by (Pehrman and Jones, 2014, Moghaddam, 2007, Hortman, 2004). These authors concluded that larger torque can be obtained with larger reluctance difference between the \(d\) and \(q\) field aligned differences.

2.3 Stator analysis

It has been shown by many researchers that the stator of a standard three-phase AC machine is similar to that of AC three-phase electrical drives except if the stator has been designed for a different purpose. The stator of a synchronous reluctance machine is constructed using thin steel lamination stamped together into shapes for winding and rotor housing. The stator windings are then inserted into designated slots of the laminated stator core. The SynRM machine coils are insulated from laminations using insulated paper and string to separate the coils.

It is worth mentioning that the stator laminated core used in this work comprises of 36 slot magnetic steel laminations with winding configurations made of distributed single layer concentric coil spread into three slots per pole and per phase as three-phase full pitch. The machine model specifications are presented in Table 2.1. For 4-pole 1.5kW SynRM, a frame size of 90L conventional three phase IM was used as shown in Appendix A (Figure A1) for conceptualise and to model a SSSynRM. Stator coil is energised using three-phase current so that magnetic flux can be induced to attract the rotor steel from the air-gap. The interaction
between the stator and the rotor field produces reluctance torque that rotates the rotor of the SynRM machine. The magnetisation present in the main field and during torque production are both stator current trying to minimise the experienced field distortion when controlling the current angle between the current vector of the stator winding and the rotor d-axis (Moghaddam, 2007, Hrabovcova et al., 2005).

Table 2. 1: Ratings and General Specifications

<table>
<thead>
<tr>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator slot pitch</td>
<td>10° mech</td>
</tr>
<tr>
<td>Airgap length</td>
<td>0.45 mm</td>
</tr>
<tr>
<td>Stack length</td>
<td>112.00 mm</td>
</tr>
<tr>
<td>Number of pole pair</td>
<td>2</td>
</tr>
<tr>
<td>Number of barriers per pole</td>
<td>3</td>
</tr>
<tr>
<td>Number of stator slots</td>
<td>36</td>
</tr>
<tr>
<td>Outer stator radius</td>
<td>67.50 mm</td>
</tr>
<tr>
<td>Inner stator radius</td>
<td>40.00 mm</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>39.55 mm</td>
</tr>
<tr>
<td>Shaft radius</td>
<td>14.50 mm</td>
</tr>
</tbody>
</table>

2.4 Classification of Rotor geometry and its development

Basically, synchronous reluctance machines are into classified into three anisotropic rotor structures (simple-salient rotor, axial-laminated anisotropy rotor and transverse-laminated anisotropy rotor).

2.4.1 Simple Salient Rotor:

The simple salient rotor type was first introduced for synchronous reluctance machines obtained from the removal of iron materials from the rotor of a conventional induction machine motor in the diagonal section as shown in Figure 2.2. The simple salient rotor was well reviewed and investigated using fixed-frequency operation in cage windings. Simple salient machine is used for different applications as detailed by (Rakgati and Kamper, 2004, Lipo and Krause, 1967, Kudła and Miksiewicz, 2004).
The construction technique for this type of rotor is simple, easy to manufacture and very strong for applications where high speed is required. The saliency-ratio of this type of machine is determined by increasing the air-gap length towards the $q$-axis. Despite the merits and effort contributed by the simple salient rotor machine, the machine still suffers low saliency-ratio recorded as 3.8. The low saliency-ratio contributes to the poor performance of the machine, thereby discouraging many machine researchers for its recommendation till construction of segmented rotor, and rotor with flux barriers technique was introduced to achieve better saliency ratio.

2.4.2 Axially laminated rotor type.

Axially-laminated rotor is a four-pole rotor type of synchronous reluctance machine presented in Figure 2.3. The rotor is constructed in a way that the axially-laminated steel sheets are bent into a U-like stacked shape and run axially with pole holder, to which the rotor is connected and separated from each other by means of electrically and magnetically sheets of laminated materials. The U-like or V-like rotor shape is a concept introduced to minimise the magnetic reluctance present in the direct axis ($d$-axis) and maximise the magnetic reluctance present in the quadrature axis ($q$-axis) respectively. During the early days of axially-laminated rotor
invention, it was reported by (Cruickshank et al., 1972), that the saliency-ratio of the motor was recorded he reported as 5.6. Further development was contributed by (Rao, 1976) to increase the saliency-ratio to 6.8 when operating the motor with rotor cage. However, the introduction of power electronic technique confirmed that axially-laminated rotor can be designed to operate at high saliency-ratio if the conducting cage present in the rotor structure is eliminated (Lipo, 1991). (Fratta et al., 1990, Xu et al., 1991) proposed an axially-laminated rotor capable of producing a saliency ratio of 12 while (Boldea et al., 1994) reported a saliency ratio of 21.

Meanwhile, reviewed articles show that axially-laminated rotor without a cage possesses a high saliency, high efficiency, high torque and a good power factor when running applications using vector-control power electronic technique or using the external drive technique.

In contrast, axially-laminated rotor anisotropy is awkward with mass production as axially-laminated rotors are not easy to skew and unsuitable for line-start due to its un-guaranteed mechanical strength and also regarded unsuitable for high-speed applications.

Figure 2.2: Axially-Laminated Four-Pole Rotor Design
2.4.3 Transversely-laminated Rotor

A transversely-laminated rotor is a four-pole machine with punched laminations done in a traditional way similar to that of induction machines (Kamper and Volsdhenk, 1994) as depicted in Figure 2.4. In addition, the standard mass production techniques used in transversely-laminated rotor is similar to that of induction machines. The punch techniques are designed to allow the thin ribs connecting the rotor segments to each other for better rotor laminations. The laminated steel sheets are designed as a flux guide to minimise the direction of flux in $q$-axis and maximise the $d$-axis path. The rotor laminations are then stacked together for skewing and to eliminate the harmonics present in the stator slot.

The construction of transversely-laminated rotor type is considered cheaper than the axially-laminated type and more guaranteed for its mechanical strength and for industrial mass production. On the other hand, the torque, efficiency and power factor in transversely-laminated rotor type underperforms that of axially-laminated rotor type (Vagati et al., 2000, Vagati et al., 1999).

Figure 2.3: Transversely-Laminated Four-Pole Rotor Design.
2.5 Synchronous Reluctance Motor Control Techniques.

SynRM without a rotor cage winding needs an external drive like frequency converter or a prime-mover for start-ups and synchronisation. Recent developments in power electronics and control techniques show that synchronous reluctance machine over perform with their high performance industrial drives for variable speed application (Vagati, 1994, Boldea, 1996). Many machine designers focused on optimising the rotor configuration of the SynRM machine in order to obtain high saliency-ratio. Modern synchronous reluctance machines are driven by the inverters and do not require starting rotor cage windings. Using power electronics technology, the motor can operate with high efficiency at normal speed, high torque, high power density, rapid dynamic response, steady power operation over a long period of time, reduced torque ripples, reliable and robust (Nagrial et al., 2011, Kamper et al., 1996, Chiba and Fukao, 1992).

The development in synchronous reluctance machine rotor design has been studied and investigated by several authors. The use of power electronics and other modern techniques has proven the high performance of the machine in terms of maximising the torque, power factor, increased efficiency and high saliency-ratio. From research study, it was verified that the starting and synchronization of the motor at a good torque and speed performance can be achieved using power electronic technology. (Kamper and Volsdhenk, 1994) finds that with the absence of rotor cage, the synchronous reluctance motors exhibited the highest possible performance indexes with converters and other modern control techniques. The control techniques (maximum torque per ampere (MTPA), maximum rate of change of torque (MRT), maximum power factor (MPF), maximum efficiency (ME), etc.) have been used by different research scholars to maximise the performance of synchronous reluctance motors.
Electric circuits were used to show the advantages of synchronous reluctance machine over induction machines, especially in multiphase-synchronous reluctance machines. Electric circuits reduce the current harmonics and phase current in rotors. It has also been used to maintain torque improvement in synchronous machines. These above-mentioned advantages with electrical circuit technique were analysed and detailed by (Toliyat et al., 1992, Toliyat et al., 1998, Rakgati and Karnper, 2004, Obe and Senju, 2006, Jones, 2002). Synchronous reluctance machines were designed to obtain the highest possible saliency-ratio but not applicable where high speed applications are required due to mechanical strength.

However, applications that require constant speed are common in the industries. Synchronous reluctance motors are recommended when constant speed is desired because it over-performs other AC drives. Synchronous Reluctance machines have been proposed and used by several authors because of its merits for high performance generator application (Boldea et al., 1993), general purpose industrial drive application (Staton et al., 1993), compressor fan and pump applications (Murakami et al., 1999, Villani et al., 2018), servo drive with high-speed application (Vagati et al., 2000), flywheel energy storage application (Hofmann et al., 2004) electric vehicle application (Malan and Kamper, 2001), low-speed wind and hydro-power generation applications (Malan and Kamper, 2001, Ojo et al., 2003).

Synchronous reluctance motor is not a self-starting motor due to the absence of conducting materials in the rotor structure, hence has no starting torque characteristics. In some applications like Fan or pump where high efficiency is commonly preferred, a self-starting synchronous reluctance motor is advised to eliminate some additional losses and extra cost due to converter and other external drives.

Hybrid motor has the capability to start up without incorporating external drives and can operate at high efficiency during steady-state operation (Pehrman and Jones, 2014).
Incorporating squirrel cage in the rotor structure can be used to start-up the synchronous reluctance motor and make it function like induction motors.

2.6 Applications of Self-Starting Synchronous Reluctance Machines

In this research work, the cage bars are positioned in the air-flux barrier of the rotor structure to avoid compromising the saliency-ratio of the machine and guiding the iron-flux pathways.

The technologies satisfying SynRM performance usually incorporate modern technology like variable speed drives (VSD) for starting purposes and to operate successfully in a situation where constant speed application is desired. Brief historical evolution of different self-starting applications can be analysed as follow:

2.6.1 Segmented rotor design.

Segmented rotor design has the same salient pole motor constructions, like conventional salient pole synchronous reluctance motor which its cage windings are eliminated. Segmented rotor is designed to fit the size frame of an equivalent induction motor as shown in Figure 2.5. Segmented rotor was developed and implemented by (Lawrenson and Gupta, 1967, Honsinger, 1971) respectively. It was reported by (Lipo, 1991), where the obtained saliency-ratio of the rotor was up to 5 and its performance was better than conventional rotor design type.

Segmented rotor has been used as line-start synchronous reluctance motor with little and unsaturated saliency of 10 and power factor of 0.7. Despite its merits, segmented rotor was given less attention due to its high complexity and high cost of production. Recent developments with rotor design configuration introduced the use of flux barriers to improve the performance of the motor.
2.6.2 Flux barrier-cage Rotor configuration filled with Aluminium.

During experimental analysis, a researcher (Honsinger) confirmed that segmented rotor is very complex to design and exhibit a high manufacturing cost, he further investigated and developed a rotor with flux barrier to improve the machine’s performance (Honsinger, 1972). A rotor with flux barrier was introduced using fitted magnetic ribes to increase the mechanical strength in the machine. The developed rotor configuration can be seen in Figure 2.6. The Honsinger rotor prototype starts directly at low starting current and can synchronise at a rated speed. The Honsinger rotor prototype exhibited a power factor of 0.67 and an efficiency of 81.6% as reported but there was no report issued on its saliency. On the other hand, reviewers reported that the Honsinger rotor configuration experienced some torque pulsation, unnecessary noise, eddy current losses and induced harmonics in air-gap flux. After so much effort to improve the performance of the rotor, the highest salienc-ratio reported was 4.83 using one flux barrier rotor structure per pole. Further contribution was done by (Kamper et al., 1996, Bomela and Kamper, 2002) using external drives.
2.6.3 Line-start Permanent Magnet Assisted Synchronous Reluctance Motor configuration.

Permanent magnet is a good alternative to overcome the unnecessary rotor copper losses produced by line-start induction motors (IMs) (Isfahani and Vaez-Zadeh, 2009). Permanent magnet operates in two different modes; asynchronous mode during starting like line-start induction motor and synchronous during when the motor runs at synchronous speed (Kim et al., 2009). Permanent magnets are usually inserted inside the air flux barriers of a rotor slots in order to assist the synchronous reluctance motor (see Figure 2.7). In [Lipo, 1991], it was presented that the existence of permanent magnet in rotor flux barriers increases the saliency-ratio of the motor. It was also confirmed by (Schmidt et al., 2002) that SynRMs with permanent magnets exhibit a higher performance through comparison to determine the behaviour of two SynRMs with inserted permanent magnet and without permanent magnet.

