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Input for young children’s number concept development

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201241203

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Doctor of Philosophy

in

Education

at the

University of Johannesburg

Supervisor: Prof Elizabeth Henning

Co-supervisors: Prof Caroline Fitzpatrick and Prof Lara Ragpot

Date: November 2018
DECLARATION

I declare that this is my original research for the purpose of the thesis, Input for young children’s number concept development.

Name: 

Signature: Date:
SUMMARY

Previous studies have indicated that various cognitive skills contribute to early number concept development and that such pre-school skills predict mathematical competence in Grade 1. In this study I argue that children’s number concept development at the beginning of Grade 1 can partly be explained by their level of numerical competence at the beginning of Grade R, as well as their language development, classroom engagement and logical reasoning. Many Grade 1 children struggle to develop mathematical competence, possibly because of the development of other cognitive skills. I propose that early grade teachers should practice a pedagogy that has taken cognisance of how cognitive skills develop and contribute to number concept development. In this study I investigated possible concurrent and predictive associations between Grade R and Grade 1 children’s number concept development and contributing cognitive skills, namely their mathematics-specific vocabulary, classroom engagement and logical reasoning.

In the analysis of literature I explain each variable, namely 1) number concept development, 2) mathematics-specific vocabulary, 3) classroom engagement as a manifestation of executive functions and 4) logical reasoning. I describe, from a theoretical point of view, how each cognitive skill contributes to early number concept development, reasoning that each skill is ‘input’ for learning number concepts and that cognitive ‘input’ is, in turned, analyzed/filtered by an ‘input analyzer’. The literature study (Chapter 2) is concluded with a discussion of how teachers can integrate knowledge of contributing constructs of number concept development in their daily teaching and assessment.

The study was designed to integrate quantitative- and qualitative data in such a way that qualitative findings could be utilized to support and explain quantitative findings. 59 Grade R Sesotho and isiZulu speaking children’s early number concept development, mathematics-specific vocabulary, classroom engagement and logical reasoning were assessed in the beginning of 2017. The same children’s early number concept development was assessed in the beginning of 2018 when they were in Grade 1. The Grade 1 assessments were conducted in English (since English is their

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1The ‘Reception’ year.
language of instruction from Grade 1 onwards) and the children’s home language (isiZulu or Sesotho) to compare attainment on these two assessments.

Statistical analysis yielded that there was a decrease in the mean of number concept development from Grade R (tested in home language) to Grade 1 when the children were tested in English, but an increase in the mean score when the children’s number concept development were tested in their home language in Grade 1. A correlational analysis also showed that mathematics-specific vocabulary in Grade R had the strongest correlation with number concept development scores in Grade 1 (tested in English). A multiple regression analysis indicated that each of the Grade R cognitive skills independently contributed to this sample’s Grade R number concept development, but that the cognitive skills also collectively predicted number concept development in Grade 1. No significant gender differences on the assessments of early number concept development, mathematics-specific vocabulary, classroom engagement or logical reasoning were detected. isiZulu speaking children outperformed Sesotho speaking children on maths conceptual development.

These findings raised questions in terms of language policy in schools. I conclude the study with future directions for research and suggestions for policy and educational practice which relates to the findings and contributions of the study.
ACKNOWLEDGEMENTS

This study is dedicated to my parents with huge gratitude for everything they have taught me. What I know about teaching and learning, I have learnt from my mother who is an embodiment of the theoretical knowledge I acquired during this study, while my father has taught me persistence, work ethic and compassion for and dedication to children who live in sub-urban and township areas.

I acknowledge the following individuals and groups who have contributed to the completion of this thesis:

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My husband, Rikus, and my friend, Marion, who were the two critical readers for this thesis.

The Grade R and Grade 1 teachers at Funda UJabule who provided valuable insights during interviews.

Funda UJabule Primary for welcoming me during the test periods.

Nozipho and Delia at the Centre for Education Practice Research.

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<tr>
<th>Acronym</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANA</td>
<td>Annual National assessment</td>
</tr>
<tr>
<td>AMS</td>
<td>Approximate Magnitude System</td>
</tr>
<tr>
<td>AJRMSTE</td>
<td>African Journal of Research in Mathematics, Science and Technology Education</td>
</tr>
<tr>
<td>CAPS</td>
<td>Curriculum and Assessment Policy Statement</td>
</tr>
<tr>
<td>CE</td>
<td>Classroom Engagement</td>
</tr>
<tr>
<td>CEPR</td>
<td>Center for Education Practice Research</td>
</tr>
<tr>
<td>CFT</td>
<td>Culture Fair Test</td>
</tr>
<tr>
<td>DBE</td>
<td>Department of Basic Education</td>
</tr>
<tr>
<td>DoE</td>
<td>Department of Education</td>
</tr>
<tr>
<td>DCCS</td>
<td>Dimensional Change Card Sort</td>
</tr>
<tr>
<td>DHET</td>
<td>Department of Higher Education and Training</td>
</tr>
<tr>
<td>ECD</td>
<td>Early Childhood Development</td>
</tr>
<tr>
<td>ECE</td>
<td>Early Childhood Education</td>
</tr>
<tr>
<td>EF</td>
<td>Executive functions</td>
</tr>
<tr>
<td>ERIC</td>
<td>Education Resources Information Centre</td>
</tr>
<tr>
<td>HL</td>
<td>Home Language</td>
</tr>
<tr>
<td>HPCSA</td>
<td>Health Professions Council of South Africa</td>
</tr>
<tr>
<td>IBM</td>
<td>International Business Machines</td>
</tr>
<tr>
<td>IIEP</td>
<td>International Institute for Educational Planning</td>
</tr>
<tr>
<td>IPS</td>
<td>Intraparietal Sulcus</td>
</tr>
<tr>
<td>JPTD</td>
<td>Junior Primary Teaching Diploma</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
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<tr>
<td>JSAIS</td>
<td>Junior Individual Scales</td>
</tr>
<tr>
<td>KiP</td>
<td>Knowledge-in-Pieces</td>
</tr>
<tr>
<td>LAD</td>
<td>Language Acquisition Device</td>
</tr>
<tr>
<td>LR</td>
<td>Logical Reasoning</td>
</tr>
<tr>
<td>MARKO-D</td>
<td>Mathematics and arithmetic competence diagnostic (as translated from German)</td>
</tr>
<tr>
<td>MARKO-D SA</td>
<td>MARKO-D SA was adapted for local use</td>
</tr>
<tr>
<td>MLD</td>
<td>Mathematical Learning Difficulties</td>
</tr>
<tr>
<td>MMLT</td>
<td>Meerkat Maths Language Test</td>
</tr>
<tr>
<td>MSL</td>
<td>Mathematics-Specific Language</td>
</tr>
<tr>
<td>NCD</td>
<td>Number Concept Development</td>
</tr>
<tr>
<td>NIAF</td>
<td>National Integrated Assessment Framework</td>
</tr>
<tr>
<td>NRF</td>
<td>National Research Foundation</td>
</tr>
<tr>
<td>OTS</td>
<td>Object Tracking System</td>
</tr>
<tr>
<td>PCK</td>
<td>Pedagogical Content Knowledge</td>
</tr>
<tr>
<td>SAQMEC</td>
<td>Southern and Eastern Africa Consortium for monitoring Educational Quality</td>
</tr>
<tr>
<td>SADC</td>
<td>South African Development Community</td>
</tr>
<tr>
<td>SFPTEP</td>
<td>Strengthening Foundation Phase Teacher Education Programme</td>
</tr>
<tr>
<td>SPSS</td>
<td>Statistical Package for the Social Sciences</td>
</tr>
<tr>
<td>ST</td>
<td>Student teacher</td>
</tr>
<tr>
<td>T1/2/3</td>
<td>Teacher 1, 2 or 3</td>
</tr>
<tr>
<td>TIMSS</td>
<td>Trends in International Mathematics and Science Study</td>
</tr>
<tr>
<td>TCR</td>
<td>Teachers College Report</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>UNESCO</td>
<td>United Nations Educational, Scientific and Cultural Organization</td>
</tr>
<tr>
<td>VASSI</td>
<td>A mathematics proficiency test developed by Dr Colleen Vassiliou</td>
</tr>
<tr>
<td>WCST</td>
<td>Wisconsin Card Sorting Test</td>
</tr>
<tr>
<td>WIAT-II</td>
<td>Wechsler Individual Achievement Test</td>
</tr>
<tr>
<td>WISC-IV</td>
<td>Wechsler Intelligence Scale for Children – Fourth UK Edition</td>
</tr>
<tr>
<td>WM</td>
<td>Working Memory</td>
</tr>
</tbody>
</table>
ALPHABETICAL LIST OF TERMINOLOGY

Classroom engagement: The degree of attention, curiosity, participation, interest and passion that children show during classroom activities.

Concepts: The most basic unit of thought or idea about a construct.

Conceptual change: Change in the development of conceptual system – also through language and a change in discourse.

Conceptual systems: When different concepts are integrated, one can say that these concepts form a sort of framework. This framework or system can be compared to a spider web, where all the concepts are connected to form a functional unit – a conceptual system.

Conspecific: An animal or human (or in the context of this study, an object) relating to the same species or group.

Descriptive statistics: A set of descriptive values that summarizes the data set.

Diagnostic instruments/ tests: Tests that can be used to determine (diagnose) difficulties. In the context of this study, the aim is to diagnose difficulty in number concept development, a lack of mathematics-specific vocabulary, poor classroom engagement and poor logical reasoning in early grades. Since all these skills contribute to numerical competence, the aim is to diagnose skills that may influence numerical competence negatively in Grade 1.

Executive functions: Executive functions (EFs; also called executive control or cognitive control) refer to a family of top-down mental processes needed when you have to concentrate and pay attention, when going on automatic or relying on instinct or intuition would be ill-advised, insufficient, or impossible.

Foundation phase: Grade R (reception year) to Grade 3.

Fritz model: A specific development model that Prof Annemarie Fritz and her colleagues developed by integrating various models that explain early number concept
development. The model consists of five number concept development levels that follows hierarchically on each other.

Grade 12/Matric: Final school exam written by learners; examination taken to certify completion of 12 years of schooling.

Grade R: The ‘Reception year’ is the year of schooling before Grade 1. ‘Pre-school’ includes Grade R.

Imputed data: Substituted statistically determined values that replace missing data.

Inferential statistics: Statistical procedures that infer properties of the sample.

Input analyzer: The input analyzer then analyzes the specific type of ‘input’. For instance, perceptual input is analyzed by a ‘perceptual input analyzer’ and innate input analyzers allows us to identify conspecifics or sort objects according to similarities. Input analyzers identify attributes of input and analyze the content of the input.

Input: In the context of this study, I define ‘input’ as cognitive skills that collectively and independently contribute to (or are input for) number concept development. The input (or contributor) for number concepts are analysed by an ‘input analyzer’.

Intuitive theories: Children, and adults, intuitively form theories, governed by how concepts (in the context of this study – number concepts) connect to other constructs, such as language and the evidence of the theory. Vocabulary of a theory is important because language (and in particular, vocabulary) interact with the formation of concepts relating to the theory.

Logical reasoning: To use a systematic line of thought to arrive at a decision or conclusion.

Mathematical learning difficulty: A spectrum of difficulties which prevents children to develop number concepts with DD at the extreme end of the continuum.

Mathematical learning disability (developmental dyscalculia [DD]): A significant impairment in mathematical skills that prevents some children to develop number concepts in the same way as their peers; a developmental disorder which affects the development of mathematical skills, while other areas of development are still intact.
Number sense: The sense or ability to understand, approximate and manipulate numerical quantities.

Number talk/mathematics-specific language: Vocabulary which relates specifically to mathematical concepts.

Numerical competence: To understand the relationships between numbers to do mathematical calculations.

Pedagogy: The way in which teachers teach and the principles that underlie their knowledge of teaching. In the context of this study the term refers to teachers’ knowledge of cognitive development.

Subitizing: To quickly label small quantities through observation instead of counting.

Symbolic distance effect: Magnitudes that are closer are compared with less accuracy than those numbers that are more distant in magnitude.

Teaching school: A school which, in collaboration with a university, serves as a training site for prospective teachers by welcoming education students into the classrooms while applying research-based teaching.

Translanguaging: The use of a single language system (or idiolect) that functions beyond named languages and enables children to employ their whole linguistic repertoire which may consist of multiple named languages.

Working memory: An active mental system that regulates cognitive behaviour by temporarily storing and manipulating information.
CHAPTER 1: 
INTRODUCTION AND ORIENTATION TO THE STUDY

1.1. Background and motivation for the study

The increased interest in mathematics education in South Africa is indicative of the need to explain the country’s weak performance on international and regional assessments. The final school examination in Grade 12 shows that most learners do not perform well in Mathematics (Department of Basic Education [DBE], 2016, 2017, 2018; Spaull, 2017). Recently, the attention of education officials and researchers has turned to the early grades. This ‘early grades turn’ represents an international trend, with research articles, journals\(^2\), handbooks, and textbooks increasing its coverage of early mathematics learning. In South Africa, this is evidenced by the creation of two First Rand Foundation Numeracy Research and Development Chairs and several South African Research Chairs, funded by the Department of Science and Technology along with the increased focus on mathematics education at local universities\(^3\).

The results of Grade 12 (matric) form a disturbing picture; the average score for mathematics in 2016 was 51.1% and only 3% of the learners achieved a distinction (DBE, 2016). In 2017, 51.9% passed mathematics (DBE, 2017, 2018). In addition to poor matric results, the outcomes of other national and international tests have also drawn attention to the academic difficulties of South African learners. The results of the Annual National Assessment (ANA)\(^4\), which were published by the DBE, indicate that the average test score for grade 3 in 2014 was 65% and the average score for Grade 6 children in the same year was only 35% (DBE, 2014). The ANA received much critique because of its poor year-to-year calibration (van den Berg, 2016).

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\(^3\) Rhodes University (Numeracy Education and Mathematics Education); University of Johannesburg (Integrated studies of learning Language, Mathematics and Science in Primary Schools); University of the Western Cape (Mathematics Education); University of Witwatersrand (Mathematics Education and Numeracy Education)

\(^4\) ANA is currently under review and is likely to be replaced by the National Integrated Assessment Framework (NIAF) or may be suspended.
Although these may not all be reliable statistics, it does indicate that many children and youth in South Africa do not learn mathematics well. Furthermore, SACMEQ\(^5\) conducted a study in the South African Development Community (SADC) region with a sample of 9083 Grade 6 learners and 1488 teachers from 392 schools across Southern Africa in 2007 (SACMEQ, 2010) and repeated the test five years later. The results of the 2007 test indicated that only 5.5% of the Grade 6s could complete single step addition and subtraction operations successfully (level 1 out of 8) and only 5.9% of the Grade 6 learners in South Africa were ‘mathematically skilled’ (level 6 out of 8) (SACMEQ, 2010). The SACMEQ results in 2013 were also discouraging (Spaull, 2018).

South Africa has also been taking part in a very large student assessment study since 1995 - the Trends in International Mathematics and Science Study (TIMSS). In 2015 South Africa’s Grade 5 learners took part in this study, whilst, in the other participating countries, Grade 4s wrote the tests. South Africa’s Grade 5 learners were rated 48\(^{th}\) out of 49 countries. In addition to the results of these tests, the Global Information Technology report of 2014 (www.weforum.org) indicates that the quality of mathematics and science education in South Africa is the poorest of the 148 countries. Although there has been a debate whether this is accurate, it is still distressing. The TIMSS results suggest that foundational education is failing to provide sufficient learning opportunities for many children in South Africa. Spaull and Kotze (2015) found that not only do just the top 16% of Grade 3 children in South Africa perform at Grade 3 level, but also that as they move to higher grades in the school system the children who struggled in the foundation phase are likely to struggle even more in higher grades.

In 2011 the South African Department of Higher Education and Training (DHET) received R141 million from the European Commission to initiate one of the flagship programmes of the DHET, namely the Strengthening Foundation Phase Teacher Education Programme (SFPTEP). The aim of that program was to “attract and deliver more capable and effective foundation phase teachers, particularly teachers who

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\(^5\) Southern and Eastern Africa Consortium for monitoring Educational Quality - a consortium of education ministries, policy-makers and researchers who, in conjunction with the United Nations Educational, Scientific and Cultural Organization (UNESCO) International Institute for Educational Planning (IIEP), aim to improve the research capacity and technical skills of educational planners.
teach in indigenous languages" (http://www.dhetnews.co.za/eu-teacher-education-programme/) in order to improve the quality of teaching in the foundation phase and also to educate a new, expanded corps of young teachers. Some of the goals of the SFPTEP were research, programme development and material development projects at 20 universities (http://www.dhetnews.co.za/eu-teacher-education-programme/), aiming to improve both teacher education and, ultimately, early grades education. Although the average of learners who could do basic maths increased from 24% in 2011 to 34% in 2015 (Spaull, 2017), there is still a long road ahead in South Africa to equip teachers with knowledge and specifically pedagogical knowledge as related to mathematics.

In response to the aims of the SFPTEP program and because of a ‘mismatch’ between what the national curriculum aspires to and what is achieved in the majority of schools (Henning, 2013a), a research community in the Center for Education Practice Research (CEPR) at the University of Johannesburg adapted a German mathematics diagnostic test for use with South African children and published a number of articles on the development of young children’s mathematical concept development (e.g. Fritz, Ricken, Balzer, Willmes & Leutner, 2012; Fritz, Ehlert & Balzer, 2013; Henning, 2012a, 2012b, 2013a, 2013b, 2014; Henning & Ragpot, 2015). Currently, this center is also piloting an intervention program for teachers of the foundation phase that can assist them in improving their teaching of mathematics in Grade R and Grade 1.

My own motivation to conduct research in early childhood education (ECE, which is the age-group of birth to nine years) was driven by the belief that initial mathematical concepts and skills develop mostly hierarchically, along the theory of

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6 This study investigates the development of early numerical competence. Depending on the context of the paragraph or section of the thesis text, the development of numerical competence will also be described as mathematical competence development / number concept development / additive number concept development / additive number relations competence development. When I use the term ‘number concept development’, I refer to the construct of a child’s development. When I write about ‘cognitive skills’, I refer to children’s general cognitive competencies, including number concept development, language development, executive functions and logical reasoning. When referring to the test results, I refer to the achievement on the tests that measured the constructs of number concept development, mathematics-specific vocabulary, classroom engagement (manifestation of executive functions) and logical reasoning.
constructivism and the role of ‘necessary knowledge’ as postulated by Jean Piaget (Smith, 2017). That is, if children have not developed a deep understanding of the most basic number characteristics and the relationships between numbers, they will not be able to develop further mathematical understanding, which means that they will be at a continuous disadvantage.

In 2010, I started my teaching career as a Grade 1 teacher. I later taught Grade 2 and Grade 3 children. During my first few years as a foundation phase teacher, I realized that, at the time, there were no locally standardized mathematics tests to reliably assess what young children know when they enter school and in their early learning at school. I also realized that my own teacher training was insufficient. Although I majored in mathematics, my education did not include studies of mathematical cognition of young children. Therefore, I joined the CEPR to investigate young children’s learning and to possibly be part of a team that would develop assessment and teaching instruments.

In 2015, I completed a master’s degree, titled “The workability of the Afrikaans translation of a mathematics diagnostic test for the foundation phase” (de Villiers, 2015). My study was part of a research project that translated and adapted a German numeracy diagnostic test for the foundation phase (MARKO-D7 test) into four South African languages. This is an interview-based test, which was translated and adapted, informed by a specific theoretical framework (Fritz et al., 2012, 2013). This framework describes how young children form early number concepts and their additive relations. In my master’s dissertation I presented the argument that the same type of theory which informed the design of the test should inform foundation phase teachers’ pedagogy.

I now make the claim that a theoretical foundation is not only necessary in the design of a diagnostic test (Borsboom, Mellenbergh & van Heerden, 2004; Henning, 2013b; Herholdt, 2017), but such a theoretical body of knowledge should also ground teachers’ pedagogical content knowledge (PCK). PCK is the synthesis or integration of teachers’ pedagogical knowledge (what teachers know about how to teach) with their content knowledge (what teachers know about what they teach) (Cochran, 1997;
Shulman, 1986). Henning (2013b) argued that the content knowledge of a foundation phase teacher should include the psychology of the developing child. Foundation phase teaching, like all other teaching, “goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching” (Schulman, 1986:9). Knowledge of what children must learn is not sufficient. For children to develop numerical competence, teachers’ knowledge of mathematics (what they teach) must be coherent with how children develop and learn. For that, I claim that teachers’ PCK must be coherent, fluent and integrated. Hence, knowledge of how children learn and what the various constituents of their learning of early numeracy are, is important for teachers. In this regard, an argument in this thesis is that foundation phase teachers should be experts on how young children learn – including how they learn and remember information, use procedures and strategies and how they develop concepts. For example, if teachers do not know how to assess an individual’s competence, or how to ‘diagnose’ an individual child’s numeracy attainment, for example, they are unlikely to understand a child’s progress, or lack thereof (Mononen & Aunio, 2016).

In mathematics learning, factual (informational) knowledge is easily retrieved from memory and is crucial. Procedural knowledge, such as knowledge of algorithms or various methods to calculate and to use strategies for problem-solving, is equally important. If children want to implement a more advanced procedure, by decomposing the numbers, they need conceptual knowledge. They must understand the cardinality of each number, be able to decompose quantities and make use of their knowledge of relationships between numbers to use such strategies. For this purpose, conceptual knowledge is important. In this study, I argue that all three types of knowledge (factual-, procedural- and conceptual knowledge) are important during early number concept development and, therefore, teachers’ PCK should include all three components of number knowledge.