In (ARORA, 2015), some approaches to achieve a successful synchronisation using permanent magnets in synchronous reluctance motors was suggested. This approach was based on the
torque produced by cage (Cage torque) and the torque produced from the presence of magnet (braking torque) which were considered as factors for determining the success rate of the motor.

In (Miller, 1984), he showed that magnet alignment torque components contributed more to the synchronising ability of synchronous torque component than the reluctance torque components. The outputs of both torques components result to a successful starting capability with synchronous reluctance motors. Though there were some disruption of demagnetisation by magnets to line-start permanent magnet synchronous motors.

The irreversible demagnetisation properties of permanent magnet was studied using 2D FEM to investigate the currents and voltages produced during starting of LSPMaSynRM (Kang et al., 2003). The irreversible demagnetisation influences the performance of LSPMaSynRM during start-up because of armature reaction. (Behbahanifard and Sadoughi, 2015).

Line start permanent magnets is known to have one rotating speed due to the fixed magnetic polarity of permanent magnets (Aliabad and Ghoroghchian, 2016). The author analysed the approach on how to overcome the problem by designing a two-speed line-start synchronous motor with different magnetic polarity point of view: permanent magnetic and magnetic reluctance. He concluded that, the motor works as PM at one speed and as a synchronous reluctance motor at another speed.

The influencing factor due to armature reaction in permanent magnets was studied and believed that demagnetisation is frequent when the starting process is taking longer time than expected. The longer start-up process was believed to be caused by the inappropriate estimate of some starting variable (inertia, load, voltage etc). (Lu et al., 2011),
The effect of demagnetisation in permanent-magnets synchronous motors were studied and the artificial neutral network (ANN) to investigate the cause of demagnetisation and faults detections in the machine (Lu et al., 2012).

Researcher (Shen) proposed a technique that can protect permanent magnet from demagnetisation. In his work, it was validated that using double-cage rotors and magnetic-barriers configurations, magnets can be protected from deterioration (Shen et al., 2013).

Harmonics is another disadvantage of LSPMaSynRMs. Work was done by (Kurihara et al., 1994, ZAWILAK and Zawilak, 2008) to investigate the harmonic contents of flux density and other variables (e.g. current and electromagnetic torque) using finite element software package. The suppression of harmonic in LSPMaSynRMs was done by (Rong and Manfeng, 2011) using finite element analysis and was practically recorded that harmonic content can be suppressed to meet the appropriate line-start magnet motors requirement.

![Figure 2. 6: Line-start Permanent Magnet Assisted Synchronous Reluctance Motor](image)

Even though, permanent magnet reluctance motors met all the requirements in field of AC drives, the implementation was considered expensive due to inadequate resources and high cost
of rare-earth permanent magnet materials. Permanent magnet machines have complicated design problem when considering high asynchronous and synchronous torque. The average transient torque is low due to the breaking torque in the magnet. Similarly, high transient torque oscillation is not advisable in permanent magnet motors because of mechanical strength.

2.6.4 Line-start Synchronous Reluctance Motor (LSSynRM)

For constant speed applications and where self-start-up is required, LSSynRM is considered as alternative to permanent magnet motor due to the high cost of manufacturing permanent magnet motor.

Also, line-start synchronous reluctance motors are good and desirable alternative solution for high efficiency start-up AC drive applications. LSSynRMs can be traced back to the 60s (Honsinger, 1972), until researchers came up with multiple flux barrier rotor of synchronous reluctance motors for vector-control drives for better saliency than the LSSynRMs (Vagati et al., 1992). Recent design with line-start synchronous reluctance motors seems promising with improved performances. Hence, researchers now focus on how to improve the line-start synchronous reluctance motor without compromising the formal.

In (Gamba et al., 2013), traditional idea was used to achieve starting torque ability of line-start synchronous reluctance motor by just filling up the rotor flux barriers completely with Aluminium and short-circuiting the Aluminium cage at both ends as shown in Figure 2.7. In (Gamba et al., 2015), two rotor geometrics were designed and compared, one partially-filled flux barrier (PaF) with Aluminium and the other was fully-filled flux barrier (FuF) with Aluminium. In his experimental results report, it suggests that the PaF has a higher substantial starting torque than the FuF motor but deteriorated in reluctance torque and efficiency were experienced. It was also concluded from his study that machines with higher reluctance torque
has better pull-in properties than machines with higher cage torque. This strategy does not compromise the pull-in and pull-out torque of the motor. However, the issue related to the saliency of the motor was not reported.

Also, the start-up capability of line-start synchronous reluctance motor was investigated by (Yeswanth, 2015). His perception was filling the rotor flux barrier with Aluminium and copper in different ways. To verify his findings, four different rotor model concepts were designed, and the performance indexes of the motors were analysed, though issue related to saliency ratio of the motor was not reported.

In (Tampio et al., 2016), there was an emphasis on the concept that deals with field magnetism and decomposition torque to analyse the line-start synchronous reluctance machine and was referred to as the direct-online synchronous reluctance machine (DOLSynRM). In their study, the rotor-flux barrier configuration was fully filled with Aluminium, connected to each other with end-ring to form the rotor cage. These authors focused on starting, synchronising aspect and the variable characteristics that hinder the successful starting and synchronisation of a motor.

Thang optimised a rotor design for a line-start single phase synchronous reluctance machine for high saliency with trade-off between saliency ratio and synchronisation ability (Tang et al., 2017). In his approach, he uses a distributed cage bars very close to air gap for starting purpose and three flux barriers per each pole for the direction of magnetic fields. He also did analysis on uneven distribution of air-gap which concludes his work that, the saliency of the motor improved form 3.7 (symmetrical distribution of air-gap) to 5.12 (unsymmetrical distribution of air-gap) of the motor with the same specification.
A two pole submersible multi-barriers three phase line-start Synchronous reluctance motor with two rotor geometries was proposed and investigated by (Baka et al., 2018). He uses the same concept as (Gamba et al., 2013), to achieve the starting capability of the motor by filling the flux barriers with Aluminium. In his work, he did analysis on improvement of efficiency and saliencies which were recorded in different motor load operations.

A high efficiency line-start synchronous reluctance motor for fan and pump application was designed and analysed by (Villani et al., 2018). The author designed two similar rotor geometries, one with three flux barriers (3FB) and the other with four flux barriers (4FB) both were filled with Aluminium for starting application. In his analysis, he concluded that the solution with 4FB has higher efficiency and as well saliency ratio of about 7 while the solution with 3FB has higher starting capability because of the volume of Aluminium which is accommodated by the rotor space thereby having saliency of about 6.

Huang used approach of applying the traditional cage barriers filled with self-made solidified Aluminium paste for self-starting of a synchronous reluctance motor (Huang et al., 2018).
In his experimental results, he concluded that “the higher filling rate of the barriers is proportional to the vibration decreasing, but the saliency ratio and reluctance torque would also decreased”. He also analysed that the output torque and efficiency could be improved with a lower filling rate.

2.7 Summary

Synchronous Reluctance Machine (SynRM) is a worldwide research interest in field of AC drives because of its numerous advantages. Generally, SynRMs are known to have good efficiency at low torque ripple, no copper losses and produces a constant power over a long period of time. SynRM with axially laminated rotor structure possess high Saliency ratio, torque efficiency and a better power factor but unsuitable for line-start due to un-guaranteed mechanical strength. SynRM with permanent magnet assisted has a high salience ratio and an attractive torque, efficiency and a good power factor characteristics but very expensive due to high cost of permanent magnets.

Synchronous reluctance machine has been reported in many research outputs of AC machine drives that, its performances depend on the performance indices $L_d$ and $L_q$, hence the efficiency, power factor and the machine saliency ratio of a synchronous reluctance machine depends on the machine indices. Machine researchers focused mainly on maximising the performance indices in order to achieve the highest possible targeting parameters like efficiency, power factor and the saliency of the machine for economic solution.

Different control techniques with synchronous reluctance machines have been studied to introduce more economical and effective synchronous reluctance machine prototype.

The use of electric circuit technology helps to drive a synchronous reluctance machine to a higher saliency as was discussed in this chapter. The design of this type of motor is very simple
and easy but exhibited a low mechanical structure and are considered unsuitable in a case where high-speed application are required.

Self-starting synchronous reluctance machines are becoming more utilised in industries because of its simplicity and cheapness. It does not require any external device for start purposes. Permanent-magnet synchronous reluctance machines are considered effective but expensive as it requires permanent-magnets. Hence, line-start synchronous reluctance machines without magnets have been of high research interest in the field of electrical machine drives. Line-start synchronous reluctance machine filled with Aluminium in the rotor-flux barriers exhibit a good steady-start performance but weaker start-up properties.

This dissertation focused mainly on a self-start synchronous reluctance motor with distributed bars in rotor structure and comparing the starting torque improvement of SynRM-A against SynRM-B.
CHAPTER 3
DY NAMIC MODEL OF A CONVENTIONAL SynRM

3.1 Introduction

In this chapter, a dynamic model of a conventional synchronous reluctance motor in abc machine variables is presented. The model is intended to determine the machine parameters using hypothetical background studies to be applied with the proposed machine model.

To model a three-phase synchronous reluctance machine, the general voltage and flux linkage equations need to be established in order to explore the developed dynamic and steady state behaviour of the proposed machine. Using this approach, the understanding of inductance establishment was verified successfully.

3.2 Dynamic Model of the Conventional Synchronous Reluctance Motor Variables.

A four-pole three-phase star-connected conventional synchronous reluctance machine as depicted in Figure 3.1 as a model introduced in this chapter using abc variable. The identical stator-windings are sinusoidal distributed and displaced 120-electrical degree with equivalent number of turns Ns and resistance Rs, as shown in Figure 3.1b. The symmetrical three-phase winding carries a balanced three-phase current, thus establishing a rotating mmf in the air gap and the mmf interaction between the stator and rotor generates electromagnetic torque (Lyshevski, 2018).
The voltage equations of the synchronous reluctance machine are derived from Kirchhoff’s law and explained using equations (3.1) to (3.3) as: (Krause et al., 2013), (Lyshevski, 2018),
\[
V_{as} = r_s i_{as} + \frac{d}{dt} \lambda_{as} \tag{3.1}
\]

\[
V_{bs} = r_s i_{bs} + \frac{d}{dt} \lambda_{bs} \tag{3.2}
\]

\[
V_{cs} = r_s i_{cs} + \frac{d}{dt} \lambda_{cs} \tag{3.3}
\]

where \( V_{abc} \) denotes the balanced three-phase voltage variables associated with the stator, \( r_{abc} \) is the stator winding resistance per phase which can be written as(Krause et al., 2013);

\[
r_s = r_{as} = r_{bs} = r_{cs} = \text{diag} \left[ r_s, r_s, r_s \right] \tag{3.4}
\]

Which can be equally written as:

\[
R_{abc} = \begin{bmatrix}
  r_{as} & 0 & 0 \\
  0 & r_{bs} & 0 \\
  0 & 0 & r_{cs}
\end{bmatrix} \tag{3.5}
\]

Note

\[
\left( f_{abc} \right)^T = \begin{bmatrix}
  f_{as} \\
  f_{bs} \\
  f_{cs}
\end{bmatrix} \tag{3.6}
\]

The \( f \) variable can be a representative of the phase voltages, currents or flux linkages as used for the dynamic machine modelling. The subscript \( s \) represents the variables associated with the stator \( abc \) windings.

The interaction of inductance and current results to a flux linkage. Hence, the stator flux-linkage mathematical equation is expressed as(Krause et al., 2013):
\[ \lambda_{abc} = L_{abc}i_{abc} \]  \hspace{1cm} (3.7)

The self and mutual inductances of \textit{abc} variable windings are represented as \( L_{abc} \) and can be written in matrix form as (Lyshevski, 2018):

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{cs}
\end{bmatrix} =
\begin{bmatrix}
L_{aas} & L_{abs} & L_{acs} \\
L_{bas} & L_{bbs} & L_{bcs} \\
L_{cas} & L_{cbs} & L_{cscs}
\end{bmatrix}
\]

\hspace{1cm} (3.8)

The current in the \textit{abc} windings variables can be written as:

\[ i_{abc} = [i_{as} \ i_{bs} \ i_{cs}] \]

\hspace{1cm} (3.9)

The diagonal elements in equation (3.8) comprises of leakage and magnetising inductance components of the stator windings and can be represented as \( L_s \) and \( L_m \) respectively. The self and mutual inductances of stator windings are the functions of the machine electrical angular displacement (\( \theta_r \)).

Where

\[ \theta_r = \frac{p}{2} \theta_{rm} \]

\hspace{1cm} (3.10)

Equation (3.10) shows the relationship between the electrical-angular displacement and the rotor-angular displacement. The need to compute machine inductances are of great importance to evaluate the machine performances.

\textbf{3.3 Dynamic Model of a SynRM in rotating reference frame.}

The machine inductances can be computed using different methods (e.g. the winding function theory, finite element analysis, field and other circuit theory approaches) to get better
inductance variables. Winding function theory approach was used by (Krause et al., 2013) and (Ogunjuyigbe et al., 2017) to calculate the self and mutual inductances present between the main and auxiliary windings of a synchronous reluctance machine.

From Figure 3.1, the three-phase stator windings were identical and displaced within 120-degrees electrical position. The direct axis and quadrature axis inductances depend on the alignment of each axis with the stator magnetic \textit{a-phase} winding (as). The \textit{dq-axis} were fixed with the rotor and rotates with an angular velocity \( w \). In this aspect, the angular velocity \( w \) is the same with the rotor angular velocity (\( w_r \)), that is \( \nu = w_r \).

Hence, inductances of electrical machines can be calculated as (Lyshevski, 2018):

\[
L_m(\theta_r) = L_{mA} - L_{mB} \cos 2\theta_r
\]  

(3.11)

Where \( L_m(\theta_r) \) is the magnetising inductance, variables (\( L_{mA} \) and \( L_{mB} \)) represent the average and half-average magnetising inductance respectively. Variables \( L_{mA} \) and \( L_{mB} \) are used to determine the maximum and minimum values of the machine magnetising inductances.

The mathematical expression for self-inductances is expressed as(Lyshevski, 2018):

\[
L_{asas} = L_{aA} + L_m
\]  

(3.12)

\[
L_{asas} = L_{aA} + L_{mq} \quad \text{for} \quad \theta_r \text{ is minimum}
\]  

(3.13)

\[
L_{asas} = L_{aA} + L_{md} \quad \text{for} \quad \theta_r \text{ is maximum}
\]  

(3.14)
While the minimum and maximum magnetising inductances are denoted by $L_{mq}$ and $L_{md}$ respectively and are determined by the position of electrical angular displacement ($\theta_r$) of the machine.

The self-inductance of the $a$-phase stator windings is combined using equation (3.11) and equation (3.12) as (Lyshevski, 2018):

$$L_{ass} = L_{ls} + L_{mah} - L_{mb} \cos 2\theta_r$$

(3.15)

Phase $b$ and $c$ windings are expressed in the same manner as $a$-phase but putting into consideration that $b$- and $c$-phases are 120-degrees electrically away from each other as shown in Figure 3.1. The self-inductances of phase-$b$ and phase-$c$ windings corresponds to be:

$$L_{bsb} = L_{ls} + L_{mah} - L_{mb} \cos \left(\theta_r - \frac{2\pi}{3}\right)$$

(3.16)

$$L_{csb} = L_{ls} + L_{mah} - L_{mb} \cos \left(\theta_r + \frac{2\pi}{3}\right)$$

(3.17)

Similarly, the mutual inductances are the periodic functions of electrical-angular displacement ($\theta_r$) and the mutual inductance is the average magnetising inductance. The mutual inductances of the stator windings can be express as (Lyshevski, 2018):

$$L_{asbs} = L_{bsa} = -\frac{1}{2} L_{mah} - L_{mb} \cos 2\left(\theta_r - \frac{\pi}{3}\right)$$

(3.18)

$$L_{csbs} = L_{csa} = -\frac{1}{2} L_{mah} - L_{mb} \cos 2\left(\theta_r + \frac{\pi}{3}\right)$$

(3.19)
Using equation (3.15) – (3.20), the inductance matrix of the stator windings can be express as:

\[
L_{a,b,c} = L_{c,b,a} = -\frac{1}{2}L_{mA} - L_{mb} \cos 2\left(\theta_r + \frac{\pi}{3}\right)
\]

(3.20)

Where:

\[
L_{mk} = \frac{1}{3}\left(L_{mq} + L_{md}\right)
\]

(3.22)

\[
L_{mb} = \frac{1}{3}\left(L_{md} - L_{mq}\right)
\]

(3.23)

The expression for flux linkage of \textit{abcs}-winding given in equation (3.7) is expanded as (Krause et al., 2013), (Lyshevski, 2018):

\[
\lambda_{a,b,c} = 
\begin{bmatrix}
L_{ts} + L_{nta} - L_{ntb} \cos 2\theta_r - \frac{1}{2}L_{nta} - L_{ntb} \cos 2\left(\theta_r - \frac{\pi}{3}\right) - \frac{1}{2}L_{nta} - L_{ntb} \cos 2\left(\theta_r + \frac{\pi}{3}\right) \\
- \frac{1}{2}L_{nta} - L_{ntb} \cos 2\left(\theta_r - \frac{\pi}{3}\right) L_{ta} + L_{nta} - L_{ntb} \cos 2\left(\theta_r - \frac{2\pi}{3}\right) - \frac{1}{2}L_{nta} - L_{ntb} \cos 2\theta_r \\
- \frac{1}{2}L_{nta} - L_{ntb} \cos 2\left(\theta_r + \frac{\pi}{3}\right) - \frac{1}{2}L_{nta} - L_{ntb} \cos 22\theta_r L_{ta} + L_{nta} - L_{ntb} \cos 2\left(\theta_r + \frac{2\pi}{3}\right)
\end{bmatrix}
\]

(3.24)

It is well observed in equation (3.21) that machine inductance is a function of rotor position varying with time. The time-dependent coefficients complicate the computation of voltage equation variables that was expressed earlier using equations (3.1) – (3.3). However, the
complexity of this machine variables can be simplified using the theory of reference frame transformation (Krause et al., 2013).

3.4 Theory of reference frame transformation of the conventional SynRM.

The $abc$ stator variables are transformed to a $dqo$ reference frame which are fixed to the rotor of the machine to eliminate the time-varying inductances effect in the machine. Due to the complex differential equations with the stator, a change of variables is used to reduce the complexity of the machine variables.

The vector notation in equation (3.6) is used to formulate a change of variable expression that transforms the stator quantities into arbitrary-reference frame. The change of transformation variables was expressed by (Krause et al., 2013) as:

$$f_{qdos} = x(t_r) f_{abcs}$$ (3.25)

Where:

$$f_{qdos} = [f_{qs}, f_{ds}, f_{os}]$$ (3.26)

and

$$f_{abcs} = [f_{as}, f_{bs}, f_{cs}]^T$$ (3.27)

The transformation matrix $x(t_r)$ is expressed as:
\[
x(t_r) = \begin{bmatrix}
\cos(\theta_r) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\
\sin(\theta_r) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]  
(3.28)

And the inverse transformation matrix \((x(t_r)^{-1})\) is expressed as:

\[
f_{qdos} = x(t_r)^{-1} f_{abcs}
\]  
(3.29)

\[
x(t_r)^{-1} = \begin{bmatrix}
\cos(\theta_r) & \sin(\theta_r) & 1 \\
\cos\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & 1 \\
\cos\left(\theta_r + \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) & 1
\end{bmatrix}
\]  
(3.30)

The dynamic equation in the \textit{dq reference (rotor) frame} can be computed using matrix notations. The transformation of the stator quantities to rotor-referenced frame is achieved by multiplying equation (3.1) and (3.2) with an appropriate matrix transformation. Hence the stator voltage equation can be express as(Krause et al., 2013), (Lyshevski, 2018):

\[
x(t_r) x(t_r)^{-1} V_{qdos} = x(t_r) r_x(t_r)^{-1} i_{qdos} + x(t_r) \frac{d}{dt} x(t_r)^{-1} \lambda_{qdos}
\]  
(3.31)

Where:

\[
x(t_r) x(t_r)^{-1} = 1
\]  
(3.32)

Then:

36
The second-term of the equation (3.33) on the right-hand side is further expanded as:

\[ x(t_r) \frac{d}{dt} x(t_r)^{-1} \lambda_{qdos} = x(t_r) \left[ \left( \frac{d}{dt} x(t_r)^{-1} \right) \lambda_{qdos} + x(t_r)^{-1} \frac{d}{dt} \lambda_{qdos} \right] \]  

(3.34)

Where the first and second term of equation (3.34) on the right-hand side is evaluated and expressed as:

\[ x(t_r) x(t_r)^{-1} \lambda_{qdos} = wr \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda_{qdos} \]  

(3.35)

And

\[ x(t_r) x(t_r)^{-1} \frac{d}{dt} \lambda_{qdos} = \frac{d}{dt} \lambda_{qdos} \]  

(3.36)

When equation (3.35) and (3.36) were substituted into equation (3.34), equation (3.37) was realised as (Lyshevski, 2018):

\[ x(t_r) \frac{d}{dt} x(t_r)^{-1} \lambda_{qdos} = wr \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda_{qdos} + \frac{d}{dt} \lambda_{qdos} \]  

(3.37)

At this stage, the stator-voltage equation can be expressed using rotor \textit{dq reference frame} by substituting (3.37) into equation (3.33) to give equation (3.38) as:

\[ V'_{qdos} = r_s i'_{qdos} + w_r \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda'_{qdos} + \frac{d}{dt} \lambda'_{qdos} \]  

(3.38)
Using the same transformation process that achieved the stator voltage equation in arbitrary reference frame (3.31), the flux linkage in equation (3.7) can also be expressed as (Krause et al., 2013);(Lyshevski, 2018):

$$\lambda_{qdos} = x(t_r) L_r x(t_r)^{-1} i_{qdos}$$  \hfill (3.39)

Where:

$$x(t_r) L_r x(t_r)^{-1} = \begin{bmatrix} L_{qs} + L_{mq} & 0 & 0 \\ 0 & L_{qs} + L_{md} & 0 \\ 0 & 0 & L_{qs} \end{bmatrix}$$  \hfill (3.40)

The flux-linkage described in equation (3.39) is given by:

$$\lambda_{qdos} = \begin{bmatrix} L_{qs} & 0 & 0 \\ 0 & L_{ds} & 0 \\ 0 & 0 & L_{os} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \end{bmatrix}$$  \hfill (3.41)

Hence the \textit{d-} and \textit{q-axis} self and mutual inductances can be expressed as (Krause et al., 2013);(Lyshevski, 2018):

$$L_{qs} = L_{qs} + L_{mq}$$  \hfill (3.42)

$$L_{ds} = L_{ds} + L_{md}$$  \hfill (3.43)

$$L_{os} = L_{os}$$  \hfill (3.44)

$$L_{mq} = \frac{3}{2} (L_{mA} - L_{mB})$$  \hfill (3.45)

$$L_{md} = \frac{3}{2} (L_{mA} + L_{mB})$$  \hfill (3.46)
From equations (3.38) – (3.46), the $d$- and $q$-axis voltage equations that described the operation of electrical machine in rotating reference frame is expressed as (Krause et al., 2013):

$$V_{qs}^r = r_{qs}i_{qs}^r + w_r \lambda_{qs}^r + \frac{d}{dt} \lambda_{qs}^r$$

(3.47)

$$V_{ds}^r = r_{ds}i_{ds}^r - w_r \lambda_{qs}^r + \frac{d}{dt} \lambda_{ds}^r$$

(3.48)

$$V_{os}^r = r_{os}i_{os}^r + \frac{d}{dt} \lambda_{os}^r$$

(3.49)

Where the flux linkages are given by equations (3.50) – (3.52),

$$\lambda_{qs} = L_{qs}i_{qs} = (L_{qs} + L_{mq})i_{qs}^r$$

(3.50)

$$\lambda_{ds} = L_{ds}i_{ds} = (L_{ds} + L_{ma})i_{ds}^r$$

(3.51)

$$\lambda_{os} = L_{os}i_{os}^r$$

(3.52)

3.5 Equivalent circuit of the Conventional SynRM.

The $qd$ equivalent circuit of a synchronous reluctance machine discussed in this chapter was described using equations (3.47) – (3.52). The machine torque and its magnetising flux are established through $qd$ equivalent circuit and was demonstrated using Figures 3.2a and 3.2b respectively ((Krause et al., 2013);(Lyshevski, 2018));
3.6 Electromagnetic Torque Equation of the Conventional SynRM

The mechanical and electromagnetic torque equation are described by the rotor dynamic mathematical equation as (Krause et al., 2013):

$$\frac{dw_r}{dt} = \frac{p}{2J}(T_{em} - T_L)$$  \hspace{1cm} (3.53)

The number of poles is denoted with $p$, $J$ is the total inertia of the rotor, $T_{em}$ is the electromagnetic torque developed and $T_L$ is the load torque.