Dunne, Craig and Long (2012) explain how teachers adapt their teaching as their understanding of mathematical thinking and knowledge of the process of numerical development increases:

The assessment of mathematical proficiency is a complex task. The particular challenges inherent in this process depend on a number of factors, including
the definition of what constitutes mathematical proficiency, an understanding of how children learn and the approach adopted as to the purpose and function of teaching (Dunne et al., 2012:1).

In the second chapter I discuss several theories of numerical cognition and concept development, which I then integrate to propose that teachers should have enough of this type of theoretical knowledge for an enriched PCK of early math teaching.

1.2. Early grades foundations for mathematics learning

One of the admission requirements of the ‘teaching’ school (Appendix A) in Soweto, where this study was conducted, is that all children must attend Grade R (kindergarten) at this school, arguing that it makes specialized provision for optimal preparation for formal education. In a country with a serious early childhood development (ECD) societal problem, there has been an upsurge in attention to Grade R education. Van den Berg (2015), in a large scale-study of Grade R provision and quality, found that middle class children benefit most from the Grade R experience. Gustafsson (2010:12) found that only 61% of children in quintile 1\(^8\) attended preschool, compared to 92% of children in quintile 5. In Grade R children develop ‘pre-numeracy’ skills such as classification, differentiation, identifying patterns, seriation, comparison and one-to-one correspondence, all of which are prerequisites for numerical learning (Fritz, 2016). An effective Grade R curriculum also offers children the chance to develop crucial executive function skills, which support classroom engagement and all learning that requires higher cognitive functioning (Fitzpatrick, 2014; Levine & Baillargeon, 2016). In addition, children develop mathematics-specific vocabulary during Grade R and are likely to learn how to reason logically. In this research I argue that, because cognitive skills\(^9\) that contribute to numerical competence are presumed to be developed in Grade R, Grade R assessments and teachers’ PCK are crucial.

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\(^8\) Quintiles are ranked according to the total household income, with quintile 1 being the lowest income.

\(^9\) This study uses the term ‘cognitive skills’ to group four constructs, namely achievement in number concept development, mathematics-specific vocabulary, classroom engagement (manifesting executive functions) and logical reasoning.
1.2.1. Cognitive skills, mathematics-specific vocabulary and numeracy attainment in Grade R

For this thesis I was particularly interested in how assessment results of contributing cognitive skills of Grade R children, who are taught in their home language (HL), can predict numerical competence at the beginning of in Grade 1 (tested in their HL) and also in the language of instruction, namely English. In other words, I wanted to determine if the skills and concepts children present at the outset of Grade R could be associated with their performance on a number concept development test at the beginning of Grade 1. I was particularly interested to see how they would perform on the same test (the MARKO-D SA\textsuperscript{10}) in Grade R and in Grade 1 (the latter in both a ‘home language’ and English version of the test). This study thus examined possible concurrent associations between Grade R children’s 1) mathematics-specific vocabulary\textsuperscript{11} (MSV), 2) classroom engagement\textsuperscript{12} (CE), and 3) logical reasoning (LR) with their achievement on a Grade R number concept development (NCD) test; and possible predictive associations between their Grade R cognitive skills (NCD, MSV, CE and LR) with their achievement on a number concept development test in Grade 1 (tested in home language and English). I wanted to understand how measures of distinct Grade R cognitive skills could jointly predict achievement on a number concept development test in Grade 1. The children were from Sesotho and isiZulu speaking families. Figure 1.1 illustrates the possible concurrent and predictive associations that I wanted to investigate.

![Diagram illustrating possible concurrent and predictive associations](image-url)

**Figure 1.1: Possible concurrent and predictive associations**

\begin{itemize}
\item Mathematics-specific vocabulary
\item Classroom engagement (manifests EF)
\item Logical reasoning
\end{itemize}

\begin{itemize}
\item Number concept development achievement
\end{itemize}

\begin{itemize}
\item Grade R (assessed in HL)
\item Grade 1 (assessed in HL and English)
\end{itemize}

\textsuperscript{10} MARKO-D SA refers to the South African version of the German MARKO-D.
\textsuperscript{11} Also described as ‘number talk’.
\textsuperscript{12} A behavioral manifestation of executive functions (EF).
This study investigated several issues, the most important one being the utility of measures of distinct cognitive skills to identify which test outcomes predict successful number concept development in Grade 1. I also wanted to determine how assessments of these skills could arguably contribute to teachers’ PCK. With that, I viewed the Grade R education of the children as the naturalistic ‘intervention’ that 59 young children experienced. Numerical competence, mathematics-specific vocabulary, classroom engagement and logical reasoning of the Grade R cohort of 2017 were assessed at the beginning of the 2017 school year, with the test conducted in the children’s home language. A follow-up assessment of the 2017 Grade R learners’ numerical competence was conducted in the beginning of the 2018 school year when they were in Grade 1. In this round the children’s number concepts were assessed twice – in their home language as well as in an English version. This ‘double testing’ came about because I also wished to find out what the difference in test outcomes would be when the children were tested in the two linguistically different versions of the instrument. I assumed that the teachers had ‘code-switched’ between English and the children’s home language, because the school’s policy is to introduce English terminology in the Grade R year along with the home language vocabulary. With Henning, (2012a) I agree that urban children enter school in a ‘linguistic maze’. Hence, I wished to find out how the participants were finding their way in the ‘maze’, with regards to their learning of early number concepts. This ‘maze’ is not uncommon in South Africa.

In this country teachers use one of 11 official languages to teach foundation phase children. From Grade 4 children are only taught in English or Afrikaans. However, some children in the foundation phase attend a school with a language of instruction that is not their ‘home language’, or their ‘primary language’ (Herholdt, 2017). Many South African children also grow up in multilingual households and multilingual communities (Henning, 2016; Setati & Adler, 2000). Henning and Dampier (2012) referred to this transfer between languages as a ‘liminal space’ in which children have to find a way to make sense of different languages. Often children remain “betwixt and between languages of learning” (Henning & Dampier, 2012:100), due to their exposure to different languages at a young age.

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13 [https://www.education.gov.za/Portals/0/Documents/Reports/Status%20of%20LOLT.pdf](https://www.education.gov.za/Portals/0/Documents/Reports/Status%20of%20LOLT.pdf)
In many homes, especially in rural areas, a restricted linguistic code\(^\text{14}\) (a ‘named’ language with a specific community discourse) (Bernstein, 1971) is used. Some children (usually middle class) are exposed to a variety of ‘restricted codes,’ which, together, form an ‘elaborated linguistic code’ which enables them to switch between various language registers and speech genres (Bezuidenhout, Henning, Fitzpatrick & Ragpot, in press). In homes and schools with elaborated codes, children are likely to be more exposed to mathematics-specific language (Levine & Baillargeon, 2016). Because of the linguistic ‘maze’ (Henning, Gravett & Van Rensburg, 2005), many South African children have to face during early education, I expected, at the outset of the study, that mathematics-specific vocabulary in Grade R to be a possible predictor of South African Grade 1 children’s numerical competence - especially because this study’s participating children’s language of instruction changes from their home language to English in the beginning of Grade 1. I knew that the teachers in Grade R had started to prepare them for mathematics learning in English.

1.2.2. Early number concept development

Substantial research has been conducted to describe how young children’s number concepts develop (Dehaene, 2011; Feigenson, Dehaene & Spelke, 2004; Fritz et al., 2012, 2013; Gelman & Galistel, 1978; Piaget, 1952; Spelke & Kinzler, 2007; Wynn, 1990, 1992). Such research has indicated that mathematical concepts build hierarchically on each other to form future networks, and that early number concept development is a strong predictor for later achievement in mathematics (Chinn, 2015; Desoete, 2015; Geary, 1994). Children who struggle to develop early number concepts usually develop mathematical understanding more slowly than their peers (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). However, several researchers (e.g. Desoete, 2015; Dowker, 2008) have argued that difficulties in number concept development are not only due to difficulty in forming number concepts during early childhood, but that there are many contributing factors that influence number concept development and later performance on mathematical assessments (Chinn, 2015).

One could argue that the four cognitive skills that I have included in the design of this study (NCD, MSV, CE and LR) are necessary ‘input’ during Grade R, so that children can develop numerical competence in Grade 1. Figure 1.2 illustrates how the

\(^{14}\) See Section 2.4.4.
four cognitive skills that develop in Grade R possibly contribute to numerical competence in Grade 1. In referring to ‘input’ I include observable empirical input, while keeping in mind the mental processes of input as described by cognitive developmental psychologists, such as Susan Carey (Carey, 2009).

Figure 1.2: ‘Input’ for number concept development

1.2.2.1. Language as input and medium

Several researchers have investigated the role of language as an important ‘input’ in learning mathematics (Dowker & Nuerk, 2016; Levine & Baillargeon, 2016; Vygotsky, 1978/1993; Wagner, Tillman & Barner, 2016). One of the researchers that contributed to my understanding of the role of language in learning, is Lev Vygotsky (Kozulin, 1990; Vygotsky, 1978/1933, 1986/1992). Vygotsky proposed that children’s cognitive development takes place largely because they interact with their environment, which includes many symbols and signs (Vygotsky, 1978/1993, 1986/1992; Kozulin, 1990). A major set of symbols is language, which serves as a representative agent. They learn language and communicate about their experiences and observations, but also experience and observe because they understand language (Nazzi & Gopnik, 2001; Rogoff, 1990) and its codifications. Thus, they learn symbolically, because, apart from their own mental representations, they also utilise symbolic representations, such as
words and sentences and mathematical notation signs (Henning & Ragpot, 2015; Spelke, 2012; Gopnik & Meltzhoff, 1997). Language creates reality, but also represents reality symbolically (Bowerman & Levinson, 2001; Pavlenko, 2014).

Another researcher who influenced my understanding of children’s number concept development and the role of language, Susan Carey (2009), argues that language serves a ‘placeholding’ function that ultimately contributes to concept formation. As children represent their reality with greater ease through the placeholding of signs and symbols, they are increasingly able to construct new knowledge. They understand more concepts, because signs and symbols (together with tools such as models or materials) mediate much of children’s learning (Henning et al., 2005), in addition to perceptual experiences. Language is thus, arguably, an important mediator (scaffold) for learning. Children are (formally) introduced to number concepts by way of language (Spelke, 2003). Other researchers (Gentner & Goldin-Meadow, 2003) have argued that language mediates logical reasoning skills, when language serves as a tool or vehicle to reason about ideas and to enlarge children’s representations and understanding of their world, including their understanding of mathematical concepts.

Researchers have also set out to test the association between language and mathematic learning empirically, but research has, however, revealed contradicting findings. For example, Negen and Sarnecka (2012) found that assessments of general language use predicted number word knowledge, while Ansari, and his co-authors (Ansari et al., 2013) did not find such an association. Researchers such as Hecht, Torgesen, Wagner and Rashotte (2001) and Koponen, Aunola, Ahonen and Nurmi (2007) have connected phonological awareness to the development of young children’s mathematical ability, while Purpura and co-authors (Purpura, Hume, Sims & Lonigan, 2011) found that children’s oral language competence was the strongest predictor for mathematical ability in preschool children. Some researchers have focused on various aspects of language as predictor for various aspects in number concept development. For instance, Praet, Titeca, Ceulemans and Desoete (2013) found that expressive language was an important predictor for solving simple mathematical operations coupled with pictures, while language structure best predicted how well children solved calculations and problems not coupled with pictures.
In South Africa, researchers (Probyn, 2008; Setati, 2002, 2008) have investigated teachers and children’s opinions about how language plays a role in mathematics learning when children start with their home language as language of instruction and then switch to English as their language of instruction in Grade 4. Setati (2008) set out to describe the role of language in South African classrooms and found that: “Language-use in a multilingual educational context like South Africa is as much, if not more, a function of politics as it is of communication and thinking” (Setati, 2002:6). Setati also found that teachers and children in South Africa use linguistic code-switching as a tool of communication in multilingual South African classes – not only cognitively and pedagogically, but especially as a social product in a specific political context (Setati 2002, 2008). Probyn (2008) stressed the negative impact of using a language (English) which teachers and learners are unable to use to communicate freely. Yet, little research has been done in South Africa to examine the influence of mathematics specific vocabulary on numerical competence right at the beginning of a child’s formal school career.

1.2.2.2. Executive functions as input

Another set of cognitive skills that may contribute to number concept development is executive functions (EF), such as working memory (Cockcroft, 2015) inhibitory control, and cognitive flexibility (Morsanyi & Szücs, 2015:103). The literature on executive function research comprises discussions of how executive functions enable and support domain specific processes, such as mathematical cognition development and how the development of arithmetic skills depend on these domain general cognitive processes (Dowker, 2005; Hanich, Jordan, Kaplan & Dick, 2001; Xu, 2016; Zaitchik, Solomon, Tardiff & Bascandziec, 2016). Children with good attentional and verbal skills, including inhibitory control and cognitive flexibility and adequate working memory, generally show better arithmetic performance than those who have problems in these domains (e.g., Clark, Sheffield, Wiebe & Espy, 2013; De Smedt, Janssen, Bouwens, Verschaffel, Boets, & Ghesquiere, 2009).

Previous research has also indicated that there is a (behavioral) link between children’s executive functions and classroom engagement (Pagani, Fitzpatrick & Parent, 2012). Attention is seen to be a central component of inhibitory control and influences young children’s classroom engagement. Inhibitory control and other
executive functions, such as cognitive flexibility and working memory, seem to modulate children’s reactivity. It influences whether they take part in, or avoid an activity, how they control their behavior, cognitive activity and emotional- and social behaviour. Executive functions also allow children to self-regulate their behavior (Chang & Burns 2005; Dennis & Brotman, 2003). As young children improve their ability to control their behavior, they become adept at controlling their responses by inhibiting impulsive responses (Kochanska, Murray & Harlan, 2000). This allows children to engage in classroom activities. A more engaged child is thus, on this view, self-confident, self-controlled, cooperative and follows instructions easily (Pagani et al., 2012). They also seem to complete tasks more easily, which allows for better academic engagement. To my knowledge, no study has been conducted in South Africa to investigate possible relations between classroom engagement as behavioral manifestation of executive functions and numerical competence at the beginning of children’s school careers.

1.2.2.3. Logical reasoning as input

The literature also provides sufficient reason to believe that logical reasoning and mathematics competence are closely related (e.g. Ayalon & Even, 2008; Markovits & Lortie-Forgues, 2011). Both domains rely heavily on working memory and inhibitory processes and require the ability to draw conclusions, apply rules and process abstract and symbolic content (Morsanyi & Szücs, 2015). In this study I investigated such a possible relationship between logic and mathematical reasoning by testing Grade R and Grade 1 children’s reasoning skills, together with their numerical competence.

1.2.3. Coherence of input for number concept development

Following international trends in investigating associations between cognitive skills and number concept development, I set out to determine what the associations are between Grade R children’s achievement on a number concept development test, mathematics-specific vocabulary assessment, classroom engagement assessment and a logical reasoning test and their achievement in Grade 1 on a number concept development test. The children are from an urban South African ‘township’

15 A ‘township’ in South African social discourse refers to racially segregated living areas dating from the apartheid era.
concepts because of ‘linguistic liminality’ (Henning & Dampier, 2012). Although various factors, such as classroom engagement and logical reasoning, may contribute to number concept development, mathematics-specific vocabulary, especially, could enable or obstruct young children to develop numerical competence. I hypothesize that ‘number talk’, specifically, could be an important contributing factor to many children’s restricted exposure to linguistic codes of mathematics.

1.3. Research problem

Two concerns motivated this study, namely that 1) young South African children struggle in learning mathematics and that their inability to do mathematics only increases as they grow older; and 2) due to a lack of collaboration between research and practice, South African teachers’ knowledge of how children develop numerical competence is arguably insufficient. From my experience as both a teacher and a researcher, I believe that the nature and focus of educational research should be defined by the observed challenges in practice, in other words, educational research problems should be grounded in everyday educational problems. Educational research should address the gaps in our knowledge of the reality of teaching practice and how children learn head-on. I propose that not only should ‘gaps in the literature’ of educational science motivate research, but that it should also be driven by observed practice – even if such practice problems may not yet have been published as research. According to Snow and Donovan (2012) research in the field of education does not have ‘the luxury’ of starting from theory only, but research should reflect practitioners’ definitions of urgent issues. This is only possible if teaching practitioners and researchers work closely together in a program where research and practice guide and support each other in a coherent and interactive, reciprocal way. Although the literature on early learning of mathematics is extensive, there is still a void in the South African literature of what children in Grade 1 start with and how Grade R prepares them to develop numerical competence.

Snow and Donovan (2012) report that during a search for educational research that had influenced practice, they found some examples, but only a few had lasted for long, due to the short life of partnerships between the academy and everyday practice. Snow (2015) proposes a version of an educational research program in which
education research is embedded in practice and is on-going. She argues for the establishment of structured, and sustained research-practice partnerships:

The traditional relationship between researcher, the producer of knowledge, and the practitioner, the user of knowledge, was replaced by a commitment to the notion of two sources of knowledge (research and practice). Though the two sources might generate somewhat different types of knowledge, both types are judged to be of equal value and importance to improving educational outcomes. A corollary of the partnership model is that researchers need to acknowledge the realities of practice and practitioners need to acknowledge the commitment to rigor in research (Snow, 2015:461).

Such a research approach is in contrast with the typical organization of research in education, where a research question is formulated by scholars after which they should convince a school, or a district, or some other organization or individuals to allow them to enter the sites in order to do research. As an experienced teacher I know the pressure to complete the curriculum and that it sometimes feels as if research projects consume teaching time and add to the pressure of ‘getting through the curriculum’. The approach for which I argue, is one where research does not ‘add to the workload’ of a teacher, but aims to provide teachers with tools to teach and assess the concepts and skills described in the curriculum, with the researchers as partners in the shared enterprise. In this study I make the claim that researchers should, first and foremost, address and utilize the concerns of practicing teachers to inform the objectives of their research. When investigating number concept development, such as I did, as a researcher I had to understand how mathematics-specific vocabulary, classroom engagement and logical reasoning skills relate to the development of numerical skills and conceptual understanding of magnitude of young children. I could not do that from a distance, but had to settle in the school as a part-time ‘resident’, along with the assistants who helped me. And while I was conducting the research I had to remain mindful of the needs of the teachers to learn something from the endeavor.

1.4. Design of the study

The argument for the design of this study is twofold: Firstly, both empirical research and a suitable literature review should inform the development of materials, such as
numerical diagnostic tests and research-based intervention programs. Secondly, I acknowledge that teachers and other practitioners invent innovative ideas and excellent materials all the time. Therefore, when investigating the relationships/associations between numerical competence, mathematics-specific vocabulary, classroom engagement and logical reasoning in Grade R, coupled with numerical competence in Grade 1, a literature review, empirical research and contributions from practitioners should add to the findings of the study.

I designed a study, naming it an ‘ex post facto’ inquiry, regarding the Grade R year of education as the naturalistic intervention. I utilized various instruments, the cardinal one being the MARKO-D SA test (Henning, Balzer, Ehlert, Herholdt, Ragpot & Fritz-Stratmann, 2018), for which permission was granted by the publisher to utilize the test prior to publication. The other instruments were a custom designed mathematics vocabulary test, a classroom engagement report (as teacher reported manifestation of children’s executive functions), and a logical reasoning test that was standardized in Germany. I analyzed data from the instruments quantitatively and conducted interviews with the teachers, the data of which I analyzed qualitatively.

1.5. Research question and objectives

This study’s hypothesis is that there will be notable associations between the independent variables (achievement on NCD, MSV, CE and LR in Grade R) and the dependent variable (NCD achievement in Grade 1) and that, apart from achievement on a number concept development test in Grade R, mathematics-specific vocabulary will be the greatest predictor of performance on the Grade 1 number concept test. The reason for this hypothesis is that the language policy at Funda UJabule determines that children learn in their home language in Grade R (while slowly starting to code switch between English and isiZulu/Sesotho), but in English in Grade 1 (while still code switching between English and their home language).

Some have argued for single measure assessments, such as the MARKO-D (Fritz et al., 2013). However, poor number concept development is not the only factor influencing number cognition in early childhood. Research suggests that mathematical difficulty may have multiple origins (Desoete, 2015; Dowker, 2008; Purpura, Day,

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16 In this study, I chose to use the US (American English) spelling of the terms ‘analyze’ and ‘input analyzer’ purposefully, due to its international currency. Therefore, I use American English throughout the study.
Napoli & Hart, 2017). A number of researchers report that poor executive functions (Allan, Hume, Allan, Farrington & Lonigan, 2014), a lack of exposure to number talk\textsuperscript{17} (Levine & Baillargeon, 2016), poor core number knowledge (Reeve & Grey, 2015) and a lack of logical reasoning skills (Morsanyi & Szüs, 2015) are contributing reasons for poor number concept development.

The unit of analysis of the study is the association between isiZulu and Sesotho speaking Grade R children’s number concept development achievement, mathematics-specific vocabulary, classroom engagement and logical reasoning; and their achievement on a number concept development test in Grade 1. I use the term, ‘unit of analysis’ to emphasize the construct that I wished to examine (Trochim, 2006). The research question is: What are the concurrent and predictive associations between children’s achievement on number concept development tests (in Grade R and Grade 1) and their Grade R mathematics-specific vocabulary, classroom engagement and logical reasoning? Secondary questions are: “Does language of teaching play a role in children’s number concept development”, and “What are current Grade R and Grade 1 teachers’ perception on the importance of cognitive input on number concept development?”