Electromagnetic torque is derived from energy stored in the coupling-field and can be defined as (Krause et al., 2013):

$$W_{coe} = \frac{1}{2} [i_{abc}]^T L_s [i_{abc}]$$ \hspace{1cm} (3.54)
Variables ($i_{abcs}$ and $L_s$) are the vector current and coupling inductances of the stator and rotor circuits respectively.

Electromagnetic torque equation as described above is evaluated as (Krause et al., 2013); (Lyshevski, 2018):

$$T_{em} = \frac{p}{2} \frac{\partial W_{coe}}{\partial \theta_r}$$ (3.55)

Where $p$ is the number of poles, $W_{coe}$ is the co-energy described using equation (3.54) and $\theta_r$ is the rotor electrical radians angular displacement.

Substituting equation (3.54) into (3.55), the electromagnetic torque is expressed in terms of electrical rotor position and can be express as (Krause et al., 2013); (Lyshevski, 2018):

$$T_{em} = \frac{p}{2} [i_{abcs}]^T \left[ \frac{\partial L_s}{\partial \theta_r} \right] [i_{abcs}]$$ (3.56)

The electromagnetic torque expression in equation (3.56) is further expanded to $qdo$ variables with the application of inverse transformation ($x(t_r)^{-1}$) which is expressed as (Krause et al., 2013); (Lyshevski, 2018):

$$T_{em} = \frac{p}{2} \frac{1}{2} [i_{qdos}]^T (x(t_r)^{-1})^T \frac{dL_s(\theta_r)}{d\theta_r} x(t_r)^{-1} i_{qdos}$$ (3.57)

After evaluation and simplification of equation (3.57) with the appropriate substituted matrices, the electromagnetic torque equation resolves to equation (3.58) as (Vagati, 1994, Hortman, 2004, Kamper and Volsdhenk, 1994, Boldea, 1996).
\[
T_{em} = \frac{3}{2} \frac{p}{2} \left[ (L_{md} - L_{mq}) i'_{ds} i'_{qs} \right] 
\]
(3.58)

Equation (3.58) is further expressed as:

\[
T_{em} = \frac{3}{2} \frac{p}{2} \left( \lambda^r_{ds} i^r_{qs} - \lambda^r_{qs} i^r_{ds} \right) 
\]
(3.59)

And can be expanded in terms of current components as:

\[
T_{em} = \frac{3}{2} \frac{p}{2} (L_{ds} - L_{qs}) i_{ds} i_{qs} 
\]
(3.60)

The electromagnetic torque in equation (3.60) is a torque developed by the stator windings containing all the torque components required for a standard three-phase synchronous reluctance machine. It can be seen from the torque index \((L_{ds}, L_{qs})\), the electromagnetic torque of a synchronous reluctance machine depends on the torque-index. That is, the higher the index differences, the greater the electrical-torque obtained.

### 3.7 Summary

In this chapter, the dynamic model of a three-phase four-pole synchronous reluctance machine was reviewed. The basic concept that described the stator voltage equation of the synchronous reluctance machine was established. The fields produced by the stator winding currents were described to be sinusoidal distributed in the airgap. The three-phase transformation that simplified the reluctance machine equation in \(abc\) reference frame into \(dq\) arbitrary reference frame were reviewed and it’s equivalent circuit in \(dq\) rotor-reference frame were reviewed.

Synchronous reluctance machine commonly used in many industries because of its performances. It was also established that the performance characteristics of a synchronous
reluctance machine depends on the inductances of the machine. The synchronous reluctance
machine inductances and its torque index \((L_{ds} - L_{qr})\) which are responsible for electrical
machine performances were reviewed in this chapter, while the saliency ratio \((L_{ds} / L_{qs})\) is also
demonstrated and will be computed in the next chapter using finite element analysis.

More so, the targeting parameters (torque, power factor and efficiency) that positioned the
performance of synchronous reluctance machine in the competitive market will be considered
in next chapter using finite element analysis technique.
CHAPTER 4

FINITE ELEMENT ANALYSIS OF THE MODEL SSSynRM

4.1 Introduction

The realistic sensitivity analysis of an electrical machine anisotropy depends on its designing variables or the microscopic parameters. Thanks to software Engineers for their marvellous work done by introducing some software packages that can be used to overcome most of the engineering design problems. One of those software packages is the “ANSYS MAXWELL”.

Due to geometrical complexity of the proposed model, a two-dimensional (2D) finite element analysis package of ANSYS 16.0 was used to model and analyse the steady-state and dynamic performances of a synchronous reluctance machine with cage in the rotor structure.

Finite-element-method (FEM) has been in use for decades and was used to design and analyse electrical machines. The numerical calculations of an electrical machine components are based on the electric and magnetic field distributions computations and using Finite Element Analysis (FEA). The proposed model is done under the standard solution study using Maxwell equations (Akin, 1994, Bianchi, 2005) and (Boldea et al., 1994). FEA can be used to analyse the problems experienced with magneto-static nonlinear materials in electrical machines. Magneto-statics solver is done using finite element simulations for computing motor parameters which mostly act as functions of rotor position. The obtained machine parameters are further analysed in FEA software package to investigate the machine performances (both in magneto static and transient characteristics) (Vagati et al., 2000, Kamper et al., 1996, Bianchi, 2005). The analytical domain of the whole process is categorised into elementary subdomains, and the field equations are implemented to each of the elements from elementary subdomain.
Meanwhile, the characteristics of magnetic field distribution, inductance computation, steady-state and dynamic torque analysis of a synchronous reluctance machine having rotor-cage will be analysed using finite element analysis (FEA).

Figure 4.1: Geometrical cross-section of the proposed SynRMs (a) SynRM-A; (b) SynRM-B.
Two SynRMs with different rotor concepts are modelled. Each of the rotor perceptions were built using the same stator winding configuration and with stamped conventional steel laminations having transverse flux barriers. The two-different motor models with their rotor concepts are described as SynRM-A and SynRM-B. SynRM-A has its rotor slot just above each transverse flux barrier, thus separated with magnetic bridges as shown in Figure 4.1a. In SynRM-B, the rotor is fitted with cage bars inside the rotor air flux barrier and were separated by magnetic ribs from each other as shown in figure 4.1b. Both Synchronous Reluctance Motors (SynRM-A and SynRM-B) were simulated and their performances were compared using FEA.

A complete finite element rotor concepts model were constructed from a geometrical cross-section of a standard Induction machine using 2D finite element method (FEM). Since the stator of a standard three-phase electrical machine is the same in all AC machines with the same rating, only the rotor structure of the proposed machine is modified. The two-modified rotor geometrics are shown in Figure 4.1a and Figure 4.1b, while the quarter of the full motor structures and its finite element mesh plot generated are shown in Figure 4.2 and Figure 4.3 respectively.

The generated finite element mesh plot in Figure 4.2 and 4.3 spread all over the geometrical cross-section of the machine. The total number of nodes and elements mesh plot obtained from quarter of SynRM-A model are: 51059 and 5499, and SynRM-B are: 61132 and 10212 respectively. The mesh plot in air-gap region shown more concentration due to the speed of magnetic variation interaction. The above-mentioned field solutions were obtained by running the finite element magneto static solver at a constant supply frequency and speed of 50Hz and 1500rpm respectively, which solve the relevant Maxwell’s equations.
4.2. Effect of number rotor copper bars

4.2.1. Space harmonics due to stator slotting and number of rotor bars

The rotor bar current will produce magnetic field having a certain number of harmonic components. The space rotor harmonic orders are found by using (Boldea and Nasar, 2002).

\[ n_r = x \frac{R_s}{p_i} + 1 \]  

(4.1)
Where $x$ is any positive or negative number, $R_b$ is the number of rotor bars, $p_1$ is the number of fundamental pairs of poles. The rotor of the SSSynRM has 28 slots. Beside the fundamental component, the rotor current will produce odd space rotor slot harmonics of order -13, +15, -27, +29, -41, +43, etc.. On other hand, the space harmonics produced by the stator slotting and the stator winding phase belt space harmonics can be obtained as in (4) and (5) respectively (Boldea and Nasar, 2002).

$$v_x = x \left( \frac{Q_s}{p_1} \right) + 1 \quad (4.2)$$

$$u_x = 2x \left( \frac{\pi}{\sigma} \right) + 1 \quad (4.3)$$

Where $Q_s$ is the total number of stator slots, $\sigma$ is the phase belt angle, which is $\pi/3$ electric radian for a 36 stator slot, 3-phase, and 4-pole machine. The SynRM SSSCIM beside the fundamental, will produce stator slot space harmonics of order -17, +19, -35, +37, etc., and the phase belt space harmonics of order -5, +7, -11, +13, -17, +19, -23, +25, -29, +31, -35, +37 etc.. The sign of the harmonic order indicates the direction of the space harmonics. Those with positive sign are traveling in the same direction as the fundamental components and those with negative sign are traveling in the opposite direction.

4.2.2. Synchronous parasitic torques in SynRM

The space harmonic components originated from the rotor will produce synchronous parasitic torques when interacting with the space harmonics originated from the stator having the same order. The interaction of the 1st rotor slot space harmonic (13th order) and the fourth stator winding phase belt (13th order) harmonic will produce synchronous parasitic torque at the speed of 179 rpm.
There is not interaction between the rotor slot harmonic and the stator slot harmonic in the SSSynRM. This has given the SSSynRM the capability to have smooth operation at the beginning of the acceleration mode. To avoid synchronous parasitic torque due to interaction between the stator slot harmonics and rotor slot harmonics, (4.1) should not equate (4.2).

4.3 Flux density distribution analysis

Literally, electromagnetic field can be described as a field produced from the interaction of electrical charged bodies in its field surrounding. The source of the field is the combination of electrical and magnetic field interaction producing stationary and moving charges in a body.

The idea is to understand the actual principle of flux-density distribution. Figure 4.4 and 4.5 respectively demonstrated clearly the Characteristics flux density distributions both on the stator and rotor core of the motor model using finite element analysis (FEA). It is well observed in figure 4.4, that the flux densities saturate the tangential magnetic bridges between flux barrier and the rotor bars, and the tangential magnetic bridges near the air-gap because of high resistance in the rotor core. While in figure 4.5, the flux densities saturate the radial bridges between flux barriers on the rotor core and the tangential magnetic bridges near the air-gap.

Figure 4. 4: Flux density distribution of SynRM-A
The spatial magnetic flux density distribution along the mid-air-gap over a half four-pole synchronous reluctance machine as proposed is shown in figure 4.6 (a). Comparing the profile of the two models, SynRM-A has reduced air-gap flux density profile than its counterpart (SynRM-B). The flux density corresponding FFT results of both models are compared in figure 4.6 (b). The effect of magnet saturation is well observed in $3^{rd}$ harmonic order and its multiples, while other harmonic contents are caused by distribution windings, stator slotting and flux barriers as shown in figure 4.6 (b). Comparing the FFT results of both models for the flux density profile in figure 4.6 (b), the fundamental component, the $29^{th}$ and $33^{rd}$ harmonic order showed that SynRM-B has higher FFT amplitude than its counterpart (SynRM-A).
Figure 4.6: Comparisons: (a) airgap flux density profiles, (b) FFT of the airgap flux density profiles.

4.4 Computation and Analysis of Inductance

It’s well known that inductances are functions of the air-gap due to nonlinear behaviour of ferromagnetic materials. Also, the circulation of current in the coil of a machine depends on the inductance functionality. Furthermore, the $d$-axis magnetizing inductance $L_{md}$ and $q$-axis magnetizing inductance $L_{mq}$ can be obtained by extracting the fundamentals of airgap radial flux density along the two axes in Figs. 4.7 and 4.10. Without disregarding the assigned currents, the saliency ratio is then obtained by using (Boldea, 1996, Muteba, 2017)
\[
\frac{L_{\text{nd}}}{L_{\text{mq}}} = \frac{2i_q}{\sqrt{3}i_d} \times \frac{B_{gd1}}{B_{gq1}}
\]  
(4.4)

4.4.1 Computation of d-axis inductance

_D-axis_ inductance is calculated by applying a magnetising current to the stator winding which is set to align with the D-axis of the rotor. The current is supplied to the stator winding so that the maximum value of the distributed MMF corresponds with the polar-axis.

The applied three-phase current to the stator winding, the winding distribution and the time instant used for computing the _d-axis_ inductance was described in (Bianchi, 2005), using phasor diagrams. Equation (4.5) was used to describe the relationship between the three-phase current and time reference,

\[
i_{as}(t) = I_M \sin(wt) \\
i_{bs}(t) = I_M \sin\left(wt - \frac{2\pi}{3}\right) \\
i_{cs}(t) = I_M \sin\left(wt - \frac{4\pi}{3}\right)
\]  
(4.5)

The three-phase current in equation (4.5) is chosen in this finite element model for a fixed winding distribution, at an instant time of \(wt\), such that the distributed _MMF_ corresponds with the polar-axis (the _d-axis_).