The objectives are to:

1. Distil relevant literature that describes young children’s additive number concept development.
2. Develop a study design that is best suited to examine associations between distinct cognitive skills in Grade R and number concept development achievement Grade 1.
3. Capture children’s performance on the MARKO-D, MMLT\textsuperscript{18}, CFT\textsuperscript{19} and classroom engagement report\textsuperscript{20}.
4. Identify a relational pattern between the results of the various instruments measuring number concept development achievement, mathematics-specific

\textsuperscript{17} Number talk refers to mathematics-specific vocabulary.
\textsuperscript{18} The Meerkat Math Language Test is an author developed test which assesses mathematics-specific vocabulary.
\textsuperscript{19} The Culture Fair Test is a language and culture fair test which assesses logical reasoning.
\textsuperscript{20} The classroom engagement report is completed by teachers and indicates to what extent children engage in classroom activities.
vocabulary, classroom engagement and logical reasoning in Grade R and number concept development achievement in Grade 1.

1.6. Chapters of the thesis

Chapter 2: Input for number concept development

Chapter 2 comprises of an overview of literature that foregrounds the field of study. The authors I chose to cite in this thesis are experts who publish mostly in English about children’s mathematical learning from a range of perspectives. I searched common databases including EbscoHost, Scholar Google, TCR (Teachers College Record), ERIC (Education Resources Information Centre), Education Research Complete, Education Database (formerly named ProQuest Education Journals) and Education Source, using the following search terms: cognitive input, input analyzers, input for learning, cognitive development, conceptual change, conceptual development, number concept development, core knowledge, assessment, urban South African classrooms, executive function, inductive reasoning, logical reasoning, language and mathematics and mathematics vocabulary.

Chapter 3: Research design

In Chapter 3 I sketch the design of the study and argue for the use of specific instruments to assess four cognitive skills that, arguably, contribute to numerical competence. The study was designed to address the research question optimally. While chapter 1 sets out to describe why it is worth learning about the interactions between specific cognitive skills (Punch, 2005:10), Chapter 3 presents the ‘plan’ I devised to indicate what I planned to do, how I intended to proceed with data collection, why I opted for specific strategies (Kumar, 2005:188) and test instruments and indicates how I planned to analyze the data.

Because I decided to include practicing teachers’ views, as well as an analysis of quantitative data, I employed an integrated model of quantitative- and qualitative methods, in what has become known as ‘mixed methods’ (Onwuegbuzie & Teddlie, 2003; Tashakkori & Teddlie, 2010). In this integrated qualitative-quantitative approach I considered what Maree (2009) suggests; when quantitative- and qualitative data are collected simultaneously, a triangulation design could be used to understand the association between the independent- and dependent variables.
Chapter 4: Analysis and interpretation of the data

Chapter 4 comprises the narrative of the analysis and a summary of the results. The chapter presents the research events chronologically so that the reader can follow the “pattern of actions taken by the researcher” (Henning et al., 2005:99). I aim to provide a chapter which is “rich in its description and exposition” (Henning et al., 2005:99). In Chapter 4 I indicate how the ‘plan’ presented in Chapter 3 was executed. I show how the data of the various instruments was captured and analyzed.

Chapter 5: Discussion and conclusion

In Chapter 5 the findings of the study are discussed and conclusions are drawn about the study’s contribution to the field of early mathematics learning and about the utility value of the instruments that were used. This chapter also includes a discussion of the contribution of the study for practice, policy and research. It highlights the methodological contribution of the study. Limitations and future directions are mentioned and thereafter I conclude that knowledge derived from the findings could be useful for teachers’ PCK.

1.7. Summary

This chapter introduced the study, emphasising the need for an understanding of the number concept development level with which township children and other historically disadvantaged young learners enter school. It also showed that Grade R may contribute to children’s readiness to learn mathematics in Grade 1.
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CHAPTER 2:
NUMBER CONCEPTS IN EARLY CHILDHOOD:
DEVELOPMENT AND LEARNING

2.1. Introduction

In this chapter I discuss some of the theories that have guided the research. These range from theories of conceptual development in the broad field of mathematical cognition, to theories with a cognitive neuroscience approach and others that are from the field of developmental cognitive psychology, sociocultural theory of concept development and cognitive skills related to executive functions. In this ‘melange’, I criss-cross between theorists, making an argument for intersects that will, ultimately, serve a pedagogical purpose.

I argue for an integrated perspective of early number development, invoking the research of authors from a range of fields, such as Carey (2009), Dehaene (2011), Feigenson et al. (2004), Spelke and Kinzler (2007), who, among other ideas, agree that children are born with various ‘core concepts’, or what some refer to as ‘core knowledge systems’. One of these is core number knowledge. Together with this type of innate number sense, there are several factors that amalgamate in the forming of a child’s early number cognition. I focus on four of these, namely 1) learning of numeracy beyond innate number knowledge, 2) mathematics-specific vocabulary, 3) classroom engagement as manifestation of executive functions, and 4) logical reasoning. I propose that these serve as some of the ‘input’ contributions to which I refer in the title of the thesis.

The chapter’s structure broadly follows the path of an argument, the ‘outcome’ of which is that number concept development is reliant on some innate knowledge and mechanisms, while developing by way of language and interaction – thus by tuition and observation – as a child learns to learn (Figure 2.1). The chapter begins with a discussion of general conceptual development, followed by a brief rendering of the notion of ‘core’ cognition (of number) and Susan Carey’s work on the construct she refers to as ‘input analyzers’ (Carey, 2009). Thereafter comes a discussion of the role of language in conceptual development and a current model of early number concept development. Following on that section is an explanation of executive functions and
logical reasoning, with the former as manifested in classroom engagement. Then comes a discussion of an integrated pedagogical approach for early grade teachers. The chapter is rounded off by a synthesis of the main points made in the chapter, proposing that there are some ‘connections’ between these four components of children’s cognition.

Figure 2.1: Structure of the chapter

2.2. Conceptual development of number

A rich tapestry of previous research has shown that number concept development is the result of an interaction between various forms of input and that these input data are ‘analyzed’ by innate cognitive mechanisms which allow humans to learn and develop conceptual structures (See Section 2.2). In this section I aim to describe how humans – and only humans – develop advanced number systems that allow us to do mathematics.
2.2.1. Core knowledge: The initial state of conceptual systems

Over many years, theorists have researched different kinds of mental representations with various origins, development trajectories, and ‘conceptual roles’ that specific constituents of cognitive functioning take up. Theorists such as Quine (1960) and Piaget (1954) assumed that one can distinguish between sensory and/or perceptual representations and conceptual representations which serve as input for development. Both theorists held that primary sensory and perceptual input are transformed into conceptual representations due to a uniquely human mental capacity. Piaget (1954) explained that infants have innate sensory-motor reflexes and that all mental life is constructed from an initial set of sensory representations. Quine (1960), on the other hand, held that infants’ representational resources are comprised of innate pre-linguistic perceptual ‘vocabulary’, which develops into conceptual representations at the time when children acquire natural language.

With imaging technology, it has now been shown that the newborn brain consists of several cognitive ‘ready’ states that will respond to the environment as the infant develops, much like young chicks who learn to pick up on visual cues, because of their innate capability to recognize ‘conspecifics’, like their mother and also others in the fowl family and their species (Carey, 2009:16-17). Much like chicks, young humans have inborn capabilities to live in the world, some of which are domain-specific and others that are ‘central’, or general (Carey, 2009; Spelke, 2000). Basic number knowledge is such a domain specific capability that allows humans to represent objects and their magnitude, among other things. Such a representation is part of a domain-specific learning device and is, in this sense, like innate representations that human infants have of conspecifics. Human infants, like baby chicks, can identify conspecifics, based on appearances and movements (Carey, 2009:17). In the same way, humans also have sensory input analyzers, which allow us to accept only “some class of stimuli”, such as sensory representations, which come straight from the five human senses (Carey, 2009:27). Through sensory (and thus perceptual) input, children can identify conspecifics which let them categorize objects, based on appearances and movement – especially during the first year of a child’s life.

At the age of 10 to 12 months children begin to classify objects, based on individual properties of objects. They are now able to identify kinds of objects and group
objects according to certain properties. There is evidence that language plays a role in children’s ability to classify and identify conspecifics (Carey, 2009), because objects have by now been named. Not only is the input now also auditive, it is now symbolic (Henning & Ragpot, 2015) and rapidly children start using this audition-based ability to communicate – also to engage in ‘number talk’ (Levine & Baillargeon, 2016).

Even though Carey (2009) and Dehaene (2011) subscribe to the constructivist principles of Piaget’s work in general, they do not accept the theory that young children’s mental representations are limited to perceptual- and sensory input. They agree that children’s conceptual repertoire, rather, includes representations that articulate core cognition21, including central systems which support language learning and core number knowledge (Carey, 2009:448). Although the evidence children have for the world they live in comes from their senses, it does not mean that all knowledge must be formulated in terms of sensory or perceptual primitives. Carey argues, throughout her groundbreaking work (2009), that core representations go beyond sensory and perceptual content. diSessa (diSessa & Sherin, 1998) refers to such very basic components of cognition as p-prims, or ‘phenomenological primitives’.

Core concepts (forming core knowledge) differ in structure from other concepts in the sense that they are not acquired through observation or children’s personal intuitive theories (which are based on experience and learning). Intuitive theories22 arise before and while children build normative theories by learning. Core knowledge, on the other hand, has been woven into humans by evolution. They are not the outcome of a process of learning and development, but are present at birth (Carey, 2009). Carey has researched various core knowledges (or core cognition), while Dehaene (2009, 2011) has specialized in the core cognition of number sense and of literacy.

Deheane (2011) describes core knowledge as a sense, or intuition, for numbers and argues that we owe this intuition to an inherited capacity of perception.

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21 Elizabeth Spelke (Spelke & Kinzler, 2007) referred to ‘core knowledge’. Carey (2009) uses the term ‘core cognition’, while Feigenson et al. (2004) referred to ‘core systems of number’. Although different terminology is used, there is a general consensus that the two systems form the core/foundation for number concept development.

22Intuitive theories comprise an ontology of concepts; it is a system of laws that govern the child’s ideas about how the world works and how different concepts interact.
of approximate number of objects. Because he works in the modality of neuroscience, he can argue from evidence, that all concepts – including core concepts – are the result of specialized neuronal circuits in children’s cerebral cortex. To provide insight into humans’ innate conceptual number system, Dehaene (2011) and Carey (2009) argue that one should understand and describe animals’ sense for ‘numerosity’, which is in some ways, very much like humans’ initial set of number concepts.

2.2.2. Preverbal core number concepts

Approximate number system

Dehaene (2011) argues that by studying some animals’ representations of ‘numerosity’ or ‘numerousness’ (Dehaene, 2011:23), we can come to understand infants’ limited core representations of number. There have been claims that some animals can compute\(^{23}\), but these claims have been proven to be false, because only humans develop the ability to accurately represent larger quantities and they learn to do that with the help of language. However, the representations of animals may be similar to that of preverbal children and, therefore, Dehaene (2011) uses a metaphor of an accumulator to explain animals’ and preverbal children’s ability to compare some quantities.

An accumulator can be compared to a shepherd with no verbal mathematical vocabulary, who wants to ‘count’ his animals without using language or numbers. The shepherd could possibly use one stone or stick to represent each animal that passes him with each stick or stone representing an animal. In this way he could keep a certain number of stones or sticks in his pocket and use the same set every day to determine if all his animals are still in the flock. Alternatively, the shepherd could pour an approximate volume of water into a bucket each time an animal passes him by to determine how many animals have passed. If the water comes to a certain level, the shepherd will know that all the animals have entered the barn. If the water does not reach the expected level, the shepherd will know that all the animals are not present. If the water rises to a level above the expected level, the shepherd will know that some other animals have joined his flock which might not be his.

\(^{23}\)For example, see Dehaene [2011:4-5] for a discussion of Wilhelm von Osten’s horse that he claimed could compute.
Dehaene (2011:23) uses this metaphor to explain how it is possible that animals or young children (like the shepherd) can ‘count’ without language. He draws two conclusions from this accumulator theory:

First, animals can count, since they are able to increase an internal counter each time an external event occurs. Second, they do not count exactly as we do. The representation of numbers, contrary to ours, is a fuzzy one... The accumulator enables animals to estimate how numerous some events are, but does not allow them to compute their exact number. The animal mind can retain only fuzzy numbers (Dehaene, 2011:23).

To investigate the so-called animal accumulating ‘counting’ ability, Rumbaugh and his colleagues (Rumbach, Savage-Rumbach & Hegel, 1987) did an experiment with chimpanzees, where they showed the chimp two trays with choc chips. On the first tray they placed two piles of choc chips, one pile containing four pieces and the other pile three. On the second tray, they also placed two piles of chocolate: One pile contained five choc chips and the other pile only one. They gave the animals enough time to look at these two trays before choosing one tray and eat the content. Most of the time, the chimpanzees selected the tray with more choc chips, which lead Rumbbaugh and his colleagues to conclude that these chimpanzees have a (limited) ability to compare two sets of objects. The chimps’ performance was not without error, but the nature of these errors provided us with essential information about the animals’ representation of quantities (Dehaene, 2011:16). The cases in which the animals chose the smaller set, allowed the researchers to conclude that the accuracy of an approximate observation is influenced by two factors, namely the magnitude of the numbers/quantity and the distance between two numbers (in their sequence on the ‘mental number line’). These two factors can be summarized in terms of Weber’s law (Weber, 1834) which states: The accuracy of approximation of numbers decreases as the value of numbers increases in proportion to how close the two numbers are. The larger a number, the less accurate an animal’s (or person’s) representation of this number will be. The smaller the number, the more accurate the representation will be.

To illustrate this, one can test oneself by first comparing a set of 25 objects with a set of 28 and then comparing a set of five objects to a set of eight. It is easier to compare a set of five objects with eight objects and determine that eight is three more
than five. This phenomenon is known as the magnitude effect: the larger the numbers, the harder it is to determine the distance between two numbers (without the help of symbolic mediation). There is thus a relationship between the magnitude of numbers, the distance between two numbers and the ability to compare two sets. The second factor that affects one’s ability to estimate precisely, is known as the distance effect, which states that the distance between two numbers will affect the accuracy of the observation of the difference between the numbers. For example, it is easier to compare 8 and 16 objects than to compare 15 and 16 objects.

Many researchers (e.g. Dehaene, 2011; Feigenson et al., 2004; Spelke & Kinzler, 2007) have shown that children are able to represent the first three numbers of the counting list with accuracy and without symbolic mediation, but representations of larger numbers become ‘fuzzy’ and ‘noisy’. A simple experiment, like asking children (or adults) to represent numbers on a straight line provides us with a ‘picture’ of the phenomenon that smaller numbers are represented more accurately than larger numbers. Both children and adults tend to distribute smaller numbers further apart and larger numbers closer together (see Figure 2.2). Because of the ‘fuzziness’ of larger numbers, larger numbers seem more difficult to distinguish. Figure 2.2 is an example of how people tend to distribute numbers on a line.

![Figure 2.2: Fuzzy representations of increasing numbers (adapted from Fritz et al., 2013:45)](image)

Most adults are familiar with such a representation of numbers on a line. However, the graphic ‘number line’ is a cultural development, which is part of the symbolic number system that humans have developed to allow us to do exact calculation and arithmetic (Feigenson et al., 2004). Children do not naturally represent quantities on a straight line but are taught to do so. Young children who have not been exposed to the cultural development of a number line may represent numbers in different ways. Figure 2.3 is an example of a possible alternative to a straight-line, yet still linear, representation of numbers.
In both these figures, the distances between the representations of smaller numbers are rather large. As numbers increase in size, the distance between the representations of numbers becomes smaller. Some may have an idea about the position of ‘tens’ or even ‘hundreds’ when imagining some sort of representation of the magnitude of numbers. Thereafter, a few children may have a vague idea of the hundreds up to a thousand. In Figure 2.3 the representation of larger numbers spirals away in a never-ending list of increasing numbers. Such a representation could be aligned with Weber’s law, because the more the ratio between numbers increases, the more easily the difference between the numbers will be perceived.

At birth and prior to formal instruction, children’s approximate representation of quantities is not limited to visual arrays like representation of numbers on a line (Feigenson et al., 2004), but also include approximate representations of duration, volume, area, brightness, loudness and other dimensions (Halberda, 2016). Instead of a representation of numbers on a line (straight or spiraling), representations of number may rather be guided by the instinct to perceive objects in space. The accumulator metaphor allows us to understand that space, number and time are
connected constructs. As the water fills more space, the shepherd knows that the number of sheep is increasing. The longer it takes for the sheep to pass the shepherd, the larger the number of sheep.

Dehaene and Brannon (2011) explain the relationship between time, space and number:

...they provide “a priori intuitions” that precede and structure how we experience our environment. Indeed, these concepts are so basic to our understanding of the external world that we find it hard to imagine how any animal species could survive without possessing mechanisms for spatial navigation, temporal orienting (e.g., time-stamped memories), and elementary numerical computations (e.g., choosing the food patch with the largest expected return) (Dehaene & Brannon, 2011:iix).

All three domains – time, space and number – encode and compute quantities. Dehaene and Brannon (2011) even claim that time, space and number are so closely related that there are mechanisms (grid neurons) that are “capable of generating a unique, partially random collective code for any quantity” (Dehaene & Brannon, 2011:x). Placing numbers on a spatial ‘grid’ allows us to perceive numbers spatially. Such ‘spatial’ representations are not exact representations with even distances between numbers, but are imprecise and approximate.

To investigate the relationship between space and number, Dehaene (1993) did an experiment in which participants were instructed to hold a response key in each hand while they were shown different numbers. Some children were instructed to use the left response key when a number larger than 65 was shown and the right key when a number smaller than 65 was shown. The results showed an automatic association between numbers and space: The larger the number, the faster the right-hand responses were and vice versa.

The object tracking system

In the literature (Dehaene, 2011; Feigenson et al., 2004; Spelke & Kinzler, 2007) there is consensus that two neurocognitive systems underlie infants’ innate intuition for numerosity as described in the previous section. The one system is a system that allows us to track the numbers of small sets (one, two or three) without using language
to count. This system is known as the object tracking system (OTS). The other system is the approximate magnitude system (AMS24), which was discussed in the previous section.

To prove that children can track small objects without using language to count (using their OTS), Wynn (1990, 1992) did an experiment with 32 five-month-old infants. The infants were randomly assigned to two groups (‘1+1’ and ‘2-1’). The first group of children was presented with a single doll. By using a small screen, the object was then hidden from view. An experimenter brought a second identical doll into the infant’s view, and then placed it behind the screen (out of the infant’s sight). In the second group, the infants were presented with two dolls, which were then hidden from view by a screen. The experimenter would then remove one of the objects from the sight of the infant. In both cases, the goal was to have the infants witness either an addition- or subtraction operation without seeing the operation being performed. In both cases, after the sequences were complete, the screens were removed to reveal either one or two objects. This process was repeated six times for each infant, alternating between one-item and two-item final displays (in other words, altering the display of items between correct and incorrect number of objects). The amount of time was measured that the infants remained visually fixated on an object – while remaining attentive to the display. As part of the same study, infants were presented with cases of 1+1 = 2 or 1+1=1.

Wynn found that infants in the first group looked for a longer amount of time when one item was shown as a result (when the math implied that 1+1=1) than when two items were shown (1+1=2). Infants in the second group did the opposite, looking longer at the display with two items (2-1=2) than the display with one item (2-1=1). The same results emerged in the experiment where 1+1=2 vs 3: Infants looked longer at the impossible outcome which suggests that they were surprised by the incorrect outcome (Wynn, 1992). This experiment suggests that young children are indeed able to keep track of small numbers of objects and that they are surprised at a mathematically impossible outcome (Feigenson, 2016). Even though Wynn (1992) concluded that babies are able to distinguish between one, two and three objects and

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24 Some (e.g. Halberda, Mazzocco & Feigenson, 2008) refer to the AMS as the Approximate Number System (ANS). I prefer ‘magnitude’ to ‘number’ because young children’s representations are that of magnitude rather than numbers or numerosities.
the relationship between these three numbers, it is important to note that this ability is limited to the first three numbers. When children are still very young they do not have the ability to represent exact quantities of numbers exceeding three. Thus, it is unlikely that they will be able to do exact calculations with larger numbers than four (Dehaene, 2009:45; Feigenson et al., 2004).

Dehaene (2011:51) suggests that one can compare infants’ representations of the first three numbers to adult knowledge of red, yellow and blue. One recognizes the three different colours (or first three numbers) and knows how to combine them to form new colours like orange, green and purple (or know that one and two makes three). Similarly, children may recognize one, two and three and even know how they combine without realizing that one is larger than two or two is smaller than three. Their knowledge of combining the three numbers may not necessarily extend to knowledge of the quantity represented by numbers.

Carey (2009) describes the distinction between the numbers one, two and three as the use of the OTS and argues that it is not a sense of numerosity. She brings to our attention that, historically, the first three or four numbers have also been represented as exact numerosities in various cultural groups. The figure below shows how different cultures represented numbers in different times in history.

![Figure 2.4: Ancient cultures’ symbolic representation (redrawn from Dehaene, 2011:54)](image)

From this figure it is evident that different cultures used comparable visual representations for only the first three or four numbers. There is a correspondence between the quantities of markings and the number of objects that a specific symbol represents. Numbers larger than three or four are represented by symbols that do not show this one-to-one correspondence. The similarities in the development of numbers one, two and three across different cultures provide insight into how young children
learn the first few numbers: By using a visual object tracking device which enables us to see individual objects by subitizing (this construct is discussed in the following paragraph). The OTS allows even young children to determine the differences between one, two and three.

One cannot visually track large numbers of objects to distinguish between larger sets (without making use of counting words). It is not by chance that people in the Cuneiform, Roman and Mayan cultures (and many others, such as the Etruscan, Chinese and ancient Indian cultures) use this kind of notation. The first three numbers adhere to the principle of one-to-one correspondence because of a principle of ‘subitation’. The word, subitize, is derived from the Latin word, subitus, which means, ‘sudden’. According to Reeve and Gray (2015), to subitize means one is able to ‘suddenly’ label small numerosities without counting.

Dehaene proposes that because we cannot track sets of objects larger than three, all humans are equipped with a number processor (or number input analyzer) which allows approximate arithmetic (Dehaene, 2011).

2.2.3. Conceptual development beyond core cognition
An important question in developmental psychology is whether change from one conceptual system to another is continuous (a system, such as number knowledge, is understood to grow progressively in complexity) or discontinuous (also known as stage-like or qualitative change, meaning that one understanding is replaced by another). Carey (2009) argues for some continuity, while Piaget and Inhelder (1941) described different levels of development according to which one system is clearly distinguished from one developmental point to the next; in other words, conceptual change phases have definitive points of change.

Piaget (1952, 1970) is widely regarded as the ‘progenitor’ of conceptual change theory. Although he did not have access to the tools of neuroscience, he developed methods to inquire into children’s logical thinking and concept formation in innovative modalities. diSessa argues that the seeds of conceptual change theory were sown by Piaget’s work (diSessa, 2014). Like other leading contemporary theorists in this field, he studied children’s concept development and theorised widely in a huge oeuvre (http://www.piaget.org/). He did not investigate the constituents of concepts in the
‘grainsize’ that researchers such as Carey (2009), or diSessa (diSessa & Sherin, 1998; diSessa, 2014) do. Carey proposes that concepts are the smallest units of knowledge, while diSessa proposes the existence of smaller units, described as knowledge-in-pieces (KiP), consisting of phenomenological primitives (p-prims) that respond to natural cues, such as gravity and force (diSessa & Sherin, 1998). Piaget acknowledges this in his last interview (http://www.fondationjeanpiaget.ch/fjp/site/textes/VE/JP80Voyat interview.pdf).

2.2.3.1. Copies of the world of number

What Piaget did recognize, however, is that a child’s experienced world is ‘copied’ in the mind of the developing child. These ‘copies’ comprise a version of reality with which children make sense of their perception of reality. Children create copies of experiences, events, ideas, properties of items and even sounds (Barner & Baron, 2016) that have numerical content. These copies are conceptual representations of reality – they are the best way we can ‘store’ our observations of real life (Barner & Baron, 2016). (Number) concepts are thus individual representations of units of reality (Carey, 2009). However, these units of ‘number thoughts’ do not exist separate from other representations of reality; Carey (ibid) argues that various concepts are interconnected and form a network, or a conceptual system, which involve relations among different kinds of constituents. So, number knowledge is never only about a numerosity without connections or relations to other numerosities and a host of other concepts.

Added to this, humans are the only species that use words for mental representations of reality, such as numerosity (Carey, 2009:247). This is a crucial component of learning number and mathematics in general. We name objects, actions, sounds, events and even abstract concepts such as ideas, aspirations or ability and many mathematical concepts that are forever abstract and are represented by symbols only. Some of these are linguistic symbols. Carey (2009:5) proposes that “representations of word meanings are paradigm examples of concepts.” She emphasizes that concepts are more than sensory representations in a meta-representation, although such representations may form part of input analyzers for concepts. This is especially the case when representations are expressed in language sentences. We create syntax and string words together in sentences to be able to
communicate with meaning (Chomsky, 1965), which may be part perceptually based. In becoming knowledgeable about magnitude and its numerosities, is one such phenomenon of copying the world, with the help of language, of learning, of executive functions and reasoning. Before learning from the world, human newborns already have some numerical knowledge at their disposal, but they need significant conceptual adaptation or change as they grow and develop.

2.2.3.2. Conceptual change theory: Fostering learning

Carey (2009) argues that a theory of conceptual change should include a discussion of the processes that underlie conceptual change. She proposes that there are three processes that underlie conceptual change, namely 1) evolution, 2) cultural-historical development and 3) individual learning.

Firstly, humans have evolution to thank for the two core number systems and our input analyzers that create, for example, representations of magnitude and approximate numbers. Innate concepts, are ‘non-learned’ concepts (Carey, 2009:453) and humans alone transcend core cognition by mastering culturally constructed concepts (Gopnik, 2001; Gopnik & Metzhoff, 1997), such as those in mathematics.

Secondly, cultural-historical development allows for language and other representational resources to develop. Each generation conveys these cultural artefacts, such as language, to the next generation and new cultural processes develop. “Learning processes in a cultural-historical context create new representational resources that in turn, are expressed and maintained in language and in the cultural products they make possible” (Carey, 2009:447-448). Human language, on this view, serves as a mediator and ‘maker’ for mental representations, which will ultimately be part of the input analyzer that processes ‘incoming’ language - and also ‘incoming’ thought. It thus serves as a vehicle of new thoughts and as a tool to develop new concepts. The various (conceptual) roles of language are discussed in Section 2.4.

The third process, individual learning, explains differences in a human’s ability, skills and knowledge. The amount and quality of input for development differs from child to child and therefore we observe differences in, for instance, number concept development (Gopnik & Meltzoff, 1997:192). All three learning processes support
conceptual development and foster conceptual change. Here, I briefly explain the process of conceptual development that these three processes underlie.

The development of concepts has core knowledge as foundation (OTS and AMS for number concept development). Because of an innate ability to learn language (Chomsky, 1965), children can transcend core knowledge and learn how to link the linguistic symbols (verbal and written symbols) to the entities in the world—for instance, how to link numerals to the quantities they express (Dehaene, 2011). Children connect the number words they hear to their preverbal core conceptualization of number. With language, children can formulate their understanding of number in terms of their available conceptual and linguistic repertoire. Now, when children learn new words that represent new concepts (at different ages), such as ‘more’, ‘less’, ‘after’, ‘add’, ‘half’, ‘double’, ‘fraction’ or ‘tenth’, a process of conceptual development takes place.

When conceptual systems change or develop, the starting point is always the existing conceptual system, or the ‘necessary knowledge’ that Piaget posited (Smith, 2017). For example, when children learn a new word and the word itself serves as a placeholder for true conceptual understanding of, for instance the word ‘more’, some cognitive change takes place. At first, the child will not know what this word means, but will have a temporary placeholder-concept in a linguistic format. Slowly, through an inductive process, children’s innate input analyzers foster sense-making of the newly learnt word. This happens when children begin to understand the conceptual properties of the placeholder word (Carey, 1988, 2009), which has by now ‘morphed’ into a concept. Only once the input analyzer has allowed children to connect the placeholder word with a true understanding of the newly learnt concept, a more sophisticated and elaborated network or system of concepts emerges.

Dehaene (2011) argues that one of the most crucial moments in number concept development is when children at around the age of two begin to distinguish number words from other adjectives or nouns, verbs, pronouns, or conjunctions. Children realize that number words belong to a specific category specifically used to refer to quantities. During this period of change, children may know the count list for almost an entire year before finding out what the meanings of the words are. Only

25 The image I use to portray this visually is of a filtering process in something like a sieve through which ‘raw’ input goes to ‘become’ cognitive and find a space in a conceptual system.
when they are able to ‘give one object’ or ‘give three objects’ do we know that they understand that the word ‘one’ refers to a single object and the word ‘two’ refers to two objects and so on. When they are able to present the correct number of objects when asked for it, they have connected the vocabulary to true conceptual understanding.

Often new conceptual systems contradict beliefs that children held prior to conceptual change. Children observe the world around them and intuitively form their own theories of how the world works. These intuitive theories ground children’s beliefs and understanding of how they understand and explain their world (Carey, 2009). A process of conceptual change often involves a process of ‘theory’ change.

2.2.4. The ‘theory’ theory of conceptual development and its reciprocity with vocabulary

Gopnik and Meltzoff (1997) argue that conceptual development and the interaction between language and cognitive development in infancy is well captured by the postulates of the ‘theory’ theory. The ‘theory’ theory posits that children’s conceptual systems are theories, just like scientists’ theories, and that conceptual development can be explained with a metaphor of theory formation and theory change. Because children interact with the world, they intuitively form their own ‘theories’ about reality (Carey, 2009). These theories make it possible to explain and explore how the world works and makes it possible for children to characterize concepts’ properties, to determine coherent causal relationship between concepts, to extend their knowledge by finding new evidence and to identify false predictions. Theory formation and change leads children to the identification of new evidence to interpret experiences in a unique way. Because of different experiences and exposure, one child’s theory may be radically different from another’s (Gopnik & Meltzoff, 1997; Gopnik, 2003; Gopnik & Wellman, 2012).

The developers of the ‘theory’ theory argue that linguistic development is a source of input for theory development and theory change. In this sense, mathematics-specific vocabulary serves as input for theory development of numbers and therefore a child’s linguistic codes (Section 2.4.4) will influence how they develop and change their theories (Gopnik & Meltzoff, 1997). Children use an internalized linguistic structure, rather than external linguistic structure of an adult or other person to
formulate their theories. However, external sources of language contribute to theory development.

On the view of the ‘theory’ theorists, conceptual- and semantic change go hand in hand (Gopnik & Meltzoff, 1997). According to this perspective, semantic and cognitive development emerge reciprocally. While children are actively solving conceptual problems, their attention is drawn to the words relevant to those problems. At the same time, children can advance in understanding a language itself and discover the relevance of how words and language structures cohere to form a concept. Gopnik and Meltzoff (1997:193) suggest that one could think of a child as a physics student, or a scientist, who develops an understanding of new scientific terms and learns how to use the terms appropriately when they hear about new theoretical possibilities from a scientific innovator. Learning new terms is an important part of understanding a new scientific theory and at the same time, learning new concepts leads to the discovery of new words. Hearing the same words in different circumstances might guide children to think about meanings not considered previously. According to Xu (2016) language is a mechanism for learning. In my study, I have devoted much time and effort to get some idea of this and other mechanisms.

2.2.5. Xu’s ‘three mechanisms for learning’

Xu (2016:16) identifies three types of mechanisms for learning, which may each give rise to new concepts: 1) Bayesian inductive learning, 2) learning by thinking and 3) language as a placeholder for learning. Bayesian inductive learning is best described by Gopnik and co-authors (Gopnik, Glymour, Sobel, Schulz, Kushnir & Danks, 2004; Gopnik & Bonawitz, 2015). Similar to the ‘theory’ theory, this mechanism assists children in combining evidence and prior knowledge to revise their beliefs - which may lead to conceptual change (Xu, 2016). Some experiences and observations are consistent with children’s knowledge and others are in contradiction to what they know or believe (Feigenson, 2016). Feigenson (2016) argues that inconsistencies and failed predictions may be mechanisms used by children during the process of learning. In Bayesian learning – as children revise their beliefs, certain concepts will become more central in their reasoning than other concepts.

Secondly, a mechanism of ‘learning by thinking’ (Xu, 2016) can lead to conceptual change. With this mechanism, children go beyond data input and evidence
to extend conceptual structures. This can happen spontaneously and without much meta-cognition, or it can happen systematically and in a controlled way. Exposure to various learnable facts alone is inadequate for conceptual development. Much like how scientists such as Einstein and Galileo made great discoveries without laboratories or other comforts such as modern technology, insight and understanding happen because of ‘thinking’ with logic. The breakthroughs of the two scientists are the result of logical thinking. Their findings were not driven by new data or evidence alone, but by their ability to manipulate existing data structures in their minds until new insights emerged. Similarly, children explain situations or mentally simulate processes to solve problems or arrive at new conclusions. In this thesis, logical reasoning, facilitated by cognitive input analyzers, can therefore be considered a mechanism for conceptual change.

A third learning mechanism suggested by Xu (2016) is the inductive learning process of ‘language as a placeholder for learning’ (Xu, 2016). Because infants only begin to learn the meaning-bearing parts of language toward the end of their first year, the initial state of infants’ concepts is not language dependent (Valian, 2016). Language of thought does not exist in core concepts and yet core knowledge supports the development of language and other conceptual systems. As discussed in Section 2.4.1, newly learnt words per se do not provide the content of new concepts, but words allow children to build placeholder conceptual structures which will later represent true understanding of the concepts they represent (Xu, 2016). Words in a child’s lexicon are thus not proof of conceptual understanding, but serves as a placeholder until true conceptual understanding develops.

Whorf (1956) utilized a metaphor of embroidery to explain the placeholder role of language in learning. He explains that language is the embroidery cloth whereupon a picture of concepts is being woven. Until the embroidered picture emerges, language forms a structure – a canvas for embroidery. “Language, for all its kingly role, is in some sense a superficial embroidery upon deeper processes of consciousness, which are necessary before any communication, signaling, or symbolism whatsoever can occur” (Whorf, 1956:239). In the next section I discuss language as input for learning. In this sense language is not only the embroidery cloth or supporting structure but is also part of the embroidered picture that is beginning to emerge.
I do not argue that language alone serves as input for learning and agree with Spelke (2012), who noted that language is crucial in mathematical learning, but its role should not be overestimated. The idea of language as a mental function or mechanism for learning is concerned with what Vygotsky’s theory of language as a mental function constitutes: Children’s ‘inner dialogue’ takes place in the “theatre of our inner speech” (Kozulin, 1990:179) and plays an important role in reasoning processes.

Figure 2.5 illustrates the three learning mechanisms and their role during the process of conceptual change. In this sense, Bayesian learning, learning by thinking and language as a placeholder are the mechanisms which form the platform for input analyzers. Conceptual change brings about new conceptual systems to emerge as a product of the analysis of cognitive input.

![Figure 2.5](image)

Figure 2.5: Conceptual change is supported by different learning mechanisms

### 2.3. Susan Carey's description of ‘number input analyzers’

In this thesis I rely on the explanations of Susan Carey (Carey, 1988, 2009), especially her argument that number concept development is facilitated by children’s ‘number concept input analyzer’ functions, which she describes as evolutionarily developed. These are, in their origins, perceptual/sensory analyzers which ensure that mental representations are causally connected to entities in the world. In other words, the
‘analyzer’ function is to represent the world reliably and to perform computations to do so properly. Carey (2009:453) explains this construct as follows\(^{26}\):

Representations are mental symbols – states of the nervous system that refer to entities in the world. Although it is possible that infants think about things in the world before they’ve had any experience with them, this is not required for the representations to be innate in the sense I mean it. ‘Innate’ simply means unlearned – not the output of an associative process…not the output of any process that treats information derived from the world as evidence. What I mean for a representation to be innate is to the product of evolution, not the product of learning, and for at least some of its computational role to also be the product of evolution (Carey, 2009:453).

In my first readings of Carey’s work, I had to find a way to deal with the term ‘representation’. I concluded that one had to distinguish between, firstly, representation as oral and written language and all other forms of symbol-based communication. Such symbols first appear as perceptual cues and these sensory cues are then formatted in symbolic form, such as words. From there on I worked on the idea that the brain had to ‘do something’ with the input from the senses. I began to understand the notion of a mental input analyzer to be an innate mechanism that ‘filters’ input. Consequently, when I read Carey’s work, it made sense to me that she argues for a construct such as a ‘number input analyzer’, which is innate (Figure 2.6).

\(^{26}\)A few quotations in the study were not paraphrased - intentionally. I wished to maintain the power of the tone and style of authors, whose writing I captured in a verbatim quote. I do, however, include my interpretation of their words.
Many researchers have theorized about how humans conceptualize properties of the world around us. For instance, James (1890), Quine (1997) and Piaget (1954) posited that sensory and perceptual input are ‘transformed’ into conceptual structures. Vygotsky (1986/1992) was intrigued by the interaction between language and conceptual structures and the social origin of cognitive development. Building on Vygotsky’s theory (Vygotsky, 1978/1993, 1986/1992) and Mervis (1987) agree that there is an interaction between language input and cognitive development. In this study I adopt the view that humans are endowed with the ability to transform empirical, symbolized input (such as language data) into cognitive structures, by utilizing an innate mechanism (or mechanisms) which allows humans to reason and develop increasingly more sophisticated conceptual structures. Carey (2009) argues that some such mechanisms are:

...input analyzers that compute perceptual representations – veridical representations of the distal word. Contrary to the empiricist theories, these input analyzers are devices that do not have to be constructed by learning processes... The input analyzers that create representations of color, of depth, and so on evolved to work as they do, and evolution is a process that is responsive to veridicality. That is, we have evolution to thank for guaranteeing
that our representations of depth have the content they do and can fulfill the computational role required of them (Carey, 2009:449).

2.3.1. Input analyzers as cognitive filters

On Carey’s view, an innate input analyzer makes it possible for humans to produce veridical representations of reality and to reason about these very representations – at a representational level (such as one does when thinking about mathematical problems). From this follows the construction of concepts and change of such concepts. This is a process that happens through 1) sensory/perceptual input, including social interaction, and 2) mental or cognitive input via the input analyzer. This ‘analyzer’ makes it possible for humans to control thoughts and use working memory. It serves as filter, agent, guard and organizer. It is a filter with specific characteristics.

The term, ‘input analyzer’ thus refers to the mental entity that processes/computes one kind of mental representation of the world. Carey (2009:451) explains that different conceptual systems have different input analyzers: Concepts that arise from perception have perceptual input analyzers – mechanisms dedicated to analyzing perceptual input. She explains, as an example, that core cognition entities are served by innate input analyzers that are dedicated only to analyzing their (own) conceptual format – which is different to other concepts. So, for instance, input analyzers for spatial concepts are visual perceptual analyzers that analyze input to create representations of visual-spatial concepts. These analyzers are dedicated to forming object representations mentally.

Filtering and computing input that is already in representational form, such as language, mathematical notation, musical notation and so forth, is then a mental process of a secondary representation. For example, hearing the word ‘two’ (auditively) is ‘input-analyzed’ as sounds; seeing of the digit ‘2’ is ‘input analyzed’ by a visual input analyzer; when the word is coupled with its meaning, it is ‘input-analyzed’ by the number input analyzer. It stands to reason that these ‘analyses’ happen in milliseconds.

2.3.2. A constructivist view of ‘input’ and ‘analyzers’

If this analogical, theoretical description of cognitive activity holds, then it follows that all mental analyzers need input – irrespective of the kind of input analyzer. On this
theory, the claim of the study is that early number concept development, mathematics-specific vocabulary, classroom engagement (as outcome of executive function) and logical reasoning skills contribute individually, but especially also in unison, to early numeracy development; as such they can therefore be seen as ‘input’ for number concept development, each of which will arguably be ‘received’ by one ‘analyzer’ – or more.

During this process of ‘analyzer computation,’ I surmise, from the perspective of a constructivist epistemology (Piaget, 1970; Von Glasersfeld, 1995), that executive functions, logical reasoning and innate linguistic ability (Chomsky, 1965) make it possible to ‘assimilate’, for example, some perceptual input, and to ‘accommodate’ the (by now computed) input in an integrated fashion, having been processed via executive function analyzers too. If all goes ‘well’, the accommodation will be in a cognitive space where there is existing knowledge with which the ‘construction’ of ‘new’ knowledge can continue – leading to conceptual change. I see in the confluence of early Piagetian theory (Piaget, 1929, 1965, 1974) and Carey’s discussion of input analyzers (Carey, 2009) as a novel explanation of why some children may not learn mathematics easily.

Although I have adopted this view of ‘number concept input analyzers’, I also argue that children learn to plan, to use their working memory and learn to integrate knowledge. They process and analyze empirical input from their environment (Valian, 2016). To this end I employ the construct of executive functions – that group of mental functions that coordinate all the different processes required for (mental) input analyses. In the study, though, I employed a classroom engagement tool to capture the manifestation of executive functions.

I also employ another variable, namely, logical reasoning, which shows how children analyze and think logically (Section 2.6.3). In my argument, executive function and logical reasoning serve as important role-players during the cognitive analysis of sensory input for number concept development and for the learning that coincides with development of numeracy. Together with executive functions and logical reasoning, an innate ‘language acquisition device’ (LAD) supports number concept development. Chomsky (1965; 1986) described this as a hypothetical ‘device’ in the human mind that accounts for humans’ innate predisposition for learning language. I argue that the
development of language coalesces in some way with children’s number concept development. They learn words, including numerals and other words and phrases that capture mathematical reality, such as ‘small’, ‘many’, ‘far’, ‘near’ ‘under,’ ‘than’ and a multitude of other linguistic qualifiers and descriptions of magnitude. In this study I generally refer to these as ‘mathematics-specific vocabulary’.

I discuss executive functions in Section 2.6.1, logical reasoning in Section 2.6.3 and LAD in Section 2.3.2. Figure 2.7 sets out the main argument for this study – that four aspects of children’s learning of numeracy show some mutual association. Insofar as the ‘input’ is regarded, the study’s argument is that these mental entities are cognitive agents and are likely to feature in the forming of concepts, but the study does not interrogate how this happens – only that there is a likely association.

![Figure 2.7: Constituents of early numeracy learning](image)

**2.4. Language for learning**

The debate of how language influences concept development has long been the subject of discussion among linguists, philosophers and psychologists (Levine & Baillargeon, 2016:127). Early in the 18th century, Humboldt (1836) viewed language as a transformative organ of thought and argued that thought and language are inseparable. Cognitive linguists in the early 1900s viewed human semantic structure as a window on conceptual structure (Gentner & Goldin-Meadow, 2003). During the
same period, the dominant view of cognitive psychologists was that concepts come first and words merely name concepts. This view is in line with the Piagetian view which described the direction of influence from thought to language (Piaget & Fiúza, 1926). Although the cognitive psychologists admitted that conceptual- and semantic structures are closely related, not one of these views allowed for different semantic structures across different languages and cultures.

In the mid-1900s Whorf (1956) argued that some concepts form even before language develops and that language allows humans to organize their minds:

The world is presented in a kaleidoscope of flux of impressions which has to be organized by our minds – and this means largely by the linguistic systems of our minds (Whorf, 1956:213).

His view also acknowledged variations in different languages. The hypothesis which has come to be known as the Whorfian hypothesis states that all languages have different semantic structures, which influence the way in which one perceives and understands reality; therefore, speakers of different languages understand the world differently (Gentner & Goldin-Meadow, 2003; Whorf, 2012). This is the so-called ‘strong’ Whorfian hypothesis (Gopnik, 2001).