With respect to \(wt = \frac{\pi}{2}\), at a chosen vector current angle of:

\[
i_{ds} = i_{as} \\
i_{qs} = 0
\]  
(4.6)

The instant time phase current is expressed as (Bianchi, 2005):
Using equation (4.7), the excitation current from the finite element field simulation was obtained, and the computation of inductances were done.

\[
\begin{align*}
i_{ss} &= I_M \\
i_{ss} &= -\frac{1}{2}I_M \\
i_{ss} &= -\frac{1}{2}I_M
\end{align*}
\] (4.7)

Figure 4.7: Distribution Comparisons of air-gap: (a) flux density profile, (b) Harmonic contents
The \textit{d-axis} magnetic flux density distribution was carried out using FEA. The plot of air-gap magnetic flux density distribution and its harmonic contents against half-pole pitch compares the characteristics between SynRM-A and SynRM-B as shown in Figure 4.7. It is well observed that SynRM-B has higher effect on fundamental air-gap flux density to compare its counterpart SynRM-A, except the 5\textsuperscript{th} harmonic order which is vice-versa (see figure 4.7b). Figure 4.8 (a & b) evident the direction of flux-line distributions of the two machine models. From the radial air-gap magnetic flux distribution, the average value of the air-gap flux density ($B_g$) distributions was evaluated using integral calculator provided in the finite element software.

![Figure 4.8: d-axis field line distributions (a) SynRM-A and (b) SynRM-B.](image)

Figure 4.9 demonstrates the characteristics of \textit{d-axis} current functionality over the load angle position of the machine.
4.4.2 Computation of $Q$-Axis inductance

The $q$-axis inductance $L_q$ is computed using similar method as the $d$-axis inductance. The applied currents (phase currents) and the distributed windings are selected such that the maximum value of MMF distribution corresponds with the inter polar axis, the $q$-axis.

Here, the instants time with respect to $\frac{\pi}{2}$ at a chosen vector current angle of:

\[
\begin{align*}
    i_d &= 0 \\
    i_q &= I_M
\end{align*}
\]

The three-phase current changes with respect to a change in vector angle of $d$-axis and $q$-axis as written in equation (4.8) for the computation of the three-phase currents (Bianchi, 2005).

The $q$-axis magnetic field distribution was done using FEA, the plot of air-gap magnetic flux distribution and its harmonic contents over a half-pole pitch compares the characteristics of SynRM-A and SynRM-B are presented in Figure 4.10. It is well observed that SynRM-A has
higher effect on fundamental air-gap flux density to compare the SynRM-B. Figure 4.11 (a&b) evident the direction of flux-line distributions of the two machine models (SynRM-A and SynRM-B) respectively. Figure 4.12 demonstrates the graphical characteristics of q-axis current functionality over a load angle position of the machine.

Figure 4. 10: Comparisons of $q$-axis air-gap flux density distribution, (a) flux density profile, (b) harmonic contents.
Using the same calculation concept done with $d$-axis radial air-gap magnetic flux distribution, the average values of the air-gap flux density ($B_g$) distributions in $q$-axis field were evaluated using integral calculator provided in the finite element software environment. The magnetic field inductances of the two models SynRM-A and SynRM-B are respectively evaluated and computed as shown in Appendix C, Table (C 1&2).

Figure 4.11: $q$-axis field line distributions (a) SynRM-A and (b) SynRM-B

The saliency ratios ($\xi$) for SynRM-A and SynRM-B are evaluated by substituting the obtained values computed from radial air-gap flux density ($B_g$) and the values of magnetising
inductances \((L_{\text{adj}})\) into equation (4.3). The corresponding values of saliency ratio were obtained using different vector angle and the corresponding values of other targeting parameters (power factor and torque density for both machines) are presented in Appendix A, Table A (1&2).

Figure B4 (a & b), Appendix (B) present the graphical efficiency and power factor values of the models obtained using magneto-static field as described in sub-section (4.2) of this chapter. Efficiency of a machine is determined from input power, output power and the total losses, however the efficiency of SynRM-B is higher than the efficiency of SynRM-A, because its bars are inserted in the flux barrier of the rotor, and hence experience minimal core losses. It also has higher power factor because of induced magnetic field in the rotor, as a result aligned with the stator field at a minimal reluctance position that result a better saliency. The two graphs were obtained in the same rotor position at 20° but in different current angle positions from 30° to 75° at interval of 5°.

4.5 Steady-state and Dynamic analysis behaviours

4.5.1 Steady-state analysis.

The three-phase currents \((i_a, i_b, i_c)\) and three-phase voltages \((v_a, v_b, v_c)\) vary sinusoidal with time and were applied using the operational variables of the proposed machines under steady-state condition. The steady-state operations of this models are analysed using FEA. A self-start synchronous reluctance motor exhibits the characteristics of synchronous motors operating on full-load at a constant speed of 1500 \text{rpm}. Under this condition, the current in the rotor bar was negligible. The synchronous torque developed by these machines depend on the differences between the direct axis \((L_d - \text{axis})\) and the quadrature axis \((L_q - \text{axis})\) inductances. The average torque and percentage torque ripples were obtained using different
current space angles. The graphical representations of electromagnetic average torque and its percentage torque ripple were plotted at a constant speed of 1500 rpm in rotor position of 20° mechanical as shown in Figure 4.13 and 4.14 respectively. However, to evaluate and compare the performance of these machines, the modelled machines magnetic field were solved using the same machine specifications in Table 2.1 and simulated using FEA. The different current-vector angle excitations of the machine and its simulated results were shown in Appendix A, Table A3. Different space-vector angle (30°, 45° and 75°) are chosen to demonstrate the machine performance differences in terms of average torque, percentage torque ripple and air-gap flux density distributions as tableted in table 4.1. The performance results are represented in Figure 4.15 and 4.16 respectively. It is well observed that SynRM-B shows more average torque improvement and well-reduced torque ripple contents compared to SynRM-A. This is because, the inserted cage bars of SynRM-B in the air flux barriers of the rotor structure limits losses and heat from the machine. SynRM-A has higher starting resistance which contributes to a high torque ripple content with magnetic resistance variation between the flux barrier and the machine teeth. The air-gap flux density distribution characteristics for both machines are almost alike but showed more improvement with SynRM-B than SynRM-A.

![Figure 4. 13: Average torque as a function vector angle.](image-url)
Figure 4.14: Average torque ripple in different vector angular position

Table 4.1: Comparison of average torque and torque ripple contents of SynRM-A and SynRM-B

<table>
<thead>
<tr>
<th>Current Space Angles (°)</th>
<th>SynRM-A</th>
<th>SynRM-B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{av}$ (Nm)</td>
<td>$T_{ripple}$ (%)</td>
</tr>
<tr>
<td>30</td>
<td>7.2284</td>
<td>44</td>
</tr>
<tr>
<td>45</td>
<td>12.7838</td>
<td>39</td>
</tr>
<tr>
<td>75</td>
<td>18.2542</td>
<td>37</td>
</tr>
</tbody>
</table>
Figure 4.15: Comparisons of average torque at an angle of; (a) 30˚, (b) 45˚ and (c) 75˚ electrical
The obtained finite-element results were plotted in MATLAB environment and used to compare different characteristic behaviours of targeting variables for both magneto-static and transient field distributions. The finite element generated torque values as function of angular positions ranging from $\theta = 0^\circ$ to $\theta = 90^\circ$ at a step interval of $5^\circ$ are shown in Appendix A, table A3.
4.5.2 Dynamic analysis.

The dynamic response characteristics for these machines were performed using 2D FEA. The integral-motor dynamics using electrical and mechanical differential equations were reviewed in chapter THREE without cage winding and detailed in chapter FIVE with cage bars. The main purpose of finite element dynamic analysis is to verify the self-starting and synchronisation capability for these machines (SynRM-A and SynRM-B) and to compare their starting improvement. The simulation was achieved using sinusoidal voltage amplitude, constant frequency with phase voltage variations, and rotor initial-angular positions. The analysis of the rotor initial position at $\theta_m = 0^\circ$ angular degree was used as reference to verify the starting capability of the motor at rest and without current status. As depicted in Figure 4.17, it was shown that when a machine starts from rest, it can synchronise at a rated synchronous speed. It also can be observed that the motor can start at any initial rotor position with respect to the starting time variation. The rotor-space angular positions are varied form $\theta = 0^\circ$ to $\theta = 90^\circ$ at a step interval of $5^\circ$ as tabulated in Table A4, Appendix A. In this aspect, four different starting rotor initial position ($0^\circ$, $30^\circ$, $45^\circ$ and $75^\circ$) as shown in table 4.2 were graphically plotted to demonstrate the behaviour of the motor when the rotor position is varied. The starting torque characteristics of both machines are more or less the same, though SynRM-A has higher starting torque improvement than SynRM-B because of rotor high resistance.
Figure 4.17: Dynamic torque Comparison of SynRM-A and SynRM-B at: (a) 0°, (b) 30°, (c) 45° and 75°
Figure 4.18: Instantaneous speed comparison of SynRM-A and SynRM-B at; (a) 0˚, (b) 30˚, (c) 45˚ and 75˚

Table 4.2: Starting time of the models at a fixed current excitation.

<table>
<thead>
<tr>
<th>Rotor space angular position.</th>
<th>SynRM-A</th>
<th>SynRM-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0˚ ‘mech’</td>
<td>0.07 sec</td>
<td>0.09 sec</td>
</tr>
<tr>
<td>30˚ ‘mech’</td>
<td>0.07 sec</td>
<td>0.07 sec</td>
</tr>
<tr>
<td>45˚ ‘mech’</td>
<td>0.07 sec</td>
<td>0.06 sec</td>
</tr>
<tr>
<td>75˚ ‘mech’</td>
<td>0.06 sec</td>
<td>0.06 sec</td>
</tr>
</tbody>
</table>
The analysis in this section was done using finite element software to demonstrate and verify the working principle of the proposed machine. The obtained numerical variables within VALIDATION limit were exported and validated in CHAPTER FIVE using MATLAB. The applied three-phase voltages ($V_{ab}$, $V_{bc}$, $V_{ca}$) vary sinusoidal with time and within the operational variables of the proposed machines under steady-state functions is displayed in Figure 4.19a. The characteristics behaviour with instantaneous torque on no-load damping and step change on load-damping were demonstrated and the results were displayed in Figure 4.20. The factors influencing the load application for a direct-on-line synchronous reluctance motor were considered, hence figure 4.21 demonstrate the speed behaviour of the synchronous machine.

Figure 4.19: Sinusoidal voltage from $abcs$ variable
Figure 4. 20: Dynamic torque in a step change on no load and in variation of loads; (a) zero damping, (b) 6Nm load damping, (c) 12Nm load damping
4.6 Summary

Synchronous reluctance machines with cage bars IN and OUT of the rotor flux-barrier were analysed in this chapter using FEA. The developed non-linear finite element analysis was used to predict the characteristics behaviour of the proposed models. It was verified that the two models were able to start and synchronise, thereby performing a good combinational property of an induction motor and synchronous reluctance motor. SynRM-A has greater impact on starting and as well losses because its cage bars are in the rotor structure near the machine airgap. Its magnetic bridges saturate easily and contributed extensively to the high ripple-torque contents of the motor. SynRM-B has its cage bars inserted in the flux barrier of the rotor structure and possesses similar characteristics like its counterpart but with a reduced torque ripple content because of magnetic resistance variation between the machine flux barrier and the machine teeth. The position of the cage bars in flux barrier has lower saturation effect on the magnetic bridge.

The influence of the flux density field distributions are analysed and illustrated graphically showing that both machines have similar characteristics and a good magnetic flux leakage due to the presence of flux barrier in the rotor structure. Though SynRM-B displayed an improved
flux-density field distribution than its counterpart due to the presence of the cage bars in the flux barriers, as a result decreases the resistance to the flow of magnetic flux than SynRM-A which its cage bars were located above flux barriers in the rotor structure. The stator windings for both machines were excited in the same manner. The exciting current is sinusoidal. Similarly, the air-gap flux density generated as a function of rotor positions are also sinusoidal.

The analysis of \textit{dq-axis} inductances has been computed and the values of \( l_d \) and \( l_q \) were determined using FEA evaluation. It is well observed in figure 4.7 and figure 4.10 that \( q-axis \) has higher effect on air-gap flux density distribution and lower harmonic contents. The obtained values for the \textit{dq-axis} inductances were used in MATLAB for research validation. The purpose of this validation is to compare the outcome of \textit{abc} variable waveform, starting and synchronisation characteristics behaviour of the proposed model using different scientific approach. The obtained results from both software (Ansys’s 16.0 software and MATLAB) were compared and analysed as show in Appendix B. Figure 4.19(a) shows the replacement of the stationary reference frame \textit{(abc variables fixed to the stator)}. It was observed that during steady-state operation, the stationary reference frame was sinusoidal with time while the rotational reference frame was constant, hence its dynamic torque obtained are shown in figures 4.17, 4.18 and 4.20.

The obtained FEA inductance values were used in MATLAB for start-up transient analysis, dynamic response at a step change with the load and the synchronisation characteristics of the machine validations.
CHAPTER 5

DYNAMIC MODEL OF SynRM WITH ROTOR CAGE BARS

5.1 Introduction

This chapter focused on the development of dynamic model of a synchronous reluctance machine using rotor cage bars in the air barriers. The presence of rotor cage in the rotor geometry provided the starting torque capability for the afore-mentioned machine as proposed.

In the earlier stage of this work (chapter three), the developed mathematical equations of a synchronous reluctance machine without any conducting material in the rotor structure were derived as benchmark for the completion of SynRM with cage in the rotor structure. The inductances responsible for the machine performances were numerically derived and analysed. Due to the absence of conducting materials in the rotor structure, the machine cannot start on its own, thus cage windings/bars were required.

The purpose of this work is to validate the developed mathematical model of the machine with cage bars in the rotor structure using MATLAB, and targeting the starting torque, loading and synchronisation of the machine using dynamic and transient response to achieve better results.

The concept used in this chapter for developing the synchronous reluctance machine model with cage bar in the rotor structure was previously applied in chapter three. However, the addition of rotor cage bar in the machine will be considered in this chapter, hence, some equations will be referred to chapter three.

In addition, the equations that describe the self-starting of synchronous reluctance electrical machine behaviour will be derived and presented in this chapter. The inductances of the machine model were established and detailed in chapter three. The \( abc \) variable equations
were transformed into $dq$ reference frame for proper simplification. The $dq$ equivalent circuit will be developed using the derived equations for better understanding of the machine model.

The electromagnetic-torque equation introduced in chapter three is the torque developed using stator $abc$ windings of the machine (refer to chapter three, equation 3.60). The equations describing the rotor torque contribution for successful starting and synchronising will be discussed in this chapter. Similarly, the equations that analyse the electromagnetic combination of the torques (torque developed by the stator variables and rotor variables) are as well discussed in this chapter.

The derived equations will be explored using MATLAB to investigate and validate the transient and dynamic analysis results of the machine obtained in the previous chapter.

5.2 Motor Model

The variable used to describe the dynamic and transient analysis of a synchronous reluctance machine with cage in rotor structure using $dq$-model circuit is shown in Figure 5.1. The three-phase stationary machine is transformed into $dq$-model and represented in the rotating reference frame. In assumption, the stator windings and the caged rotor bars were sinusoidal distributed.

5.2.1 Stator voltage equation:

The voltage equations describing the behaviour of the electrical machine variables can be express in matrix form using equation (5.1) and (5.2),

\[ V_{abc} = R_{abc}I_{abc} + \frac{d}{dt}\lambda_{abc} \]  \hspace{1cm} (5.1)

\[ V_{qdr} = R_{qdr}i_{qdr} + \frac{d}{dt}\lambda_{qdr} \]  \hspace{1cm} (5.2)
where

\[
(f_{abc})^T = \begin{bmatrix} f_{as} & f_{bs} & f_{cs} \end{bmatrix}
\]

(5.3)

\[
(f_{qdr})^T = \begin{bmatrix} f_{qr} & f_{dr} \end{bmatrix}
\]

(5.4)

The symbol \( f \) in equation (5.3) and (5.4) is a representation of current, flux linkage or voltage.

The subscripts \( s \) and \( r \) in equations (5.1) to (5.4) indicate variables related to the stator and rotor winding respectively. The resistances \( R_s \) and \( R_r \) are diagonal matrices expressed using equations (5.5) and (5.6),

\[
r_{abcs} = \text{diag}[r_{as}, r_{bs}, r_{cs}]
\]

(5.5)

\[
r_{qdr} = \text{diag}[r_{qr}, r_{dr}]
\]

(5.6)

The flux linkage equations can be express as:

\[
\lambda_{abcs} = L_{abcs}i_{abcs} + L_{abcsqdr}i_{qdr}
\]

(5.7)

\[
\lambda_{qdr} = L_{qdrabcs}i_{abcs} + L_{qdrqdr}i_{qdr}
\]

(5.8)

The self-inductances and mutual-inductances of the machine can be represented in flux linkage using equations (5.9) – (5.10),

\[
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{cs} \\
\lambda_{qr} \\
\lambda_{dr}
\end{bmatrix} =
\begin{bmatrix}
L_{uas} & L_{ubs} & L_{uscs} & L_{usqr} & L_{usdr} \\
L_{lbs} & L_{lbs} & L_{lbscs} & L_{lbsqr} & L_{lbsdr} \\
L_{csas} & L_{csbs} & L_{csas} & L_{csqr} & L_{cisd} \\
L_{qras} & L_{qrs} & L_{qrs} & L_{qrsqr} & L_{qrsdr} \\
L_{dras} & L_{drbs} & L_{dras} & L_{dras} & L_{dras}
\end{bmatrix}
\begin{bmatrix}
{i_{as}} \\
{i_{bs}} \\
{i_{cs}} \\
{i_{qr}} \\
i_{dr}
\end{bmatrix}
\]

(5.9)
\[
\begin{bmatrix}
\lambda_{abc} \\
\lambda_{qdr}
\end{bmatrix} =
\begin{bmatrix}
L_s & L_{sr} \\
L_{sr} & L_r
\end{bmatrix}
\begin{bmatrix}
i_{abc} \\
i_{qdr}
\end{bmatrix} =
\begin{bmatrix}
L_{abc} & L_{abcqdr} \\
L_{qdrabc} & L_{qdr}
\end{bmatrix}
\begin{bmatrix}
i_{abc} \\
i_{qdr}
\end{bmatrix}
\]
(5.10)

Where

\[
L_{abc} =
\begin{bmatrix}
L_{aax} & L_{axb} & L_{acs} \\
L_{baa} & L_{bab} & L_{bcs} \\
L_{csa} & L_{cbs} & L_{csac}
\end{bmatrix}
\]
(5.11)

\[
L_{abcqdr} =
\begin{bmatrix}
L_{axq} & L_{axd} \\
L_{bxq} & L_{bxd} \\
L_{cxq} & L_{cxd}
\end{bmatrix}
\]
(5.12)

\[
L_{qdrabc} =
\begin{bmatrix}
L_{qax} & L_{qbx} & L_{qcs} \\
L_{dax} & L_{dbx} & L_{dcx}
\end{bmatrix}
\]
(5.13)

Equation (5.12) and (5.13) are coupling inductances that are equivalent and can be expressed as:

\[
L_{abcqdr} = (L_{qdrabc})^T =
\begin{bmatrix}
L_{axq} & L_{axd} \\
L_{bxq} & L_{bxd} \\
L_{cxq} & L_{cxd}
\end{bmatrix}
\]
(5.14)

\[
L_{qdr} =
\begin{bmatrix}
L_{qax} & L_{qdx} \\
L_{dax} & L_{ddx}
\end{bmatrix}
\]
(5.15)

It is worth mentioning that the diagonal elements in equation (5.9) are self-inductances and were associated with flux leakages.

5.2.2 Machine Inductances

The machine inductance matrix forms can be expressed as (P. Krause 2013),
Where:

\[ L_{mq} = \frac{3}{2}(L_{na} - L_{nb}) \]  

(5.19)

\[ L_{md} = \frac{3}{2}(L_{na} + L_{nb}) \]  

(5.20)

\( L_{ss} \) and \( L_{sr} \) are the leakage inductances in the stator and rotor windings respectively. \( L_{ms} \) and \( L_{mr} \) are the magnetising inductance of stator and rotor windings respectively. The self-inductances and the mutual-inductances obtained from the stator winding and the mutual-coupling inductances produced between the stator and the rotor winding are computed using equation (5.16) and (5.17) respectively. The self-inductances in the rotor winding is computed using equation (5.18). Under this condition, the electrical angle of the machine is expressed as a variable. The theory of reference transformation was used to simplify the complicated differential equations derived (Lyshevski, 2018, Krause et al., 2013).
5.2.3 Arbitrary reference frame transformation:

It is well known that, since the inductances of a synchronous reluctance machine are fixed in the rotor, its varying time can be easily eliminated (Krause et al., 2013). Hence, the stator and rotor equations of this modelled machine are referred to as the rotor reference frame. A change of variable is used to simplify the complexity of the machine variables due to the complicated differential equations obtained with the stator and rotor variables.

Using the same concept in chapter three, the vector notation in equation (3.25) can be used to formulate the expression of change-of-variable that transform the general equations into arbitrary reference frame as expressed by (Krause et al., 2013),

\[
\begin{align*}
\mathbf{f}_{qdos} &= x(t_r)\mathbf{f}_{abc} \\
\mathbf{f}_{qdos} &= [f_{q}, f_{d}, f_{o}]^T
\end{align*}
\]

Where:

\[
T_{qdos qs ds os} = \begin{bmatrix}
\cos(\theta_r) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\
\sin(\theta_r) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

And the inverse transformation matrix \( x(t_r)^{-1} \) is expressed as:

\[
\mathbf{f}_{qdos} = x(t_r)^{-1} \mathbf{f}_{abc}
\]
The dynamic equation for the rotor reference frame can be computed using matrix notations.

The transformation of the stator quantities to rotor reference frame is applied by multiplying both sides of equation (5.1) and (5.2) by an appropriate transformation matrix. Hence the stator-voltage equation can be expressed as:

\[
x(t_r)x(t_r)^{-1}V_{qdos} = x(t_r)r_x(t_r)^{-1}i_{qdos} + x(t_r)\frac{d}{dt}x(t_r)^{-1}\lambda_{qdos}
\]

Where:

\[
x(t_r)x(t_r)^{-1} = 1
\]

Then:

\[
V_{qdos} = r_xi_{qdos} + x(t_r)\frac{d}{dt}x(t_r)^{-1}\dot{\lambda}_{qdos}
\]

The second term of equation (5.28) on the right-hand side is further expanded as:

\[
x(t_r)\frac{d}{dt}x(t_r)^{-1}\lambda_{qdos} = x(t_r)\left[\left(\frac{d}{dt}x(t_r)^{-1}\right)\lambda_{qdos} + x(t_r)^{-1}\frac{d}{dt}\dot{\lambda}_{qdos}\right]
\]

Where the first and second term of equation (5.29) on the right-hand side is evaluated and expressed as:
\[ x(t_r) \frac{d}{dt} x(t_r)^{-1} \lambda_{qdos} = wr \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda_{qdos} \] (5.30)

And

\[ x(t_r) x(t_r)^{-1} \frac{d}{dt} \lambda_{qdos} = \frac{d}{dt} \lambda_{qdos} \] (5.31)

Substituting equation (5.30) and (5.31) into equation (5.29), we have:

\[ x(t_r) \frac{d}{dt} x(t_r)^{-1} \lambda_{qdos} = wr \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda_{qdos} + \frac{d}{dt} \lambda_{qdos} \] (5.32)

At this stage, the stator voltage equation can be expressed using rotor \( qd\)-reference frame by substituting equation (5.32) into equation (5.28) as an expression presented as equation (5.33),

\[ V^r_{qdos} = n_i i^r_{qdos} + wr \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda^r_{qdos} + \frac{d}{dt} \lambda^r_{qdos} \] (5.33)

Using the same \( dq \) voltage equation in (5.33) transformation, the flux linkages can be expressed as:

\[
\begin{bmatrix}
\lambda_{qds} \\
\lambda_{qdr}
\end{bmatrix} = \begin{bmatrix}
x(t_r) L_s x(t_r)^{-1} + x(t_r) L_{sr} \\
\frac{2}{3} x(t_r) L_{sr} x(t_r)^{-1} + L_r
\end{bmatrix} \begin{bmatrix}
i_{qds} \\
i_{qdr}
\end{bmatrix} \] (5.34)

Where:

\[ L_s = L_{abc} \] (5.35)
\[ L_{qr} = L_{abc \rightarrow qr} \quad (5.36) \]

\[ L_r = L_{dq} \quad (5.37) \]

Recall as stated in equation (5.19 & 5.20) that:

\[ L_{mq} = \frac{3}{2}(L_{mA} - L_{mB}) \quad (5.38) \]

\[ L_{md} = \frac{3}{2}(L_{mA} + L_{mB}) \quad (5.39) \]

Hence, the machine behaviour can be predicted by referring the rotor variables to the stator variables, using appropriate turn ratio transformations as:

\[ L_{aqr} = \frac{N_{qr}}{N_s} \left( \frac{2}{3} \right) L_{mq} \quad (5.40) \]

\[ L_{adr} = \frac{N_{dr}}{N_s} \left( \frac{2}{3} \right) L_{md} \quad (5.41) \]

\[ L_{qr} = \left( \frac{N_{qr}}{N_s} \right)^2 \left( \frac{2}{3} \right) L_{mq} \quad (5.42) \]

\[ L_{dr} = \left( \frac{N_{dr}}{N_s} \right)^2 \left( \frac{2}{3} \right) L_{md} \quad (5.43) \]

And other rotor variables like current, voltage and flux linkages can be expressed using rotor reference frame as:

\[ i_j = \frac{2}{3} \left( \frac{N_j}{N_s} \right) i_j \quad (5.44) \]
\[ v_j = \left( \frac{N_s}{N_j} \right) v_j \]  
(5.45)

\[ \lambda_j = \left( \frac{N_s}{N_j} \right) \lambda_j \]  
(5.46)

Where variable \(( j \) may be \( qr \) or \( dr \).