In this study I argue that language, along with other input data, serves as input for number concept development. However, if linguistic input in school (in terms of sound and structure) is radically different to the linguistic input children received during the first few years of their lives, it is likely that the input analyzer will find it difficult to filter input. For example, if a child’s input analyzer is used in analyzing sounds and structures from one language (for instance isiZulu or Sesotho) and a child must learn in a different language (such as English), she may struggle to interpret not only new vocabulary, but also new sounds and new syntax. Consequently, the process of conceptual development may be hindered. Currently, developmental psychologists use different theories and metaphors to explain the significance of language during conceptual change. In the next few paragraphs I describe theories that have contributed to my own understanding of linguistic influences on number concept development.
2.4.1. Process of thought becoming itself in the medium of language: Vygotsky

Lev Vygotsky (1896-1934) was one of the first modern researchers to do empirical research rather than to theorize about the role of language from a philosophical perspective. Although he did not have any modern research techniques or technology at his disposal, he was able to theorize about the integration of concepts and language, based on empirical work. Vygotsky viewed the essence of psychological development as a cultural notion, rather than a natural one. He described the unity of cognition and behavior and argued that all individual behavior has a social origin. He also described different modes of speech in childhood development, viewing language as a mental function on its own and as a mediating mechanism for other psychological processes (Kozulin, 1990:7). He was concerned with “the process of thought becoming itself in the medium of language” (Kozulin, 1990:5) and said that children’s development takes place largely because they interact with their world, which consists of language as a mediating tool (Vygotsky, 1986/1992). As children learn language and learn to communicate their observations and experiences, they also experience and observe the world because they understand language (Rogoff, 1990). Language thus serves as a mediator for the process of learning. Because language mediates learning, children can construct new knowledge and represent their reality with greater ease through the use of language (Henning, 2012a).

According to Vygotsky it is not the phonological quality of language that makes human language a language, but rather the functional use of its signs (phonological and graphemic) (Kozulin, 1990:154). Language thus creates reality but also represents reality symbolically. As children increasingly represent their reality using signs or symbols, they become able to construct new knowledge because signs and symbols (together with tools such as models or materials) mediate learning (Henning, 2012a).

To Vygotsky it was clear that concepts can be preceded by words, but words can also develop by teaching concepts. He modelled a dynamic pattern of interaction and separation between verbal and intellectual functions (Kozulin, 1990). To him the relationship between language and cognitive development could not be described as an instantaneous fusion between concepts and words, but rather as a zigzag line where concepts and words are woven into one another. Only when the process of
integration of concepts and language is complete, can a child express his/her thoughts through language. He argued that a balance between the development of concepts that serve as knowledge and language that serves as a representation of the world of knowledge (Kozulin, 1990). The following diagram illustrates this relationship between language development and conceptual development:

![Diagram](image)

The integration of language and concepts: an inductive process where concept- and language development collide to form understanding.

**Figure 2.8:** ‘Collision’ of pre-intellectual language and pre-linguistic concepts

Vygotsky distinguished between a pre-intellectual phase in the development of language (where language consists only of phonological imitations of sounds heard by young children) and a pre-linguistic phase (when concepts exist in a child’s cognitive thinking, although they still cannot be expressed through language). “At a certain moment these two developmental lines become intertwined, whereupon thought becomes verbal, and speech intellectual” (Kozulin, 1990:153).

Vygotsky also suggested that there are four stages in the sequence of the engagement between thought and language, which could also be distinguished by the different modes of speech/language27 (Vygotsky, 1986). During the ‘primitive’ stage, language is still pre-intellectual, and ‘intelligence’ (cognition) is not supported by verbal functions. When children, in the second phase (‘practical intelligence’), master problem solving at a sensory-motor level, language is still divorced from its corresponding logic. The syntax of language is still embedded in concrete tasks, while the syntax of thought at this stage is still embedded in concrete activity/operations. Therefore, Vygotsky claimed that the syntax of language comes before the ‘syntax of thought’. Only when verbal production changes from primitive communicative language to more mature verbal forms (‘egocentric speech/language’), does the aim

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27 The term ‘speech’ in Russian also refers to language and is not limited to oral language. ‘Speech’ = речь rech. “Language” = язык yazy.
of language become controlling of one’s behavior and thinking. This is the third stage. During the last stage, external symbolic means are internalised. Children are now able to reason logically and solve problems mentally. ‘Speech’ now becomes silent inner speech (inner language) which enables children to plan and use thoughts for reasoning.

The idea of a continuous process of connectedness between cognition and language emphasizes the importance of the claim that child concepts are not stagnant, but that both their concepts and language do evolve (Carey, 2009; Dehaene, 2011). It would be wrong to think that once children start using terms that they understand the meaning.

2.4.2. Language in thinking: Tool, lens and input

In the introductory chapter of Language in Mind, Gentner and Goldin-Meadow (2003) distinguish between ‘language as a lens’ (language enables children to represent the world in new ways) and ‘language as a tool’ (language enlarges children’s existing representations of their world). The two questions Gentner and Goldin-Meadow (2003) pose are whether language influences how children see the world and whether language augments children’s capacity to represent reality. They link the view of language as a lens to the Whorfian hypothesis by arguing that “the grammatical structure of a language shapes its speakers’ perception of the world” (Gentner & Goldin-Meadow, 2003:9) and thus enables children to represent reality in new ways. Similarly, they link the view of language as a tool to the views of Vygotsky: “Learning to direct one’s own mental processes with the aid of words or signs is an integral part of the process of concept formation” (Vygotsky, 1962:59) and, I would add, the development or enlargement of one’s conceptual system.

Levine and Baillargeon (2016) agree that language enables children to represent concepts that they could not previously represent and therefore the authors agree that language can be seen as a lens as described by Gentner and Goldin-Meadow (2003) because it provides a new way to ‘look at’ concepts. They also agree with Gentner and Goldin-Meadow (2003) that language provides children with a ‘toolkit’ which allows them to represent concepts which they could not previously represent. A ‘toolkit’ also makes concepts children already know more salient or accessible.
Levine and Baillargeon (2016) consider a third way in which language influences number concept development, namely language as input. They argue that the quantity and quality of language input a child receives during early development, may affect its influence on the development of concepts. Gopnik and Meltzoff (1997) support this view:

...aspects of linguistic input can have quite striking effects on conceptual development. Children who hear language relevant to a particular conceptual problem are more likely to solve that problem than children who do not (Gopnik & Melzoff, 1997:208-209).

In this sense, children who are often exposed to the accepted use of mathematics-specific vocabulary are more likely to develop conceptual understanding of the words they are exposed to. However, if a child is often exposed to one language and has to develop conceptual understanding of, for instance numbers, via a different language, the child may still struggle to understand the concepts.

Levine and her colleagues (Gunderson & Levine, 2011; Levine, Suriyakham, Rowe, Huttenlocher & Gunderson, 2010) have examined the relationship between parents’ mathematics-specific vocabulary and their children’s cardinal number knowledge. During this longitudinal research they videotaped 90-minute natural interactions between parents and 14 to 30-months-old children, once every four months. In the approximate 450 minutes of video, parents ranged from saying 4 to 257 number words. If one takes this to be the average of ‘number talk’ in different households, children are exposed to between 1200 and 100 000 number words per year. This is as Levine and Baillargeon (2016:135) say “definitely not a level playing field.” In their study they controlled for parent talk, family socioeconomic status and the child’s own number talk. Their findings revealed that the amount of mathematics-specific vocabulary children heard between 14 and 30-months-old influenced their understanding of cardinality at the age of four (48 months). Also, the quality of parents’ use of mathematics-specific vocabulary mattered (Levine et al., 2010). If the mathematics-specific vocabulary was accompanied by visual sets that were visible to the child, the mathematics-specific vocabulary was a significant predictor of cardinal number knowledge at the age of four. If there were no visible sets but only, for
instance, rote counting or counting to three when throwing the child in the air, it did not serve as a significant predictor.

In a similar experiment, other researchers found that preschool teachers’ ‘number talk’ also predicted the growth of young children’s mathematical concept development (Klibanoff, Levine, Huttnelocher, Vasilyeva & Hedges, 2006). Levine and Baillargeon (2016:136) sum it up: “A three-year-old who understands the cardinal principle is likely to have received more and better data – to have heard a lot more number talk referring to present objects than a five-year-old who has yet to understand this principle.” One can thus conclude that the quantity and quality of number talk serves as data for number concept development.

The view of language as ‘data’ input is central to the main argument for my study. Mathematics-specific vocabulary (and other cognitive functions) can be viewed as input for number concept development and the amount and quality of language (and other) input, influences number concept development. Whorf hypothesized that speakers of different languages are likely to perceive and understand the world differently due to the different ‘input’ of the different languages. Each language has its own phonemes, morphemes, semantics and syntax which may lead to subtle differences in understanding.

2.4.3. Levels of linguistic influences

Dowker and Nuerk (2016) distinguish between different linguistic levels at which number concept development may be influenced. They agree that the syntactic structure, semantic meaning, lexical structure and unique phonemes of different languages contribute to how children develop an understanding of number concepts as discussed in this section. Furthermore, they argue that the conceptual properties and other cognitive skills, such as working memory, other executive functions and logical reasoning contribute to how children develop number concept understanding. To explain the various linguistic influences on concept development, Valian (2016) provides an example.

If a two-year old child says, “want lollipop”, the child excludes the determiner (possibly ‘a’) and a subject (possibly ‘I’). She argues that children’s syntactic knowledge might be more developed than it appears to be in expressive language:
“Even when most children’s speech consists of two-word utterances, (she) suggests, their grammars already contain genuinely syntactic categories, plus operations that combine and move those categories in ways that are isomorphic with adult grammar” (Valian, 2016:261). On the other hand, children may know some words, but do not yet understand the word’s meaning. Consider, for instance, the number word ‘sixteen’, which young children may include in their uttering of the wordlist of counting numbers. Objects/elements in a set of 16 do not imply the grasping of the concept of ‘sixteen-ness’ unless a child has developed the cardinality principle of the number 16. When the word ‘sixteen’ is uttered, it does not necessarily imply that the speaker knows the meaning of the word (semantically) or the concept that it represents (both as linguistic conceptualization, as described by Dowker and Nuerk, [2016] and the numerical and magnitudinal concept).

Valian (2016) argues that there are two things that children do not know when speaking in incomplete utterances. Firstly, children do not know the language: they do not yet have knowledge of the lexical items or morphology of a particular language, they have not yet developed a rich vocabulary or learnt the grammar or syntax of the language. Secondly, very young children do not yet have the executive control processes which are required to coordinate fluent speaking. Speaking requires planning, coordinating, integrating and updating functions to speak in full sentences. Valian (2016) explains the process towards which children strive:

Speakers take into account the prior discourse, content, and the audience. Speakers choose lexical items; they put those items together according to the syntax of their language; they provide a prosody for the utterance; they pronounce each lexical item according to the phonology of the target language; they choose among alternate ways of conveying their message; they keep track of where they are; and so on. When talking to others in a conversation, speakers must take turns with the other participants, coordinate their gestures with their speech, and switch back and forth between talking and listening. They revise each of those procedures as necessary (Valian, 2016:263).

This means that although children have a sort of innate skeleton structure which facilitates language development (a LAD as described by Chomsky, 1965), the content of a specific language is yet to be learnt when they are young. Because of limited
executive functions that support language use, they are not yet able to use language in the same way as adults do. They still need to develop executive control (these two factors allow for data analysis) and learn the content of language.

Furthermore, variations in semantic structures of languages should be considered while investigating the influence on number concept development. For example, some languages, such as Afrikaans and German, invert the order of tens and units. In Afrikaans 13 is ‘dertien’ which means ‘three ten’, 14 is ‘veertien’ which means ‘four ten’, 21 is ‘een en twintig’ which is translated as ‘one and twenty’, 32 is ‘twee en dertig’ which is translated as ‘two and thirty’. English follows the same pattern up to twenty (for instance, 16=six te[e]n, 17=seven te[e]n). Thereafter the tens are presented before the units (22=twenty-two, 34=thirty-four). The counting nouns are therefore already a cause for confusion in children’s development of conceptual understanding.

Also, the structure of number names in agglutinative languages like isiZulu and Sesotho (which are the home languages of children attending Funda UJabule28 and also the language of instruction in Grade R), allows for conceptual transparency. Therefore, by using a transparency rule, Sesotho and isiZulu children can understand all number names after they have learnt the number words up to ten. For instance, yishumi nanye (11) in isiZulu means ‘ten and one’ and yishumi nambili (12) means ‘ten and two’. Leshome le motso o mong (11) in Sesotho, means ‘ten and one’ and leshome le metso e mmedi (12) means ‘eleven and one’. The disadvantage of this structure, is that some Sesotho and isiZulu number names become very long and overload children’s working memory (see Section 2.6.1). For instance, the isiZulu word for eight, isishiyagolombili (isiZulu, like French or Finnish is an agglutinative language), consists of eight syllables and is much more difficult to remember compared to the one syllable word ‘eight’ or ‘agt’ in English or Afrikaans. Also, the eight syllable leshome le motso o mong, which means ‘eleven’ in Sesotho, is more likely to overload a child’s working memory than the three syllable word ‘eleven’ or one syllable word ‘elf’, which means ‘eleven’ in Afrikaans.

28 The primary school where the research was conducted – see Appendix A.
Another example is the ‘headaches’ some words cause when translated from one language to another. For example, the word number can be translated to Afrikaans as ‘getal’ or ‘nommer’. Although the two translations are very close, the subtle differences allow for different meaning. The word ‘getal’ refers to the value or quantity of a set of objects, while ‘nommer’ may have no quantitative meaning. ‘Telephone number’ for example is translated as ‘telefoonnommer’ rather than ‘telefoongetal’. In this sense, ‘getal’ is only a series of numerals without quantitative meaning.

Adding to the argument that semantic structure of various languages influences how we understand the concepts, Chomsky (1965) argued that the syntax or word order gives meaning to language. On the Chomskyan view all humans are born with ‘universal grammar,’ which realizes in the environment of an individual. Just like most human bodies allow us to learn how to walk, see or listen, our bodies also allow for language development through a LAD. Parts of our brain are specialized to analyze language and that is congruent to what Chomsky said about an LAD. According to Chomsky (1965), the meaning of language lies in syntax and construction of sentences. Not all languages have the same syntax, yet all have a syntax which creates meaning. For example, if I instruct a child in Afrikaans to count the items (for example, buttons) on the table, I will say, “Tel asseblief hoeveel knope daar op jou tafel is.” If I translate this directly to English, it will make no sense, “Count please how many buttons there on your table are.” The meaning of this sentence gets lost because of the incorrect order of the words. If I reorder the words so that they make sense it will sound like this, “Please count how many buttons there are on the table.”

To summarize, mathematics is not only about number names in isolation, but children learn to understand number names when these words appear in sentences which adhere to a specific language’s grammatical structure and syntax. If they do not understand the linguistic properties of a language and cannot make sense of the conceptual properties that is embroidered in a mathematic specific word (conceptual properties of language), children cannot learn mathematics. They cannot learn or develop if they do not understand. Some children are exposed to rich linguistic environments and others are less exposed. Their exposure to language and in particular, mathematics-specific vocabulary – influences how and when they learn number concepts. Much of this linking is embedded in the social discourse
communities in which children live and learn. Some children live in a world with a ‘restricted code’ in terms of the discourses they can access.

2.4.4. The deployment of a child’s ‘expanded’ or ‘restricted’ repertoire

The sociologist, Basil Bernstein (1964) argued that if children are widely exposed to specific language of numeracy (or ‘number talk’ as Levine and Baillargeon [2016] call it), they develop an elaborate linguistic code that allows a child to choose from a wide range of syntactic options when speaking or interpreting language from another speaker. Because of an expanded vocabulary, when speaking or listening, children can draw from an expanded linguistic repertoire to make sense of their own or others’ thoughts or words. On the other hand, if children have limited exposure to specific language, they develop a limited linguistic code and the range of syntactic alternatives is reduced. If a listener or speaker attempts to interpret their own or others’ thoughts or words, they might not be able to do so due to a reduced range of vocabulary and language structures. Restricted- and expanded code may be distinguished by the extent to which the code inhibits (restricted code) or enables (elaborate code) the ability to symbolize thoughts verbally (Bernstein, 1971).

As children learn the specifications of a specific discourse structure, they learn language, or develop linguistic codes that regulate their verbal acts. Usually, children learn one or two named languages which are defined by their social structure or political and ethnic affiliation (Otheguy, Garcia & Reid, 2015). Named languages, such as isiZulu, Sesotho, Afrikaans or English develop over a period, within a very specific cultural environment. Children’s exposure to these culturally developed structures (which may include more than one linguistic culture) allow them to develop their own unique, personal language that emerges because of interaction with other speakers (Otheguy et al., 2015).

The unique code an individual develops because of linguistic social interaction, is referred to as an idiolect (Otheguy et al., 2015). Each person’s unique idiolect consists of lexical and grammatical features such as phonemes, morphemes, lexicons, syntax, semantic structures, systems of tenses, and so forth. An idiolect is a person’s unique system that underlies use of language and no two idiolects are similar. Exposure to a social system, like a family, cultural group or school system influences a child’s idiolect and therefore, the extent and quality of language children are exposed
to will influence the development of their idiolect. When children are exposed to more than one named language (or in the case of South Africa – to a mixture of many named languages), their idiolect may be a mixture of named languages.

Syntactic differences across various languages have also been shown to influence children’s idiolects, and specifically their understanding of number words. For example, in languages that don’t differentiate between singular and plural, number concepts develop later (Dowker & Nuerk, 2016). Other languages distinguish between singular, dual and plural and this linguistic distinction enhances (or sometimes hinders) number concept development in young children (Dowker & Nuerk, 2016). In Section 2.4.3 I provided an example of how lexical differences in number words may influence an individual’s idiolect. I have shown that isiZulu and Sesotho have some very long number words compared to English or Afrikaans. Longer number names may delay number concept development. I have also explained how inversion- and transparency properties of languages can influence an individual’s number concept development and in this context, their idiolect.

As I have shown, a child may struggle to interpret a speaker if the speaker’s idiolect is very different to that of the child. In South African classrooms, very different idiolects between child (with restricted codes) and teacher (possibly with an elaborated code which may include more than one named language) presents a huge challenge. An extreme form of linguistic restricted codes is observed in classrooms in rural areas. English (which is the language of instruction from Grade 1 at Funda UJabule) is not widely used in rural communities and yet English is the language with powerful South African currency. Children’s restricted exposure to English reduces their linguistic options when interpreting information from a teacher who explains number concepts in a language they do not understand. In the context of this study, language input affects children’s number concept development.

García (García & Lin, 2017; Otheguy et al., 2015; Vogel & García, 2017) proposes translanguaging as a possible solution to reduce the gap in classrooms between child and teacher. The idiolect is the cornerstone to sustain the idea of translanguage (Otheguy et al., 2015). In the classrooms at Funda UJabule, children’s idiolects may include a rather elaborated understanding of the isiZulu or Sesotho language, but a restricted understanding of the language of instruction, which is
English. Their restricted English code thus inhibits them to develop number concepts. Translanguaging involves full deployment of a person's idiolect which allows children to use both isiZulu or Sesotho and English.

Previously, researchers have suggested altering between two languages (for instance, isiZulu and English) in the form of code-switching. Code-switching involves going back and forth between two languages to scaffold teaching (and learning) in two different languages (García & Lin, 2017). Translanguaging theorists propose that one does not alter between clearly distinct linguistic systems which are normally deployed separately (Otheguy et al., 2015), but rather to select and deploy features form a unitary linguistic repertoire (idiolect) to convey meaning in ways that work optimally and to take part in communicative activities (Vogel & García, 2017).

When switching between linguistic codes, one preserves named language categories and, from this view, a bilingual person switches between two different systems (Vogel & García, 2017). Translanguaging differs from code-switching in the sense that translanguaging dismantles the distinct categories of named languages and uses the internal linguistic resources of a speaker (Vogel & García, 2017). Translanguaging thus relies on conceptualisation of bilingualism as dynamic integration of one semiotic system, rather than shuttling between two independent linguistic systems (Vogel & García, 2017).

Because translanguaging responds to one integrated linguistic system, rather than to two separate languages, it is more useful than code-switching. García and Lin (2017:4) explains:

It is precisely because translanguaging takes up this heteroglossic and dynamic perspective centred on the linguistic use of bilingual speakers themselves, rather than starting from the perspective of named languages (usually national or state languages), that it is a much more useful theory for bilingual education than code switching (García & Lin, 2017:4).

García and Lin (201) agree that, although different epistemologically, translanguaging and code-switching are linked because both disrupt the traditional isolation of socially and politically named languages. (Otheguy et al., 2015).
According to García and Lin (2017), the fear of using translanguaging as a teaching tool is that national languages will become 'contaminated', and yet they argue that this approach can make meaning and conceptual development more comprehensible, than persisting to teach in one colonial language. According to Otheguy et al. (2015), translanguaging provides smoother conceptual paths which allow children in multilingual learning environments to access and develop previously unavailable concepts. In this sense, translanguaging provides multilingual children almost the same opportunity to learn than children who attend school in their mother tongue. By using translanguaging as a pedagogical approach, children can enjoy cognitive and emotional benefits by using all of their linguistic resources (Otheguy et al., 2015).

Although translanguaging as a teaching tool makes use of an integration of various named languages, this pedagogy can develop vocabulary simultaneously in different named languages, rather than compartmentalizing the development of languages in a hierarchical order (Vogel & García, 2017). For instance, mathematics-specific vocabulary can be developed simultaneously in isiZulu or Sesotho and English, rather than developing vocabulary in isiZulu or Sesotho first, followed by English vocabulary later. Even if teaching is done in the dominant or prescribed language (such as English), children are allowed to constantly make sense and use other languages (such as isiZulu or Sesotho) by integrating what they already know in their home language with emergent English (García & Seltzer, 2016).