The \( dq \)-axes voltage equation that described the operation of the proposed electrical machine in rotor reference frame after proper transformations is expanded by substituting equations (5.40) - (5.46) into equation (5.33) and (5.34) as:

\[ V_{qs}^r = r_s i_{qs}^r + w_r \lambda_{ds}^r + \frac{d}{dt} \lambda_{qs}^r \]  
(5.47)

\[ V_{ds}^r = r_s i_{ds}^r - w_r \lambda_{qs}^r + \frac{d}{dt} \lambda_{ds}^r \]  
(5.48)

\[ V_{os}^r = r_s i_{os}^r + \frac{d}{dt} \lambda_{os}^r \]  
(5.49)

\[ 0 = r_{qr} i_{qr}^r + \frac{d}{dt} \lambda_{qr}^r \]  
(5.50)

\[ 0 = r_{dr} i_{dr}^r + \frac{d}{dt} \lambda_{dr}^r \]  
(5.51)

Similarly, its flux linkages can be written as:

\[ \lambda_{qs}^r = L_{qs} i_{qs}^r + L_{qsqr} i_{qr}^r \]  
(5.52)

\[ \lambda_{ds}^r = L_{ds} i_{ds}^r + L_{dsdr} i_{dr}^r \]  
(5.53)
\begin{align}
\lambda_{qs}^r &= L_{qs} i_{qs}^r + L_{mq} (i_{qs}^r + i_{qr}^r) \\
\lambda_{ds}^r &= L_{ds} i_{ds}^r + L_{md} (i_{ds}^r + i_{dr}^r) \\
\lambda_{os}^r &= L_{os} i_{os}^r
\end{align} (5.56)

\begin{align}
\lambda_{qr}^r &= L_{qr} i_{qr}^r + L_{mq} (i_{qs}^r + i_{qr}^r) \\
\lambda_{dr}^r &= L_{dr} i_{dr}^r + L_{md} (i_{ds}^r + i_{dr}^r)
\end{align} (5.57)

\subsection{Equivalent circuit}

To draw the equivalent circuit, the magnetising leakage inductances as shown in chapter three were put into consideration. The voltage equations (5.47) – (5.51) remain unchanged while the flux-linkages equations (5.52) – (5.55) were further expanded as:

\begin{align}
\lambda_{qs}^r &= L_{qs} i_{qs}^r + L_{mq} (i_{qs}^r + i_{qr}^r) \\
\lambda_{ds}^r &= L_{ds} i_{ds}^r + L_{md} (i_{ds}^r + i_{dr}^r) \\
\lambda_{os}^r &= L_{os} i_{os}^r
\end{align} (5.56)

\begin{align}
\lambda_{qr}^r &= L_{qr} i_{qr}^r + L_{mq} (i_{qs}^r + i_{qr}^r) \\
\lambda_{dr}^r &= L_{dr} i_{dr}^r + L_{md} (i_{ds}^r + i_{dr}^r)
\end{align} (5.57)

The dq-equivalent circuit presented in Figure (5.1) is the synchronous reluctance machine with cage bars in rotor structure as proposed and discussed in this thesis.
5.2.5 Torque Equation

The electromagnetic torque equation derived from Co-energy concept (Fletcher et al., 1994) is expressed as:

\[
W_{coe} = \frac{1}{2} \begin{bmatrix} i_{abc} \end{bmatrix}^T L_s \begin{bmatrix} i_{abc} \end{bmatrix} \quad (5.61)
\]

The electromagnetic torque equation is resolved to equation (5.62) as:
\[
T_{em} = \frac{p}{2} \frac{\partial W_{coe}}{\partial \theta} = \frac{p}{2} \left[ i_{abcs}^T L_s i_{abcs} \right] \quad (5.62)
\]

And electromagnetique (Anon., n.d.) torque equation in rotor reference frame is expressed using transformation equation \( f_{abc} = x(t) \frac{d}{d\theta} f_{qdo} \) as:

\[
T_{em} = \frac{p}{2} \left[ (x(t))^{-1} i_{qdo}^T \left( \frac{d}{d\theta} L_s x(t) \frac{d}{d\theta} i_{qdo} + \frac{d}{d\theta} L_s i_{qdr} + \frac{d}{d\theta} L_s i_{qdr}^T \right) + i_{qdr}^T L_s i_{qdr} \right] \quad (5.63)
\]

Where \( L_s, L_{sr} \) and \( L_r \) as described in equations (5.35) – (5.37) are coupling inductances of the stator and rotor circuits accompanied with vector current.

Resolving equation (5.63) with appropriate matrices, the expression for the electromagnetic torque developed is written as:

\[
T_{em} = \frac{3}{4} p \left[ \lambda_{ds}^r i_{qs}^r - \lambda_{qs}^r i_{ds}^r \right] \quad (5.64)
\]

The electromagnetic torque expression in equation (5.64) can further be expanded to demonstrate the dynamic torque equation developed by the machine with respect to current and can be expressed as:

\[
T_{em} = \frac{3}{4} p \left[ \left( L_{ds} - L_{qs} \right) i_{ds}^r i_{qs}^r + \left( L_{mqs} i_{ds}^r i_{ds}^r - L_{mqs} i_{qs}^r i_{ds}^r \right) \right] \quad (5.65)
\]

Equation (5.65) comprises of two torque components (synchronous reluctance torque and the asynchronous torque). The synchronous torque was developed from the \( abc \) winding (chapter three) while the asynchronous torque was developed from the interaction between the cage bars and the \( abc \) windings. The torque developed by the cage helps for starting and synchronisation.
of the machine. It also dampens the oscillations that occurs during starting or any interruption occurring due to load step change.

The developed electromagnetic torque and the rotor speed are related by rotor dynamic equation, which is expressed in differential form as:

\[
\frac{dw_r}{dt} = \frac{p}{2J} \left( T_{em} - T_L \right)
\]

(5.66)

Where \( p \) is the number of poles, \( J \) is the total inertia of the rotor, \( T_{em} \) is the electromagnetic torque developed by the motor, and \( T_L \) is the load torque.

5.3 Summary

The derived mathematical model presents the fundamental principles of an asynchronous function and the dynamic torque production for a self-starting synchronous reluctance machine. The transformation of \( abc \) variable to \( dq\)-reference-frames were performed to investigate the performance of machine during dynamic and torque production. Obtained results confirmed an improvement with the characteristics performance of the machine using such configuration. The developed dynamic equations in this chapter will be used in chapter six to validate the theoretical analysis associated with this work and will be compared with the FEA technique that was done in chapter four.
CHAPTER 6
MATLAB/SIMULINK IMPLEMENTATION AND VALIDATION

6.1 Introduction

In this chapter, the derived dynamic equations for self-starting synchronous reluctance machine with rotor cage will be modelled using MATLAB. The main aim of this chapter is to investigate the transient and dynamic response of the proposed machine and to validate the results obtained from; electrical machines theorem, dynamic derivations and Finite Element simulations. The mathematical fundamentals that was set up from chapter three to chapter five will be finalised in this chapter.

6.2 Equations for Simulation

It has been shown in chapter five from equations (5.56) – (5.60), that the flux linkages can be expressed using $dq$-axis platform. These equations are further formulated so that flux linkages can be expressed as state variables while currents are dependent variables which are written as:

\[
\begin{align*}
\lambda_{qs} & = \frac{\lambda_{qs}}{L_{ts}} \\
i_{dq} & = \frac{\lambda_{dr} - \lambda_{m}}{L_{ts}} \\
i_{ds} & = \frac{\lambda_{ds} - \lambda_{md}}{L_{ts}} \\
i_{qr} & = \frac{\lambda_{qr} - \lambda_{mq}}{L_{rq}} \\
i_{dr} & = \frac{\lambda_{dr} - \lambda_{md}}{L_{sr}}
\end{align*}
\]

Where $\lambda_{mq}$ and $\lambda_{md}$ are $dq$-axis magnetizing flux linkages and can be expressed as:
\[ \lambda_{mq} = L_{mq} \left( i_{qs}^r + i_{qr}^r \right) \]  
\[ \lambda_{md} = L_{md} \left( i_{ds}^r + i_{dr}^r \right) \]

The Simulink model was designed by incorporating equations (6.1) – (6.4) into the voltage equations (5.47) – (5.51) as discussed in chapter five, while the integral form of the manipulated machine variables expressed as state variables is presented using equations (6.7) – (6.14),

\[ \lambda_{qs}^r = \int \left[ V_{qs}^r + \frac{r_s}{L_s} \left( \lambda_{mq} - \lambda_{qs}^r \right) - w_i \lambda_{ds} \right] dt \]  
\[ \lambda_{ds}^r = \int \left[ V_{ds}^r + \frac{r_s}{L_s} \left( \lambda_{md} - \lambda_{ds}^r \right) - w_i \lambda_{qs} \right] dt \]  
\[ \lambda_{qr}^r = \int \frac{r_{qr}}{L_{qr}} \left( \lambda_{mg} - \lambda_{qr}^r \right) dt \]  
\[ \lambda_{dr}^r = \int \frac{r_{dr}}{L_{dr}} \left( \lambda_{md} - \lambda_{dr}^r \right) dt \]

Where

\[ \lambda_{mq} = L_a \left( \frac{\lambda_{qs}}{L_s} + \frac{\lambda_{qr}}{L_r} \right) \]  
\[ \lambda_{md} = L_a \left( \frac{\lambda_{ds}}{L_s} + \frac{\lambda_{dr}}{L_r} \right) \]

And

\[ L_a = \left( \frac{1}{L_m} + \frac{1}{L_s} + \frac{1}{L_r} \right)^{-1} \]
A Simulink model is built using equations (5.65), (5.66) and (6.1) – (6.14). The developed model was simulated in MATLAB/Simulink environment to investigate and demonstrate the characteristics of a self-starting synchronous reluctance machine with distributed cage bars in the rotor structure. The Simulink model has a stator winding connected at a frequency of 50 Hz and a rated voltage supply. The complete Simulink model is presented in Figure 6.1 showing only the main subsystems of the model. Though the following sub-sections will display the contents associated with each subsystem.

Figure 6.1: Complete Simulink model of a Self-Start Synchronous Reluctance machine with cage bars in the rotor structure.
6.3 Simulink network implementation

The inputs of a self-starting synchronous reluctance machine are the same to the inputs of induction machine and can be described using the three-phase voltages, the desired frequency and the load torques. The expected outputs are; the dynamic torque with its step-in load change and the rotor speed of the machine. It is required that all the stationary (three-phase) variables be transformed into rotational reference (two-phase) frame.

6.3.1 abcs to dq internal detail subsystem block.

The Simulink blocks for the transformation of the three-phase ‘abcs’ variables into two-phase ‘dq’ rotating frame is demonstrated in Figure 6.2.

![Simulink transformation implementation from abcs variables to dq variable.](image)

The fundamental Simulink block functions used as depicted in Figure 6.2 show the transformation of three-phase voltage variable to dq rotational reference frame. The set of equations that described and implemented as shown in figure 6.2 were derived from equation (5.33). The obtained simulated result for the above Simulink transformation (Fig. 6.2) is presented in Figure 6.3, it demonstrated a three-phase voltage as input against time (t) in seconds.
The simulated results in Figure 6.3 shown the sinusoidal behaviour of the applied three-phase voltages.