2.5. A constructivist model for number concept development

Many researchers have proposed models to describe young children’s mathematical development from a constructivist perspective (Brannon, 2002; Gelman & Gallistel, 1978; Resnick, 1989; Wynn, 1990, 1992). Some models differ in terminology and conceptualization, but the theories of the key constituents of number concept development do not differ substantially (Fritz et al., 2013) and most theories are based on Piaget’s constructivism theory (Piaget, 1970). Some researchers describe how young children learn to count (LeCorre & Carey, 2007; Sarnecka & Carey, 2008; Wynn, 1992) and how they learn the concept of number cardinality (Frye, Braisby, Lowe, Maroudas & Nicholls, 1989; Sarnecka & Wright, 2013). Others are dedicated to
understanding how children learn the symbolic notation of mathematics (Henning & Ragpot, 2015; Lyons & Ansari, 2015).

Fritz and her colleagues (Fritz et al., 2012, 2013) compared many of these theoretical models and research findings to empirically validate a comprehensive theoretical model that describes five levels of early number concept development. A one-dimensional Rasch analysis (Rasch, 1960) confirmed the sequential structure of the five levels of performance (Henning et al., 2018). The fundamental assumption of the model was that a complex construct, like mathematics, is constituted of different numerical concepts (or levels of development) that build hierarchically (or constructively) on each other (Fritz et al., 2013). Each level is characterized by the development of a particular concept and together the conceptual levels form a continuum of numerical development.

The theoretical model was used to design a diagnostic instrument that assesses four to eight year-old children’s number concept development by determining children’s mathematical ability related to the five levels of the theoretical model (Fritz et al., 2012, 2013). The test is used to identify children who need special attention in number concept development. The theoretical model assists the assessor to determine on which conceptual level the child’s number concepts are so that intervention can start on that given level (Henning et al., 2018). It is also suggested that the hierarchical model can be used to inform general teaching or specialized intervention. All teaching should thus focus on identifying children’s current concepts, after which the subsequent concepts are introduced (Henning et al., 2018). The test is described in Chapter 3, Section 3.4.1.

The ‘Fritz model’ is built on the assumption that number concepts build on the foundations of the two core number systems: AMS and OTS. According to Spelke (2000), core number knowledge not only provides a solid foundation for the development of number concepts, but also continues throughout one’s lifespan:

First, core systems continue to exist in older children and adults, giving rise to domain-specific, task-specific, and encapsulated representations like those found in infants. Second, core systems serve as building blocks for the development of new cognitive skills (Spelke, 2000:1233).
Fritz and her colleagues (Fritz et al., 2012, 2013) describe five levels of concept development that build on the foundation of core number knowledge: counting, ordinality, cardinality, class inclusion and relationality.

Level 1: Counting

Around the age of two, children begin to distinguish number words from nouns, verbs, prepositions and other parts of speech. They realize that number words refer to numbers, but do not link each word to a specific quantity yet. “It is possible that children understand that number words denote numerosity before they have assigned any number words with precise meaning and well before they become competent counters” (Slusser & Sarnecka, 2011:42). The first concept in the development of number concepts, is knowledge of the countable nouns. Children learn the sequence of the number words and realize that these numerals can be used for enumerating sets. They develop an understanding of the meaning of each specific word and learn that sets of objects or fingers can be counted by linking each number word in succession to objects or fingers.

Children learn to count for different reasons. Parents often let children repeatedly count to five or ten because they are proud that their children ‘can already count’. Repetition in this way is important, because children learn that the number words always have the same order. By repeating the ‘count list’, they learn the stable order principle (Gelman & Gallistel, 1978). Young children often close their eyes and count to ten to let time pass (Sarnecka & Carey, 2008) to play hide-and-seek. They count objects to determine the quantity of sets of elements by using the one-to-one-correspondence principle (Gelman & Gallistel, 1978). An alternative and more challenging task (as described by Level 3 of this model) is the task to count out a given number of objects within a larger quantity (Fritz et al, 2013). In other words, children first only repeat a rhyme or ‘sing a song’ to learn the order of the number words.

Level 2: Ordinality

Gradually children begin to construct a representation in which numbers are organized as gradually increasing quantities (Fritz et al., 2013). Because there is no evidence that a ‘mental number line’ must be a representation of numbers on a straight line (Fritz et al., 2013), one could also think of this representation as a number list. A
qualitative representation of numbers (as explained by Level 1 of the Fritz model) does not indicate any ordering relationships between numbers (Fritz et al., 2013), but only lists a few numbers which always have the same sequence. With the count list and counting principles as a basis, children begin to develop an understanding of the sequential connection between numbers as seen on the ‘mental number line’ (Fritz et al., 2013). This characteristic of the number line enables children to identify preceding and succeeding numbers. They learn concepts like ‘before’, ‘after’ and ‘in between’. Numbers can thus be compared to each other based on their position on the number line. Children can now also solve several simple addition and subtraction tasks by counting forwards or backwards (Fritz et al., 2013) within a limited number range (Ehlert & Fritz, 2013).

Level 3: Cardinality

After acquiring the ‘ordinal’ property of numbers, children learn that each number represents a specific quantity and that each number can be decomposed into a specific number of elements (Fritz et al., 2013). They understand that each number’s ‘manyness’ differs from other numbers’ ‘manyness’. Numbers are understood as increasing cardinalities, rather than rising numbers due to their position on a number line. By counting out objects, children apply the cardinal principle which states that “the [numeral] applied to the final item in the set represents the number of items in the set” (Gelman & Gallistel, 1978:80). True cardinal understanding thus requires an integration of the number of objects that is counted, and grasping the concept of a ‘whole’ that consists of a specific number of objects.

Sarnecka and Carey (2008) distinguish between children who only grab a few objects without knowing how many they grab (grabbers) and children who can count out a certain amount of objects (subset-knowers). They refer to a child who can only accurately count out one object, a ‘one-knower’; accurately count out two objects ‘two-knower’ and so on. Thus, a child who can count out N objects is a ‘N-knower’ or ‘subset-knower’. LeCorre and Carey (2007) argue that children utilize their core knowledge to map the first four numbers onto representations of small sets. When children are able to generate the correct number of objects for five or above, it is clear that they can apply the counting principles (including the cardinal principle) to sets with more than only three or four objects and are thus referred to as ‘cardinal principle-

Upon understanding of number as a quantity, children are able to determine cardinality of sets with ease and know that the order of the objects being counted are irrelevant (Fritz et al., 2012). Now, children also have an improved idea about the successor function: the cardinality of each number is generated by either adding one more to the preceding number or by taking one away from the succeeding number (Sarnecka & Carey, 2008). They can now determine N+2, N+3 or N−4, N−5 and so on, not by using knowledge about the number line, but by using the decomposability attribute of numbers.

Fritz et al., (2013) see the acquisition of the cardinal principle as a key prerequisite for effective calculating strategies. By using their knowledge about numbers’ cardinality, children can solve addition problems by counting a second quantity onto the first rather than to count all. A true understanding of cardinal value allows the child to understand class inclusion.

Level 4: Part-part-whole

Acquisition of a comprehensive part-whole concept is seen as an essential step in number concept development (Langhorst, Ehlert & Fritz, 2012). Understanding that numbers can be decomposed into smaller parts, but they also comprise compositions of other numbers, enables children to interpret quantitative relationships and enables a flexible understanding or operation (Langhorst et al., 2012). The number five, for instance, can be understood as ‘five ones’ or as part of ‘six ones’ or ‘seven ones’. Each number is seen as a unit of units (Fritz et al., 2013). Because of the fixed relationship between numbers, children can determine any part in an ‘a+b=c’ or ‘c-b=a’ equation if two of the three values are known (Fritz et al., 2013). Now children can flexibly change two parts of the whole without changing the whole itself. For instance, the (whole) number nine can flexibly be seen as two parts: eight and one or four and five, and so on, without changing the value of nine.

Upon understanding the part-part-whole concept, children learn that addition and subtraction tasks are complementary. “In summary, the part-part-whole concept
can be understood as the prerequisite for an integration of several algebraic principles, such as the commutative law, the complementarity of addition and subtraction, and the understanding that (natural) numbers are composite units” (Fritz et al., 2013).

The following diagram illustrates how one can use the part-part-whole concept to teach the relationship between numbers. It is important to notice that although two distinct ‘parts’ of a number are visible, the child can always also see the ‘whole’ number. The objects used to show the whole are not divided into smaller sets, but rather another set of objects is used to represent the ‘parts’ while the ‘whole’ is still visible.

Figure 2.9: Part-part-whole representation of number seven

Level 5: Relationality

A deep understanding of natural numbers means that children understand the concepts of ordinality, cardinality and the relationship between the two (Fritz et al., 2013). The mental number line is now seen as increasing cardinal units. Each number word represents a cardinal number that is one unit more than the previous number (Fuson, 1992). The number line is understood as precise metrical units between succeeding numbers with even distances between numbers. The relation between segments on the number line is also now understood. Because the distance between one and four is equal to the distance between five and eight or nine and twelve, there is a relationship between these segments on the line.

Figure 2.10: Relationality between numbers (adapted from Fritz et al., 2013:59)
When provided with a picture consisting of two rows with three dots in one row and five dots in the second row, children can now be asked, “how many more are there in the second row?” or “how many fewer are there in the first row?”

Figure 2.11: The relationship between two quantities

Also, for instance, when children are presented with ten red counters and ten blue counters, the teacher can ask, “Give me eight counters. There must be two more blue ones than red ones.” If children consistently provide correct answers for such tasks, one can conclude that they have a true understanding of relationality.

In terms of the discussion whether development is continuous or stage-like (thus discontinuous), I would argue that both perspectives are useful to understand number concept development. I argue that, although the five phases of number concept development which I’ve discussed in this section can be distinguished and that one concept is preceded by previously developed concepts, distinct ‘levels’ do not develop separately from preceding and succeeding concepts. Concepts on a higher level, do not only begin to develop once the child completely understands the concepts which are defined by the previous level. Rather, conceptual development is a continuous inductive process where children might begin to understand concepts of a higher level while their understanding of a lower level’s concepts is still developing. For instance, children may begin to understand the concept of place value while still developing their understanding of the relationality between numbers. However, when one looks at their conceptual change over a period, the initial conceptual system will be changed and completely reorganized and will be incommensurable with previous conceptual systems. Therefore, different phases or levels can be described and distinguished, but development is continuous and inductive.
2.6. Additional constituents of early numeracy attainment

Although I acknowledge the important role of previously developed number concepts in some hierarchy as well as the role of language, this study also aims to show whether other cognitive constituents, such as classroom engagement, which manifest executive functions and logical reasoning skills, contribute to number concept development. In this section, I argue that variance in number concept development, can (among other reasons) be explained by the fact that individual children vary in their engagement in classroom activities and ability to reason logically. I wanted to find out if classroom engagement and logical reasoning skills in Grade R collectively predict number concept development in Grade 1 (see Figure 1.1).

2.6.1. Executive functions consolidate classroom engagement and early number concept development

Executive functions support conceptual development and are an important indicator for school readiness (Blair & Razza, 2007; Diamond, 2013; Fitzpatrick & Pagani, 2012). Children with high executive functions might have the potential to achieve high grades in mathematics, while poor executive control may inhibit children’s ability to learn and to express their understanding of certain concepts. In this sense, executive functions play an important role when cognitive input is analyzed during conceptual development. Describing preschool children’s executive functions could indicate the level of engagement in the classroom (Fitzpatrick & Pagagni, 2012), which has been shown to contribute to number concept development (Fitzpatrick & Pagani, 2012).

Previous research has shown that executive functions (cognitive flexibility, inhibitory control and working memory) and number concepts are correlated (e.g. Cragg & Gilmore, 2014; Prager, Sera & Carlson, 2016) and that executive functions predict conceptual development (Blankson & Blair, 2016; Zaitchik, Iqbal & Carey, 2014). Executive functions are general mechanisms that regulate the operation of cognitive processes and modulate human cognition (Miyake, Friedman, Emerson, Witzki & Howarter, 2000)29. Although some researchers (e.g. Norman & Shallice, 1986) have argued that executive function is a unified concept, this study’s definition of executive functions is grounded in the work of researchers who postulate that there

29 In my interpretation in this study I see executive function as a specific input analyzer.
are three or four subcomponents of executive functions (e.g. Bull & Scerif, 2001; Bull & Lee, 2014; Diamond, 2013; Fitzpatrick, 2014; Miyake et. al., 2000). The three often postulated core executive functions are working memory (for input updating); cognitive flexibility (also referred to as set switching, mental flexibility, or mental set shifting); and inhibitory control (including self-control [behavioral inhibition] and interference control [selective attention and cognitive inhibition]).

The cognitive function of ‘cognitive flexibility’ – often referred to as shifting of mental sets, attention switching or task switching – enables children to shift back and forth between multiple tasks (Miyake et al., 2000). The ability to switch between tasks plays a role in helping children hold and refocus attention towards relevant stimuli (Fitzpatrick, 2014). Switching between sets is often difficult when the same input or stimuli requires different reactions (Cragg & Chefalier, 2012). For instance, 11 objects can be represented in many different ways. The input analyzer uses cognitive flexibility to distinguish between the representations of 11. The number 11 can be seen as the number just after 10 or before 12; as a combination of smaller numbers such as 10+1 or 5+6; as a part of larger numbers such as 12, 13, 14 and so on; spoken or written word or an Arabic symbol. Cognitive flexibility involves the ability to know when to use which representation of the concept of ‘elevenness’ – depending on the context of a task.

Flexibility of cognitive tasks enables children to flexibly select among potentially relevant input and to shift between multiple tasks, operations or mental sets. Attention switching enables children to switch their focus from, for instance, addition to subtraction. When Grade 1 children, for example, have to compute 6+2 and thereafter 4-2, some children might still add four to two because of a lack of cognitive flexibility. They still need practice in choosing a correct response to input. The input analyzer might not be able to support such a switching task yet and therefore the child may continue to add instead of subtract. Miyake et al. (2000) explain that “the shifting process involves the disengagement of an irrelevant task set and the subsequent active engagement of a relevant task set.” In the example, children must disengage in an addition task and actively engage in a subtraction task. Often, teachers are under the impression that children do not understand a certain procedure, while the inability to switch between tasks (such as addition and subtraction) creates a bottleneck in the processing of input and as a result the child gives the incorrect answer. I argue that
one must therefore distinguish between children’s knowledge (or lack of knowledge) of number content and executive control which allows children to develop numerical competency.

The most commonly used tests to measure cognitive flexibility are the Stroop tests (Golden & Freshwater, 1978), Wisconsin Card Sorting Test (WCST; Grant & Berg, 1948) and The Dimensional Change Card Sort (DCCS; Frye, Zelazo & Palfai, 1995; Zelazo, 2006). During tests that measure cognitive flexibility, children are usually presented with cards displaying shapes of different colours. First, children must sort the cards according to colour, shape or number. After a few rounds of using this criterion, they have to switch and sort the cards by a different dimension. Although other areas of the brain might also be involved in these tasks, neuropsychological and neuro-physiological research indicates that shifting between tasks involves the prefrontal lobes of the brain (Miyake et al., 2000:56). The prefrontal cortex coordinates multiple neural activities and monitors and integrates activities across multiple neural structures (Funahashi & Andreau, 2013). The prefrontal cortex controls information processing by sending signals to posterior cortices and is therefore involved in executive functions (Funahashi & Andreau, 2013).

Being able to refocus attention on a different task or switch between tasks, means that one must inhibit the automatic inclination to continue with a previous learned response. Inhibitory control enables children to choose to change activities or to inhibit an automated response. Because humans are creatures of habit we naturally tend to react to stimuli in our environment. To inhibit an incorrect response, we need inhibitory control to be able to choose our actions and to change our activities when necessary. Instead of only reacting to conditioned responses, inhibitory control enables humans to control their thoughts, actions and emotions. A significant part of children’s behavior is on ‘auto-pilot’ since this is more efficient for saving energy. However, it is sometimes necessary to over-ride auto-pilot and inhibit automated responses. It is almost like taking a different route back from school because there was an accident on the road. Often children must change their automated responses to adapt to a given situation. Diamond (2013) distinguishes between inhibitory control of attention (the choice to focus on one thing and suppress attention to other stimuli); cognitive inhibition (resisting unwanted thoughts or memories) that supports working memory; and self-control (including control over one’s emotions and behavior by
resisting temptations, completing a task despite distractions and delaying gratification). The use of inhibition of attention, cognition and self-control supports effective learning.

In this context, the input analyzer enables children to deliberately inhibit dominant, automatic, or prepotent responses (Miyake et al., 2000) and to override automatic responses in favor of more adaptive, goal-directed or effortful behavior (Fitzpatrick, 2014). The ability to damp down the activation of competing responses or conflicting representations (Carey, Zaichik & Bascandziev, 2015; Miyake et al., 2000) enables the input analyzer to select the appropriate input for a specific task. Similar to cognitive flexibility, inhibitory control is linked to the development of the prefrontal cortex and its tandem development is optimal from birth to around six years, then slowing down into adulthood (Müller & Kerns, 2015). Typically, inhibitory control is measured by a go/no-go task where children are required to either respond (by pressing a designated key used for testing inhibition) or withhold a response (by not pressing a designated key) depending on whether a go stimulus or a no-go stimulus is presented (Verbruggen & Logan, 2008).

The third often mentioned executive function is working memory. Children store and process information relevant to a task at hand in their working memory. Updating working memory requires monitoring and coding input which is relevant to a task and appropriately revising input held in the memory by replacing old information with new, more relevant information (Miyake et al., 2000:57). In this view, incoming information (including language input) is analyzed, stored and updated by using one’s executive functions. By holding and adapting information, children can make sense of anything that unfolds over time in order to keep track of what happened earlier in comparison to what will happen later; make plans; do any mathematical calculation; incorporate new information; consider alternatives, etc. Working memory is thus needed for all reasoning activities (Diamond, 2013).

According to McLeod (2012) working memory comprises a central executive, visuo-spatial sketch pad (also referred to as visuo-spatial working memory) and phonological loop (or phonological working memory). The central executive drives the working memory system and allocates input to the phonological loop and the visuo-spatial sketchpad (McLeod, 2012). The central executive thus coordinates all data
input and decides what the working memory should pay attention to. Baddeley (1986) explains that the central executive controls the system rather than to function as a ‘store’. The phonological working memory deals with written and spoken language. The phonological loop ‘circulates’ language in one’s mind. It is almost like repeating certain information (like a telephone number) in your head. As long as you keep on ‘repeating’ the information in your head, you are able to retrieve it from your working memory. The visuo-spatial sketch pad analyses visual and spatial information and allows us to keep track of where we are in relation to other objects (Baddeley, 1997).

A subset of the WISC-IV\textsuperscript{30} is commonly used to assess working memory. The working memory index subscale on the WISC-IV requires children to process, recall or manipulate verbal sequences which are presented orally. The sub tests in the scale include: Digit Span (digits Forward: Auditory short-term memory and digits Backward: Auditory working memory); Letter-Number Sequencing, which assesses shorter string lengths; and Arithmetic which assesses auditory short-term memory, auditory working memory and fact retrieval (Jepson, 2008). Some tasks (such as conceptual development, reading comprehension and multistep arithmetical problems) require involvement from the central executive, while other tasks draw strongly on the phonological loop (such as reading, vocabulary acquisition and arithmetical word problems) or visuo-spatial sketch pad (such as learning numbers and interpreting graphs) (Cockcroft, 2015; Davidson, Amso, Anderson & Diamond, 2006).

2.6.2. Executive functions manifest in classroom engagement

Although the literature describes different parts of executive functions and even breaks working memory up into smaller parts, one must consider the descriptions of different parts of the executor control system as a model to describe how humans deal with input. The literature highlights that the three components of executive function (cognitive flexibility, inhibition and working memory) are interconnected and work together. Processes like planning, problem-solving and reasoning, seem to involve a combination of the three executive processes (Diamond, 2013). Diamond (2013) explains that working memory and inhibitory control support each other as the child must keep in mind what information is relevant and what to inhibit. Switching between

\textsuperscript{30} Wechsler Intelligence Scale for Children – Fourth UK Edition.
tasks also requires inhibition of prior tasks while keeping relevant information in your memory.

Fitzpatrick and Pagani (2012) argue that classroom engagement is a behavioral manifestation of executive functions and that classroom engagement can therefore be used when a researcher wants to investigate the relationship between executive functions and another cognitive skills (like number concept development). Children with good cognitive flexibility are likely to be able to disengage from a complete task and re-engage in a new task which allows them to engage in all tasks from the outset, while children with poor cognitive flexibility may find it hard to engage in a new task. This means that they only take part in parts of activities instead of the whole activity. Blair (2002) argues that attention is an integral part of inhibitory control, which on its own accounts for variance in children’s cognitive skills. Fitzpatrick and Pagani (2012:716) argue that “the executive attention system seems pivotal in modulating reactivity, including approach, avoidance, and inhibition, and effortful control of behavior in response to cognitive, emotional, and social situational demands on self-regulation.” All the skills they mention are likely to not only have an influence on classroom engagement, but also on number concept development. When young children’s effortful control increases, they become more able to inhibit impulsive and improper responses. As their attention and self-control improves, they become more engaged in the classroom and are consequently able to engage in learning activities which in turn is likely to lead to the development of concepts.

Increasing working memory also allows children to become more engaged in classroom activities. For instance, if children are able to integrate auditory input and produce meaningful language (phonemes, words or sentences) by storing, maintaining and rehearsing auditory information, using their phonological loop (a subcomponent of working memory) (Cockcraft, 2015), the more likely it is that they will be more engaged in cognitive activities that require actively holding and manipulating information in their minds. Similarly, storing and rehearsing visual/spatial information by using their visuo-spatial sketchpad (another subcomponent of working memory), children are more likely to engage in activities that require manipulation of visual information (Cockcraft, 2015). Based on these examples, classroom engagement can be used as a ‘placeholder measure’ for executive functions.
2.6.3. Logical reasoning as contributor for numerical competence

The fourth contributor for number concept development which I argue, is logical reasoning. Being able to memorize and retrieve factual number knowledge, apply learnt strategies to calculate, inhibit unnecessary information, switch between tasks, or even have a good mathematics-specific vocabulary is not sufficient to develop numerical competence. Children must be able to reason about why a specific method for calculation is needed to solve a specific numerical (and other mathematical) problem(s). They must be able to apply their conceptual knowledge and consolidate number knowledge when responding to an instruction like, “Give me eight counters. There must be two more red counters than blue ones” (Item 42 on the MARKO-D SA). I propose that core number systems, number knowledge, working memory, language and so forth all operate in harmony and are actively involved when the child reasons about how to provide a correct response. Logical reasoning is actively involved when the input analyzer assembles input to produce the correct ‘output’ or answer.

In Section 2.4.1 I referred to the concept of ‘inner speech’ (the Vygotsian term for thinking with language) as a mechanism for learning. Vygotsky described ‘inner speech’ as “the generation of thought in the form of word meanings” (Kozulin, 1990:267). Inner speech, or ‘speech-for-oneself’, allows for a process of thought formation which is reflected in a sort of dialogue between ‘meaning’ and ‘sense’ – two epistemologically relevant terms (Kozulin, 1990).

The interaction between sense and meaning constitutes the inner dialogue between two different ‘subjects’ of one thought. One subject accommodates her thought to the pre-existing system of meanings, while the other immediately turns them into idiosyncratic sense, which later will be transformed again into intelligible words (Kozulin, 1990: 268).

As a result of inner speech activity in this cognitive modality, new knowledge or new understandings are generated. In this study I use the Vygotskian metaphor and connect it to what I understand to be logical reasoning. It consolidates all forms of input and produces understanding. Such new understandings emerge as a result of a ‘dialogue’ between already existing ideas and newly created thoughts which still lack definition (Kozulin, 1990). Logical reasoning is thus a process of “thought becoming itself out of what is not yet thought” (Kozulin, 1990:267). During the process of logical
reasoning thoughts are not yet expressed by meaningful words and language is not yet infused with meaning. The process of logical reasoning results in the alignment of thoughts and words – when language becomes infused with meaning and thoughts can be expressed by meaningful words. During logical thinking the child becomes the ‘source of thinking’.

Although reasoning about abstract concepts, such as mathematical concepts, is extremely difficult for young children (Evans, Handley & Harper, 2001), some researchers have found a connection between logical reasoning and mathematical tasks (Handley, Capon, Beveridge, Dennis & Evans, 2004). Morsanyi and Szüs (2015) investigated this type of link. They found that mathematical thinking and logical reasoning both involve complex cognitive processes, the skills of which are dependent on executive functions such as working memory and inhibitory control. They also found that logical reasoning and mathematics require the retrieval and application of normative rules to draw conclusions and process abstract or symbolic content.

Other researchers found that mathematics and logical reasoning are not connected. For example, Butterworth, Varma & Laurillard (2011:180) found that verbal processing, visual identification of digits and words, quantity representation, memory and reasoning are distributed among different brain regions. They also indicated that because some brain damaged patients’ number concepts, language, memory and reasoning worked independently (Butterworth, 1999:181), these different skills do not rely on each other. However, in more recent work Butterworth et al. suggest that mathematical abilities rest on “a swarm of highly specialized neuronal networks communicating through multiple parallel pathways” (Butterworth et al., 2011:179), and that although mathematical abilities and reasoning skills depend on separate brain circuits, the development of number concepts, language, memory and reasoning are “necessary for acquiring the conceptual tools provided by culture” (Butterworth, 1999:181). Because mathematics is a cultural development, logical reasoning requires collaboration between distinct brain areas. Because of associations between mathematics and logical reasoning, I argue that logical reasoning can consolidate input for number concept development.

To conclude this section of the chapter, I argue that individual differences in number concept understanding is a result of differences in children’s early number
concept development, a variation in exposure to mathematical language, varying executive functions and logical reasoning. For children to develop these contributing skills in preschool in order to develop numerical competence in Grade 1, teachers should have knowledge of how the various cognitive skills develop and contribute to number concept development. In the next section I pay attention to teacher knowledge and how teachers can effectively identify children who might be at risk of struggling with number concepts due to insufficient early number concept development, a lack of exposure to mathematics-specific vocabulary, poor executive functions or poor reasoning skills.

2.7. An integrated pedagogical approach

Any study in the field of education, I would argue, should include application of theory in practice. In Chapter 1 I reasoned that research about the developing child will only have an impact on children if the fields of education practice and research collaborate. The question, then, is how the knowledge that emerges from studies like this one can be applied to education practice.

If one accepts that number concept development is compromised if children do not understand the language of mediation sufficiently, are not engaged in class, and cannot reason logically, it is fair to argue that teachers should know how to develop children’s mathematics-specific vocabulary, engage children in class and teach them reasoning skills. A teacher who ignores the role of these cognitive skills is unlikely to realise that some children will not develop numerical competence unless these cognitive skills are also developed. A claim of this study is thus that all teachers – practicing- and prospective – should insert the explicit teaching of a variety of cognitive skills in their pedagogy.

To me, it seems obvious that teachers should have knowledge about children’s cognitive development and possible challenges they may face. Yet, Henning (2013b:1) found that a “discourse of teachers’ expressed knowledge about their practice was embedded in the language of policy, curriculum, teaching methods, and the omniscience of the ANA test in South Africa, with very few discourse markers representing knowledge of child cognition.” In Radebe’s study (2018) there was little evidence that teachers had been able to infuse their knowledge into classroom practice – even teachers that had been participating in professional development,
targeting conceptual development in early mathematics (Radebe, 2018). These findings indicate that teachers’ focus is not on the holistic, developing child and early identification of mathematical learning difficulties, but rather on the implementation of the curriculum, national assessments (at the time) and teaching methods.

This thesis calls for a change in perspective: A shift from the curriculum and assessment to the inclusion of relevant components of cognitive, developmental psychology, which ought to be part of a teacher’s PCK portfolio. Such an approach to mathematics pedagogy would focus on the whole developing child and how assessment of predicting cognitive skills can support the identification of children who struggle with mathematics and, specifically number concept development.

2.7.1. Constructivist pedagogy and cognitive psychology for teachers

A mix of some cognitive psychology ideas with loosely defined constructivist tenets for teaching have often been reduced to a simple formula of ‘constructivist pedagogy’ (Darling-Hammond & Bransford, 2005; Phillips, 2000). If one considers the explanation about ‘input analyzers’ as described by Carey (2009) which is central to my study, it is almost inevitable to wonder about the popular notion of an educationist ‘constructivist pedagogy’. In my interpretation of children’s input, the teacher, the methods she employs, and the teaching tools or instruments are not data for input analyzers. The teacher herself is not the source of knowledge and the teaching tools are not the primary mediators between teaching and learning. In the theory of ‘input analyzers’, as proposed by Carey (2009), I would argue that the ‘mediator’ in learning is the individual child herself, – receiving the cues from a social source, but whose input analyzers stand by – ready to extract from the ‘world’ only what her analyzers are able to compute, process and digest.

The main point of critique in this study is thus one of militating against the approach that starts with the question, “How should a teacher teach so that children will learn optimally?” I propose, rather, that the starting point for teachers should be to understand how children develop and what social input they need to form new concepts. In terms of this study, I would rather ask, “How might the input received from ‘the world’ be analyzed and processed by an individual learner’s input analyzer?” I would work from that and try to be a truly learner-centered theorist of pedagogy.
From my own experience as a teacher I make a tentative claim that when teachers integrate this kind of knowledge (about number concept development and mathematical cognition in general) and discourse in their personal pedagogical approach, they may be able to select or design a suitable ‘method’ and ‘instrument’ in such a way so that children can develop new number conceptual systems on which they can build the solutions to mathematical problems. Teachers are, however, dependent on research by the scientists of learning and developing children. Some of these scientists work with behavioral data, while others rely on the technology of neuroscience.

From a developmental scientific view, teachers ought to know how new conceptual representations emerge in development (Carey, 2009). For instance, teachers should recognise variance between children and of more ‘learned’ people like adults, for instance. A claim like “a preschool child’s conceptual understanding differs from an adult’s” seems highly obvious but thinking about why there is difference, has significant implications for teaching. Carey (1988) argues that developmental psychologists (and I would add, teachers) must know what the initial state of learning (core knowledge) is and how more developed systems, which transcend the capacity of what was previously available, are created. Furthermore, teachers should define the process of change (which is supported and managed by an individual’s executive control) from their own experience and include a theory of change in their approach to teaching number concepts.

In addition, developmental neuroscientists argue that “a theory of education could only be derived from understanding the mind that is to be educated” (Premack & Premack, 2003:227). Dehaene (2011) argues that applications from cognitive neuroscience to education are imperative to the process of understanding how concepts originate and develop. Because great strides have been made over the last 20 years to understand how the human brain works, non-invasive techniques which measure brain function during cognitive tasks enable the scientific study of the human brain. According to Gazzaniga, Ivry and Mangun (2002), cognitive neuroscience is the study of how the brain enables the mind. Ansari, Coch and De Smedt (2011:40) define cognitive neuroscience as “an interdisciplinary science that draws on results from cognitive psychology, neuroscience, sociology, and anthropology to generate a better understanding of the brain bases of cognitive processes.” The goal of cognitive
neuroscience is to “constrain cognitive, psychological theories with neuroscientific data, thereby shaping such theories to be more biologically plausible” (Ansari et al., 2011:37).

Although applications of cognitive neuroscience to developmental psychology and education are not only possible, but important, Ansari et al. (2011) warn against unrealistic expectations from educationalists toward the application of neuroscience in the classroom, arguing that high expectations are unrealistic. According to Thomas (2013:24), no neuroscientific technique will be “magic bullet solutions to revolutionise education across the lifespan” but findings from studies in the field of cognitive neuroscience will “represent an accumulation of small effects that can combine to optimise learning” (Thomas, 2013:24). Even though cognitive neuroscience may only offer small influences on education and developmental psychology, joint contributions from neuroscience and developmental psychology may influence educational researchers’ understanding of conceptual development (Thomas, 2013).

One of the main applications of research in both developmental psychology and developmental neuroscience, is to show how to use predictors to identify children who might experience difficulties in developing number concepts and to identify what the specific reasons for those difficulties may be. In the context of South Africa’s poor results (or output) on national and international assessments (DBE, 2014; SAQMEC31, 2010; TIMMS, 2015), I would argue that an application of the ‘input theory’ to early identification of possible hindrances in number concept development is particularly relevant.

2.7.2. Early identification of possible mathematical difficulties

Various researchers have highlighted the importance of early identification of children who struggle to develop number concepts (Chinn, 2015; Ehlert & Fritz, 2013) and early intervention (Ehlert & Fritz, 2013; Mononen & Aunio, 2014). Research has been dedicated to determining the correlation between early number concept development and potential early markers for children at risk (Desoete, Ceulemans, De Weerdt & Pieters, 2012; Desoete, 2015; Purpura, Schmitt & Ganley, 2017). Stock, Desoete & Roeyers (2010) found that 87.50% of children at risk for mathematical learning

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difficulties can be detected based on their preschool abilities. To identify these difficulties at an early age, researchers such as Desoete (2015), Reeve and Gray (2015) suggest that teachers of young children, who have the responsibility to identify mathematical learning difficulties as soon as possible, should not only look for poor mathematical performance, but also for other contributing factors which serve as input for number concept development. Although these learners might need continuous additional support (Mononen & Aunio, 2014), early identification and intervention can play a critical role in helping these children achieve their full potential (Ehelert & Fritz, 2013). Reliable predictors of mathematical achievement can be targeted through focused preventive intervention programs aimed at children who might struggle with number concepts (Clements & Sarama, 2011; Ehelert & Fritz, 2013; Mononen & Aunio, 2016). These intervention programs not only have to focus on strengthening number concepts, but also on strengthening additional constituents, such as mathematics-specific vocabulary, classroom engagement and logical reasoning.

With regard to children who struggle to learn mathematics, Chinn (2015) argues that a clear definition of the acronym ‘MLD’ is required. The acronym can refer to ‘mathematical learning disabilities’ or ‘mathematical learning difficulties’. Changing the word ‘disability’ to ‘difficulty’ has significant implications. ‘Mathematical learning difficulties’ includes a wide range of difficulties which are closely related to other domain specific and -general factors. The term ‘difficulty’ also suggests that there is a spectrum or continuum of difficulties, which includes a disability to calculate, namely developmental dyscalculia. When I refer to MLD in this study I do not refer to dyscalculia alone, but to the broader spectrum of mathematical learning difficulties, which may be the result of poor input for learning.

Chinn (2015:1) writes in the introduction of an international handbook of dyscalculia and mathematical learning difficulties that compared to research on language difficulties, the field of MLD is still in its infancy. Yet, most researchers report a prevalence of 2-14%32 of dyscalculia (Desoete, 2015; Dowker, 2005). Bugden and Ansari (2015) define developmental dyscalculia as:

32 Others indicated that 5-7% (Butterworth et al., 2011) or 3-6% (Wilson & Dehaene, 2007) of children have dyscalculia which falls within the range of 2-14% but narrows it down.
A specific disorder that is characterized by impairments in learning basic arithmetic facts, processing numerical magnitude, and performing accurate and fluent calculations. These difficulties must be quantifiably below what is expected for the individual’s chronological age, and must not be caused by poor education or daily activities or by intellectual impairments (Bugden & Ansari, 2015:19).

Butterworth et al. (2011) suggest that developmental dyscalculia can be highly selective and that it might affect learners with normal intelligence and normal working memory, although it seems to co-occur with other developmental disorders. While dyscalculia has an estimated prevalence of about 5 to 7% (Butterworth et al., 2011), not all children have a disability. If the word disability is replaced with difficulty, the construct changes. The notion of mathematical learning difficulty does not imply a disability in mathematics. As I have argued, some children struggle because of poor classroom engagement, a language difficulty, poor reasoning skills or the effects of poor education (Dehaene, 2011). These can be defined as difficulties rather than disabilities. Others have a disability. As Chinn (2015) remarks: MLD is a spectrum and it often co-occurs with other difficulties such as poor classroom engagement, linguistic barriers or poor logical reasoning.

I argue that although the field of developmental neuroscience is still a young and developing field, teachers should have basic knowledge of how neuroscience can inform the teaching of number concepts, assessing number concepts and all contributing cognitive skills or provide an intervention. Dehaene (2011) claims that the classroom should be laboratories for research. Therefore, I argue that a basic understanding of developmental neuroscience can be of great value for prospective teachers and should therefore be included in teacher training.

2.7.3. Applying the ‘input analyzer’ theory to teacher education

The connection between education and cognitive neuroscience is not a one-way link where findings from neuroscience influence education. Instead, when thinking about how education can be ‘influenced’ by neuroscience, Ansari et al. (2011) call for a bi-directional collaboration between education and cognitive neuroscience. In this section I argue that collaboration can be established by including basic knowledge about the brain in both practicing- and prospective teachers’ pedagogical training.
Thomas (2013) uses the analogy of educational neuroscience and the history of medicine to define the impact of neuroscience on education. He acknowledges that such an analogy is not simplistic and might have its shortcomings, but concludes that the comparison between education and medicine will provide us with an understanding of how cognitive neuroscience will influence education. For the first 18 centuries medical practices relied on trial and error and the application of ‘folk’ theories of biology (Thomas, 2013). Without a doubt, these treatments included successful, but also harmful treatments. Slowly, natural scientists and anatomists began to understand how organs functioned and what the purpose of different organs are. Thanks to the contributions of science to medicine, the modern world has a multidisciplinary understanding of the human body. Doctors are trained to diagnose and treat many conditions. Medicine has become part of our daily lives.

Such an analogy proposes that research and practice in education can possibly be influenced by findings in the field of developmental neuroscience. Thomas (2013) explains:

The analogy between educational neuroscience and medicine proposes that the teachers of today are the healers of yesteryear, with current teaching shaped by an accumulation of cultural knowledge of what practices seem to work; and the neuroscientists of today are the natural scientists and anatomists of the 19th century. In the future, the analogy implies, teaching will have been transformed and underpinned by a foundation of scientific understanding on the nature of biological learning mechanisms (Thomas (2013:24).

2.7.3.1. What teachers should know about numbers and the brain

When arguing for the inclusion of basic cognitive neuroscience in teacher training, I do not claim that teachers should be experts in the field of neurology. However, I believe that teachers should be aware of findings from cognitive neuroscience due to such findings possibly influencing their pedagogy and their understanding of ‘input analyzers’. I begin this section with a summary of what teachers should know about developmental neuroscience.

Figure 2.11 shows the interaction between education, behavior, cognition and biology. The diagram suggests that genetics and core knowledge play a role in
mathematical abilities. On a biological level, different areas of the brain are activated during mathematical tasks, depending on the nature of the task. For example, when tasks involve the acquisition of new written mathematical skills, there is a clear collaboration between the frontal lobe (which is responsible for executive functions and working memory), the parietal lobe (including the intraparietal sulcus [IPS] and angular gyrus) and the occipital temporal area (which is responsible for processing symbolic formats).

On a cognitive level, such a task requires the representation and manipulation of numbers which are associated with spatial abilities and involves number symbols, fact retrieval and knowledge of facts, principles and procedures. Cognitive activity during this task results in behavioural arithmetical tasks which might involve simple or more complex arithmetical tasks. Neuroscience has implications for education on all three levels, namely biologically, cognitively and behaviorally.

Figure 2.12: The interconnections between behavioral-, cognitive- and biological role-players in an educational context (Butterworth et al., 2011:1048)
Results from various experiments confirm that the parietal lobes, specifically the IPS, are involved during number processing. Whenever a person attends to a number, whether during a complex or simple number task, IPS activates in neuroimaging. The IPS, for instance activates when a person is asked to think of a specific number, but does not activate when a person has to think about a letter or a specific patch of color (Dehaene, 2011: 239). It seems as if no person can think of numbers without activating the IPS. It is thus clear that the IPS is involved when new arithmetical facts are learned (Butterworth et al., 2011), when magnitude or symbolic numbers are represented (Dehaene, 2011), during estimation (Dehaene, 2011) and mental maths (Gelman & Butterworth, 2005).

During more complex tasks there is an interaction between the IPS and the left angular gyrus. The horizontal segment of the IPS posterior superior parietal sulcus are involved in estimation and approximation. Because language is also involved in exact calculation tasks, complex tasks involving exact calculation also activate the left angular gyrus. Dehaene (2011) explains the involvement of the different areas:

Some arithmetic calculations require specific thinking about quantities, while others only require rote memory of arithmetic facts. For instance, most of us have a stored ‘mental table’ of multiplication facts, but somehow have to compute the answer for a subtraction of two digits, because we do not know the answer by heart. Even within the same operation, such as addition, we can adopt one of two attitudes: either try to retrieve the result from verbal memory, or try to compute it by manipulating quantities (Dehaene, 2011: 241).

I opt for including such basic knowledge of the content of neuroscience in both practicing- and prospective teachers’ pedagogical knowledge. If teachers know that the angular gyrus is involved during fact retrieval and that the horizontal IPS is involved in arithmetical tasks (Butterworth et al., 2011), they will understand that successful retrieval of arithmetical facts, such as multiplication or number bonds do not necessarily imply conceptual understanding, but rather the recall of previously learnt facts. It might rather imply that the semantic- or lexical memory is at work (Dehaene, 2009). If teachers are aware that young children’s prefrontal areas (which are associated with executive functions and working memory) and medial temporal areas
(associated with declarative memory) are activated during ‘routine numerical activity’ (Butterworth et al., 2011) they will naturally strengthen children’s executive functions, especially working memory, to enable children to thrive in mathematics.

Comprehending the dynamic neural organization of arithmetic will remind teachers that the focus of mathematical activities shifts during the process of learning (Butterworth et al., 2011). Knowing that older children use previously learned mathematical facts and working memory (which implicates the left angular gyrus) to do single digit addition or subtraction, while younger children still need to calculate each addition or subtraction sum by depending on their IPS, will influence foundation phase teachers’ pedagogy. They will know that they should explicitly teach conceptual understanding, but that they should also support children in learning mathematical facts (such as number bonds and time tables) by rote. Being able to recall time tables allows the working memory to concentrate on more complex calculations rather than focusing on determining an answer which the child should know as fact to simply recall.

Teachers must know that although numerical processes involve the parietal lobe (which is different to the classical language areas), the left angular gyrus is also involved during exact calculation tasks (Gelman & Butterworth, 2005). Knowing this might influence teachers’ understanding of the role of language during mathematical learning. They might also consider that there is a relationship between the value of a numeral and the physical size of the numeral in print or on screen (Dehaene & Brannon, 2011). We use distance to describe the difference between numbers by saying that, “these numbers are close to each other or far from each other.” Time is also connected to numbers. We can say, “It takes longer to move from two to eight, but not as long to move from two to three.”