### 6.3.2 Torque internal subsystem simulation implementation

The subsystem describing the dynamic torque characteristics of the proposed model is shown in Figure 6.6 and the derived equation (5.65) used for the implementation can be found in chapter five.
Figures 6.7 (a) – (c) presents the characteristics of the torque dynamic on no-load, in step change of 6 Nm and 12 Nm respectively.

![Torque Response Graphs](image)

Figure 6.5: Dynamic response of the machine in step change in load: (a) no-load, (b) 6Nm and (c) 12Nm.
6.3.4 Speed detailed subsystem simulation implementation

The equation describing the synchronous speed for the proposed machine is found in equation 6.14 and the subsystem analysing the dynamic speed is depicted in Figure 6.8.

![Figure 6.6: Subsystem that detailed the Simulink implementation of dynamic machine speed](image)

Figure 6.9, evidenced the dynamic response of the machine at a rated synchronous speed.

![Figure 6.7: Dynamic response of the machine at rated speed.](image)
6.4 Summary

A dynamic derivation of a line-start synchronous reluctance machine with rotor cage configuration has been presented and analysed.

The entire dynamic derivation built in a computer model as shown in Figure 6.1 is developed to investigate the dynamic and torque production performances of the machine. The model was built and simulated using MATLAB software.

The stationary variables obtained as demonstrated in chapter three were transformed into rotational reference frame in this chapter for easy computation and machine analysis.

The obtained simulated results showed the behaviour of the machine from $abc_s$ variables to $dq$ variables. The dynamic torque produced by the interaction of the stator and rotor field was also computed and analysed.

The dynamic-torque response to step-change in applied load was examined. Findings show that the proposed machine possesses the capability to start-up and synchronise using cage bars in the rotor structure.
CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

7.1 Introduction

This work is done to demonstrate and analyse the self-starting capability of a synchronous reluctance machine with rotor cage IN and OUT of the air barrier in the rotor structure. The rotor cage bars were symmetrically distributed in the rotor structure. As mentioned earlier in chapter one of this work it follows the drift of on-line starting electrical machine to obtain the starting torque and to compare the improved starting performance SynRM-A against SynRM-B. Also to investigate parameters like torque, power factor and efficiency of a self-starting synchronous reluctance machines using FEA. This trend is done by introducing electrical conducting cage bars in flux barriers on the rotor structure, such that starting torque will be obtain, and the torque index, power Factor and Efficiency of the machine can be performed.

The categorical developments and performances of On-line starting synchronous reluctance machines were presented in chapter two as a literature research review. The different methods of geometrical and electrical approaches used to improve the performances of this type of machines were reviewed, and its outcomes were analysed.

Chapter Three presents the basic reviewed mathematical modelling of a conventional synchronous reluctance machine which set a benchmark for the full modelling of a SSSynRM proposed in this work. The transformation of a three-phase $abc$ reference frame contents to $dq$-arbitrary reference frame was reviewed. The performance indices were demonstrated for better understanding of the machine.

Chapter Four presents the structural geometry and graphically behaviour of the model discussed in this thesis. The models were built in Ansys’s Maxwell environment specifically
in Two-Dimensional Finite Element Method (2D-FEM). The electromagnetic field Characteristics were analysed using Finite Element Analysis (FEA). The simulated results are computed, and the calculated performance indices were obtained, hence were used in Chapter Six for validation.

Chapter Five presents the mathematical modelling of a synchronous reluctance machine with cage bars in the rotor structure. It completed the mathematical derivation that started from review done in chapter three and has shown the combination of stator and rotor torque performances. The derived equations were built in MATLAB/Simulink software package for overall validation.

In chapter six, the theoretical knowledge and other scientific machine design method used in this work were validated using MATLAB/Simulink software. The performance indices obtained in finite element environment were simulated into MATLAB/Simulink environment which validates and verified the overall FEA results obtained in Chapter Four. Hence has proven the starting and synchronisation of the machine and other performance credibility.

The overall conclusion that conclude the contents of this thesis and the recommendations for future references were drawn in this chapter. Hence the conclusion of this work corresponds to the objectives listed in chapter one of this thesis.

### 7.2 Machine Model and Modelling analysis

The modelling approach of the above proposed machine model discussed in this thesis emanated from induction machine (IM) concept. Since the stator structure of a three-phase rotating electrical Machines are the same, only the rotor structure is modified. Flux barriers were introduced in rotor structure to reduce magnetic flux leakages and as well rotor losses. The cage bars were inserted IN and OUT of the flux barriers in rotor structure of the machine
for self-starting purposes and to compare which rotor concept improved in terms of starting performances against each other. As per the fulfilment requirement of a line-start electrical machines, this machine is able to start from rest and speed up to a constant speed when supplied from rated source, hence displays self-starting and synchronization capability. The theoretical knowledge of a conventional IM performances were not compromised, instead the performance of modified machine as discussed in this thesis exhibit a better performance in all targeted parameters.

Meanwhile, the basic understanding operation of synchronous reluctance machine with cage bars inserted in the rotor structure were reviewed and presented. The numerical method appropriate for computing electric and magnetic field were used due to the geometrical complexity structure of the machine. The distribution of electromagnetic field was determined, the analysis of flux density distributions, the torque both in transient and steady-state were graphically presented.

In conclusion, Finite Element Method is the appropriate numerical method used in this work for field solutions. It permits a good estimation during the analysis of electromagnetic device performances and as well allows to reduce considerably the number of prototypes.

7.3 Finite Element Method/Analysis (FEM/FEA)

The analysis of a synchronous reluctance machine with cage bars in rotor structure is studied and presented. 2D FEM is used to model the machines as discussed in this thesis and the field calculator solver is used to investigate the performance indices. Hence, the inductances were calculated based on the fundamental distribution of radial components of airgap flux density from the field solution. The transformation of abcs variables fixed on the stator to the synchronous rotating reference frame (induced dq-windings) rotating at the same speed of the
rotor were analysed and presented. The rotating reference frame equivalent circuit was
developed and simulated. The performances of the self-starting transverse-laminated
synchronous reluctance machine were investigated, and the resulted output are as follows:

**The starting torque.** This was achieved because of the presence of electrical conducting cage
bars inserted IN and OUT of flux barriers in rotor structure. These bars are responsible for
starting capability of the machine.

**The performance indices.** The presence of flux barriers in the rotor structure provide the
motor with a better saliency ratio, efficiency and as well power factor. The machine can
withstand synchronism after undergone oscillations that occurred from starting capability and
disturbance due to change in load.

Meanwhile, the output results were investigated using FEA and are validated using
MATLAB/Simulink. The above-mentioned output results were well detailed in published
IEEE international Conference papers stated as part of dissertation contributions.

### 7.4 MATLAB/Simulink Validation

This sub-chapter draws the work into to final investigation since the validation of this work
leads to the conclusion of the dissertation. However, the dynamic model of the motor was
implemented in MATLAB/Simulink environment. The validation of theoretical and finite
element environmental work as discussed in this thesis were verified. The obtained values of
$\textit{dq}$ inductances simulated in FEA environment were exported into MATLAB/Simulink model
built to test the Self-start synchronous reluctance machine as proposed. The obtained results
were computed and compared with the FEA results naming: the applied $\textit{abc}$s Voltage variables
fixed to the stator (the stationary reference frame), the transient and steady-state operation of
the machine in no-load, the transient and steady-state operation of the machine in step change at applied loads of 6Nm and 12Nm and the speed characteristic at a rated speed.

7.5 Thesis findings

The research output of this work has been reported and published. It can be found in Institute of Electrical and Electronic Engineer (IEEE) website. These publications are listed in chapter one, section 1.6.

The insertion of electrical conducting bars IN and OUT of flux barrier in the rotor structure provides a self-starting capability of a synchronous reluctance machine as proposed. Both of the machines possess similar characteristics though was observed that SynRM-A starts and synchronise faster than SynRM-B because of skin effect, hence improves in terms of starting capability against its counterpart but experience losses due to the position of the cage which is situated in rotor iron structure. In other hand, SynRM-A has higher torque ripple content due to variation of magnetic resistance between the flux barriers and teeth of the machine. The detailed solutions and its demonstrations are found in chapter four and six respectively. Rotor configuration approach used to achieve SSSynRMs from time to time are reviewed. However, the proposed rotor configuration achieved all the performances required by on-line synchronous reluctance motors without compromising its properties, hence easy to construct at low cost of manufacturing materials.

The literature review analysis of a conventional squirrel cage induction machine (IM) shown that the squirrel cage rotor has better starting torque behaviour to compare the proposed rotor with bars in the flux barrier. Though, the proposed rotor model minimises copper losses to the lowest level and thereby achieved reduced torque ripple contents. The proposed model maintained good average torque in different vector angle position due to the presence of flux
barriers which is designed to minimise the magnetic reluctance present in the direct-axis and to maximise the reluctance present in quadrature-axis.

More so, Finite Element Analysis is used to analyse the characteristics and field analysis of the proposed model, hence to demonstrate the air-gap flux density distribution and torque performances.

Furthermore, the validation of this work is done in MATLAB/Simulink platform which demonstrated the successful operation of SSSynRM fitted with cage bars in rotor structure. The obtained results from FEA and MATLAB/Simulink displayed similar characteristics which evidenced the validation of this work.

7.6 Recommendation and future work

This work presented the advantages of a self-start synchronous reluctance machine with cage bars in the rotor structure. The results obtained from finite element analysis and MATLAB/Simulink evidenced that, the proposed model (self-starting SynRM) can start and synchronise on its own and has capability of withstanding an unexpected interruption that might occur due sudden change in applied load.

However, the continuation of this work would be very motivating to build the prototype for further investigation and compared with the software simulation results. Comparing the results obtained in this work with Experimental work would provide a comprehensive understating of this motor operation.

Meanwhile, since the proposed model of this work was based on FEA/Simulink simulation, other recommended and future work that will be interesting to investigate in this type of work are:
• The analytical development prototype of this work should be optimized to improve the comprehensive understanding parameters that influence the performance of self-starting synchronous reluctance machines.

• The position of bars in flux barrier of the rotor should be investigate. One can study the effect of cage bar position in rotor flux barrier which brings about saturation of rotor ribs and assurance of mechanical strength of the motor.

• The main losses encountering in this type of work was obtain from stator copper loss, hence the stator of this type of machine should be further investigated and optimize.

• Issues related to thermal modelling, control Algorithm and performance behaviour of the synchronous machine during complete cycle operation will be very interesting to investigate in future work.
REFERENCES


LYSHEVSKI, S. E. 2018. Electromechanical systems, electric machines, and applied mechatronics, CRC press.


APPENDIX A

SIMULATION AND CALCULATED RESULTS

Table A 1: SynRM-A Magneto static dq machine simulated performance index

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<th>( PF_{max}(p.u) )</th>
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Table A 2: SynRM-B Magneto static dq machine simulated performance index

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Table A 3: Average torque and torque ripple comparison of SynRM-A and SynRM-B.

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<th>Current Space Angles (°)</th>
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<th>SynRM-B</th>
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<td>$T_{av}$ (Nm)</td>
<td>$T_{ripple}$ (%)</td>
<td>$T_{av}$ (Nm)</td>
<td>$T_{ripple}$ (%)</td>
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Table A 4: Starting time of the models at interval of 5° angular position.

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<td>5° mech</td>
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</tr>
<tr>
<td>10° mech</td>
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<td>0.07 sec</td>
<td></td>
</tr>
<tr>
<td>15° mech</td>
<td>0.06 sec</td>
<td></td>
<td>0.06 sec</td>
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<td>20° mech</td>
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<td>0.08 sec</td>
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<tr>
<td>25° mech</td>
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<td>0.08 sec</td>
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<tr>
<td>30° mech</td>
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<td>0.06 sec</td>
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<tr>
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<td>0.07 sec</td>
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<td>0.07 sec</td>
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<td>50° mech</td>
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<td>0.09 sec</td>
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</tr>
<tr>
<td>55° mech</td>
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<td>65° mech</td>
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<td>80° mech</td>
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<td>85° mech</td>
<td>0.12 sec</td>
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<tr>
<td>90° mech</td>
<td>0.07 sec</td>
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<td>0.09 sec</td>
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</tbody>
</table>
Figure B 1: Simulated Self-start SynRM; (a) efficiency and (b) Power Factor
Appendix C

Motor prototype and its modifications

Figure C 1: Quarter rotor structure of a conventional Induction Machine

Figure C 2: Cross section cage bars of (a) SynRM1 and (b) SynRM2