An example of successful collaboration between neuroscience and education is the interaction between executive functions, brain studies and a teaching approach, namely the Montessori methodology. According to Diamond (2013) it is evident in executive functions that social-, emotional- and physical- health is crucial for cognitive development. This relationship is evident in the prefrontal cortex. Diamond (2013:6) claims that the “prefrontal cortex and executive functions are the first to suffer, and suffer disproportionately, if you are lonely, sad, stressed, or not physically fit. Lonely people do not reason as well and their prefrontal cortex works less efficiently.” Thus,
developmental neuroscience proves that there is a relationship between cognitive skills, such as number concept development, reasoning and language and executive functions and that this relationship is also evident in our behavior – which manifests in classroom engagement.

When Lillard and Else-Quest (2006) evaluated Montessori education, they found that five-year old ‘Montessori children’ performed much better on executive function assessments than a control group not attending Montessori programs. Their results also indicated academic results for the ‘Montessori children’. The reason could possibly be that Montessori education has a different structure than traditional education. This type of education is characterized by multi-age classrooms, the absence of tests, instruction in both academic and social skills on individual- and small group level and a special set of materials.

Another example of collaboration between developmental neuroscience, developmental psychology and education, is the Tools of the mind program. The Tools of the mind program was designed based on Vygotsky’s insights into the development of executive functions (Diamond, Barentt, Thomas & Munro, 2007). The program, which has been refined over a period of 12 years, consists of 40 activities that aims to develop preschool children’s executive control to promote effective learning and achievement. The program can also be used in professional teacher development programs by training teachers to use continuous assessments, scaffold learning and to use individualized instruction (Bodrova & Leong, 2007). Diamond et al. (2007) found that the more challenging the executive functions program is, the more robust the influence on academic achievement. Thus, strengthening executive functions has implications for number concept development and other skills, such as reasoning and language. Neither the Montessori nor Tools of the mind programs were designed to boost executive functions, but neuroscience is now helping us understand why these programs are so effective for building young children’s brains.

2.8. Summation

Levine and Baillargeon (2016:127) argue that “the amount and quality of … input a child receives that is relevant to a concept may affect the timing and even the nature of its influence on this concept.” In this chapter I intertwined theories from developmental psychology, sociocultural theories of concept development and
developmental neuroscience to build the argument that early number concept development, mathematics-specific vocabulary, executive functions (manifested in classroom engagement) and logical reasoning are cognitive input for number concept development. These contributing constituents are analyzed by an innate input analyzer. I have argued that, together with core knowledge, this kind of input affects the timing and nature of number concept development – as Levine and Baillargeon (2016) propose. Also, I suggested that a theory about input analyzers can contribute to pedagogy in the foundation phase that grounds general teaching, assessment of number concepts and special support for children with possible MLD.

While this chapter assessed possible associations between number concept development, mathematics-specific vocabulary, classroom engagement and logical reasoning from a theoretical perspective, in Chapter 3 I describe the design of the study that allowed me to investigate possible associations between these constituents from an empirical perspective.
CHAPTER 3:
RESEARCH DESIGN

3.1. Introduction

This chapter outlines how I used research knowledge as well as practitioner knowledge and experience to design a study. I wished to better understand the possible correlations between young children's number concept development, mathematics-specific vocabulary, classroom engagement and logical reasoning. Throughout the study I argue that deficits in these skills, as assessed at the beginning of Grade R, are indicative of children’s numerical competence in Grade 1. Furthermore, I proposed that a combination of instruments, measuring number concept development achievement, mathematics-specific vocabulary, classroom engagement and logical reasoning will be effective in detecting possible MLD. In this chapter I set out how measures of distinct cognitive skills can be used to jointly predict numerical competence in Grade 1.

The study was not designed to assess causal associations, but rather to describe concurrent and predictive associations between achievement on number concept development tests (in Grade R and Grade 1) and 1) mathematics-specific vocabulary, 2) classroom engagement and 3) logical reasoning (as tested in Grade R). The objective of the study was to model possible concurrent relations between Grade R children’s MSV, CE and LR with their achievement on a Grade R NCD test; and possible predictive relations between their Grade R cognitive skills (NCD, MSV, CE and LR) and achievement on a NCD test in Grade 1 (tested in home language and English). My aim was to, ultimately, see how such assessments could inform teachers’ pedagogy.

The chapter begins with the study design, which integrates quantitative- and qualitative data to address the research question. Next, I describe how children were selected for the quantitative data collection component of the study and how the teachers were selected for interviews. Then follows a description of the instruments that were used to assess the child’s cognitive skills and the ideas of practicing teachers for the quantitative and qualitative components of this study, respectively. Section
3.4.5 foregrounds the advantages of discursively oriented semi-structured interviews. Next, I describe how data was collected. I also discuss the analytical strategy by showing how qualitative data enriched the findings of the statistical analysis.

### 3.2. Research design

The study was designed to integrate the perspectives of the researcher and practicing teachers. This integrated qualitative-quantitative approach is influenced by what Maree (2009) suggests as a ‘triangulation’ design, when both quantitative- and qualitative data are collected at the same time and integrated to best understand the phenomenon at hand. In contemporary research methodology discourse the design can also be labelled a ‘mixed methods’ design (Collins, Onwuegbuzie & Sutton, 2006; Ivankova, Creswell & Stick, 2006; Johnson & Onwuegbuzie, 2004; Onwuegbuzie & Teddlie, 2003; Tashakkori & Teddlie, 2010; Yin, 2006).

The quantitative methods provided for a mathematically objective way of testing the hypothesis (Johnson & Onwuegbuzie, 2004). Data obtained from the qualitative methods gave me insight into the teachers’ perspective as practitioners. In this design, complementary data is collected to bring together the strengths of both qualitative- and quantitative data. Although some quantitative purists (Ayer, 1959; Maxwell & Delaney, 2004; Popper, 1959; Schrag, 1992) and qualitative purists (Denzin & Lincoln, 2002; Nagel, 1986) may disagree, I agree with Johnson and Onwuegbuzie (2004:21) that by integrating quantitative- and qualitative data, a researcher can “provide stronger evidence for a conclusion through convergence and corroboration of findings”. Figure 3.1 provides a visual representation of the notion of concurrent triangulation, where the researcher integrates perspectives of different modalities of data.

![Figure 3.1: Triangulation of qualitative- and quantitative data (adapted from Maree, 2009:267)](image)

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Quantitative data collection and analysis from test scores

Qualitative data collection and analysis from interviews

Researcher integrate and compare quantitative- and qualitative data

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However, I use a design that differs slightly from a concurrent triangulation design (www.cirt.gcu.edu/research/developmentresources), where all the quantitative- and qualitative data are collected at the same time. In this study’s design the quantitative data was collected in two separate phases and the second phase is divided into two strands as shown in Figure 3.2.

The design type of the quantitative data collection be described as an ex post facto design (Simon & Goes, 2013), while the qualitative data was collected by means of discursively oriented semi-structured interviews. Although this is not a design commonly used, it allows the researcher to draw conclusions about young children’s development in a somewhat controlled way. The advantage of this design is that it makes it possible for the researcher to measure the dependent variable, namely the numerical competence attainment in Grade 1, in two different languages and to compare the two sets of data. By utilizing an ex post facto design, I was able to test the impact of Grade R skills as they occur in a natural environment, and the performance on the NCD test at two different time points. There was no specifically structured intervention program as used in a classical true experimental design. The development and learning over one year was the naturalistic ‘intervention’.

Ex post facto design
Quantitative data collection_1:
Independent variables
(NCD, MSV, CE & LR)

Quantitative data collection_2:
Dependent variable (NCD) measured in
• English and
• Home language

Quantitative analysis

Discursively orientated semi structured interviews
Qualitative data collection and analysis from interviews

Researcher integration of quantitative- and qualitative data

Figure 3.2: Integration of ex post facto design and interviews
Ivankova et al. (2006) suggest a graphic model to visualize the sequence of events during data collection, analysis and interpretation, especially when quantitative- and qualitative methods are combined. They suggest that such a visual model should mention the phases of data handling, list the procedures used during each phase and indicate what the outcomes of each phase are.

Figure 3.3 displays various phases of the research (what I did), shows the procedures I followed during data collection, analyses and interpretation (how I did it) and what the outcomes of each phase were (why I did it) (Kumar, 2005). I planned the process to follow a logical design: First, based on the literature review and expert advice, I selected the MARKO-D SA, the CFT and the classroom engagement report as instruments. I also developed the MMLT for mathematics-specific vocabulary because there was no test available to assess these skills (see Section 3.4.2). Next, I tested the 2017 Grade R cohort’s achievement on NCD, MSV, CE and LR at the beginning of 2017. I also designed the interview questions (see Section 3.4.5) to conduct the interviews in 2017. In the beginning of 2018, I assessed NCD of the Grade 1 children (the 2017 Grade R cohort) in two languages: their home language (isiZulu or Sesotho) and English. Next, I analyzed the quantitative- and qualitative data. Lastly, I interpreted the results by integrating the data sets.
Figure 3.3: Integrating quantitative- and qualitative data
During the planning I regarded the Grade R school year as the ‘naturalistic intervention’. No designed ‘intervention program’ was evaluated in this study. Rather, children’s natural development and their learning were seen as the ‘intervention’. In planning the ex post facto design, I kept in mind that the ‘intervention’ would simply comprise what happened in the children’s daily lives, specifically in their school education.

3.2.1. Ex post facto design for quantitative data collection

In an ex post facto study, or ‘after-the-fact’ research, the investigation focuses on what is observed after the fact has occurred (Grade R learning). Although an ex post facto design cannot be regarded as a true experiment, or a design experiment, it has some of the basic logic of inquiry (Simon & Goes, 2013).

Instead of taking groups that are equivalent and subjecting them to different treatments to determine differences in the dependent variables, an ex post facto experiment begins with groups that are already different in some respect and searches in retrospect for factors that brought about those differences (Simon & Goes, 2013:1).

A field ‘experiment’, as opposed to controlled experiments, takes place in the participants’ everyday lives, which allows for high ecological validity. By studying a phenomenon that occurs naturally, the conclusions of the research can be applied in the natural setting of the classroom. Studying the development of early number concepts without formal (outside) intervention allows the researcher to better understand how children’s number concepts develop in everyday life or in the ‘real world’.

The ‘independent variables’ of the study are mathematics-specific vocabulary, classroom engagement and logical reasoning. Grade R is the everyday context of the ex post facto study. The dependent variable of the study is number concept attainment as measured by the MARKO-D SA. Grade 1 children’s number concept development was assessed in English and in their home language in Grade 1 and only in their home language in Grade R. The study aimed to measure how learning and developing of mathematics-specific vocabulary, adeptness in classroom engagement and logical reasoning ability in Grade R were concurrently and prospectively associated with
number concept development during the year of schooling. The hypothesis is that mathematics-specific vocabulary, classroom engagement and logical reasoning will positively contribute to number concept development in Grade R and Grade 1.

3.2.2. Teachers’ views about children’s mathematical development

To find out what teachers’ observations are about how different cognitive skills might contribute to mathematical concept development, as well as their views on other aspects of teaching and learning (and thus to answer the question, “What are current teachers’ perception on the importance of cognitive input on number concept development”), four interviews were conducted. The four interviewees were with two Grade R teachers and two Grade 1 teachers. These participants were selected purposefully as they are the teachers of the children who were tested. Henning et al. (2005:50) argue that “the individual’s perspective is an important part of the fabric of society and of our joint knowledge of social processes and of the human condition”. The authors claim that the power of interviewing is situated in the minds of the person on the street – or in this case in the mind of the teachers in the classrooms. By interviewing more than one subject, one can find shared views between the teachers – for intersubjectivity. Henning et al. say:

This is the main aim of interview data – to bring to our attention what individuals think, feel and do and what they have to say about it in an interview, giving us their subjective reality in a “formatted” discussion, which is guided by and managed by an interviewer and later integrated into a research report (Henning et al., 2005:52).

My experience as a teacher was a benefit for this part of the research. I planned an interview protocol as described by authors in the volume by Gubrium and Holstein (2002). I decided on the genre of a semi-structured interview with specific questions that would address 1) influences on NCD; 2) Grade R NCD in preparation for Grade 1; 3) teaching school characteristics; 4) diagnostic instrument use; 5) language for mathematics learning; and 6) executive functions. The idea is not to use the interview as a data eliciting process, but to view the interview in itself as a data making process (Henning et al., 2005). Gubrium and Holstein (2002) say that an interviewer should not only listen to the content of the interview and summarize it by coding and classifying the content, but also pay attention to the evolvement of the interview itself and make
notes about the tone and the non-verbal cues. This means that the interview is seen as a discursive event with its own discourse. During the analysis of a discursively oriented interview, one has to search for a specific speech genre or sequencing of information to see how interviewees communicate their thoughts (Henning et al., 2005). The talk itself becomes a social action and data-making process during the coding and categorizing of the data.

In the Figure 3.4 I present my understanding of a discursively oriented semi-structured interview, as discussed by Henning, Van Rensburg and Smit (2004). Firstly, both the interviewer and the interviewee have a unique representation of reality, a world view or a filter which enables them to make sense of the world. Secondly, the interview process, in itself, is a data-making process and therefore the communicative relationship between interviewer and interviewee is important. The content and the language, or overall discourse, is equally important. Lastly, both interviewer and interviewee are situated in very specific social and cultural environment, which influences the data.

Figure 3.4: The interview as a data creating event

Henning et al. (2004, 2005) describe an interview as the togetherness of mindful practice. They remind us that the significance of interviewing is logged in the reading of social life. Because the interviewee not only expresses her own thoughts, but also the identity of a social and cultural group, the interviewer should be cognizant of the culture and discourse of the interviewee. In the instance of the interviews of this study, the teachers’ world including their daily practice, the social context of the school had to be taken into account. More than these factors, though, I had to take into account that, even though the teachers knew the children quite well, they inevitably were also biased, due to their many years of experience (except for one student teacher) and
may have had stereotypical views of children about whom they have to regularly write reports and document assessments.

### 3.2.3. Integrating quantitative- and qualitative data

Toward the end of the fourth chapter I integrate the findings by showing how qualitative findings can be used to amplify quantitative findings and by adding a researcher’s perspective to the findings. Such an integration takes place through an inductive process and requires a single researcher to know both sets of data well. I began this process by reading through the transcripts of the qualitative data again. Then, I listed all the quantitative findings and wrote down how each quantitative finding could be explained or supported by either a code, category, theme or even a direct quote from the qualitative data. Lastly, I indicated how I interpreted the findings from my own perspective. All the integrated findings are discussed in the fifth chapter.

### 3.3. Participants

The participants were selected through purposive intact group selection, which is a non-probability sampling method with which the researcher selects participants based on specific criteria for the study. The researcher’s knowledge of the study’s topic and the development of theory in the field of education, determine the criteria (Henning et al., 2005:71). An intact group is an already formed or naturally constituted group, such as a whole class or cohort.

Purposeful sampling was beneficial for a few reasons. Firstly, I wanted to find out how numerical concept development is possibly influenced by language when the language of instruction changes from home language to English – which is the case at Funda UJabule School. Secondly, I wanted to do research in a community that aims to improve the relationship between research and practice, the collaboration between the University of Johannesburg and Funda UJabule makes that possible. Lastly, the school is also a research school and all parties – teachers, parents, students and children – have given consent (or assent) to take part in research when they enrolled or started working at the school or university.

The relationship between the school, the provincial- and the national education department, as well as the university, allows educational scientists to do research in the ‘real world’ (Snow, 2015). The teacher training program provides opportunities for
novice teachers to practice what they learn theoretically. Practicing teachers are provided with extensive opportunities to enrich their PCK with contemporary theory through teacher training programs. Practicing teachers often contribute to ‘knowledge making’ by engaging in various research projects. Focusing on the research school allowed me to better understand which skills are the most important in contributing to disadvantaged pre-schoolers’ math achievement.

For the quantitative data collection, the Grade R- cohorts of 2017 took part in the research. The same children were tested in Grade 1 in 2018. 56% of the children were male, 48% were isiZulu speaking. The average age of participating children was 6 years and 2 months in Grade R and 7 years and 2 months in Grade 1. Although there were 65 children in Grade R in 2017, six were retained to repeat the year. In other words, six children who were tested in Grade 1 did not have Grade R scores. The sample of children who attended Grade R in 2017 and Grade 1 in 2018 totalled 59. Eight children had also been retained in Grade 1 at the end of 2017. Hence, there were 67 children in Grade 1 in 2018. The results of children who did not attend Grade R in 2017 or Grade 1 in 2018 were used to impute data for a multiple regression analysis.

For the qualitative data analysis, the two Grade R- and two Grade 1 class teachers participated in semi-structured interviews. One of the Grade 1 teachers was on maternity leave and a B.Ed. Honours student replaced her for the first term in 2018. The ‘student locum teacher’ thus had no experience, but had excellent theoretical knowledge which was evident during her interview. Her mother tongue is isiZulu, but she is also fluent in English. She obtained her B.Ed. degree at the University of Johannesburg. The second Grade 1 teacher had 18 years’ teaching experience and has a Junior Primary Teaching Diploma (JPTD). Her mother tongue is Sesotho and she is also fluent in English. Because of her experience she provided rich examples during her interview. The Grade R teachers respectively had eight and nine years teaching experience. Both of them have an ECD Level 5 qualification and are fluent in English and their home language (isiZulu or Sesotho).

3.3.1. Missing data

Data was missing due to six children who were retained in Grade R and eight children who were retained in Grade 1. Children who were retained in Grade R only had Grade
R scores, while retainers in Grade 1 only had Grade 1 scores. Multiple imputations were conducted to reduce bias that could be introduced through sample attrition (Cummings, 2013). Imputed data was created using SPSS\(^{33}\) statistical software, which uses an algorithm to estimate data through approximation and maximization. Based on conditional distributions of this study’s variables, which were estimated from available data, the algorithm created six imputed files which were pooled in a summary measure to create imputed values for missing data (Schafer, 1999). This statistical technique allowed me to analyze a data set with incomplete data by ‘filling in’ the missing values for the 14 children who weren’t tested in Grade R or Grade 1.

### 3.4. Measures

Prior to the selection of assessment instruments, I did a literature study from which it was clear that early number concepts do not develop independently, but that the development is in some way connected to other cognitive skills (Punch, 2005). From the literature study, combined with my experience as a teacher, I identified mathematics-specific vocabulary, classroom engagement (as manifestation of executive functions) and logical reasoning as important contributing cognitive skills. Before selecting test instruments, I enquired about different test instruments used to assess young children’s early number concept understanding, mathematics-specific vocabulary, classroom engagement and logical reasoning.

I had access to various iterations of the DBE’s annual national assessment of mathematics report documents. The ANA tests were designed “to determine learner performance with regard to the skills and knowledge that they have acquired as a result of teaching and learning experiences in school” (DBE, 2013:7); but Spaull and Kotze (2015) found that the ANA tests were not psychometrically calibrated, which means that the results cannot be compared across time or between grades. Van den Berg (2015) argues that the ANA tests could indicate the socioeconomic learning gaps in South Africa but cannot be used to provide information about how children’s concepts developed over time. Two curriculum-based mathematics tests, namely the VASSI Maths proficiency test (Vassiliou, 2003) and WIAT-II (Wechsler, 2003), could also be considered to test children’s mathematic skills. I decided to use the

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\(^{33}\) Statistical Package for the Social Sciences
MARKO-D SA (Henning et al., 2018) because it is a local test and it is available in the three languages relevant to the study.

In terms of mathematics-specific vocabulary, the WIAT-II and a subsection of the Junior South African Individual Scale (JSAIS) tests include some mathematics-related language. I considered these tests to assess children’s mathematics, but since the test items are spread out across the assessments and no subsection of the instruments assess mathematics-specific vocabulary per se, I did not use these instruments. I concluded that there was no instrument designed to specifically test children’s mathematics-specific vocabulary and therefore I designed the MMLT, which could assess which vocabulary a child knows and which words are yet to be learnt.

To test classroom engagement, only one instrument exists, namely the classroom engagement scale (Fitzpatrick & Pagani, 2013). Because executive functions are closely related to classroom engagement, I considered testing children’s executive functions. The only test which is currently being used in South Africa to assess executive functions, is the Missing Scan Task, which assesses children’s working memory. Several animals are shown to a child after which one of the animals is removed. The child must name the missing animal. However, this test does not assess executive functions, but rather focuses on one specific aspect, namely working memory. For this specific study, I wanted to determine the relationship between number concept development and executive functions (as a whole) as it manifests in classroom engagement. The classroom engagement report reflects children’s executive functions in a natural setting, rather than a laboratory environment. In Section 2.6.1 I discussed the association between executive functions and classroom engagement.

To test children’s logical reasoning, I considered a subsection of the JSAIS and the ‘Matrix Reasoning’ subtest in the Wechsler Intelligence Scale for Children Fourth Edition (WISC-IV). However, the JSAIS was not designed to test logical reasoning per se and the WISC test takes approximately 30 to 90 minutes to administer and the instructions are presented orally. I reasoned that oral language communication barriers may influence assessments and that the test may not yield a true score of the children’s ability to reason logically. Therefore, I chose Sections 3, 4 and 5 of the CFT,
which assesses children’s ability to identify similarities, to ‘complete the row’ and to classify objects.

3.4.1. A diagnostic test for numeracy competence in early childhood

The MARKO-D SA test is a diagnostic instrument which assesses 4 – 8 year-old children’s early number concepts up to the number 10. The test, consisting of 48 items, was originally designed in Germany and was translated, adapted, standardized and normed in South Africa. The translation of a test has often been (mistakenly) viewed primarily as a linguistic issue (Meiring, van der Vijver & Rothmann, 2006). Although much of the meaning of a language lies in the syntax and sentence construction (Chomsky, 1965), the translation of a test also includes non-linguistic issues such as cultural-, functional- and metric considerations (Peña, 2007). Therefore, a multidisciplinary team of experts worked together to ensure linguistic-, cultural-, functional- and metric equivalence as described by Peña (2007).

Linguistic equivalence ensures that each item of the test is semantically equivalent to the original test (Grisay, 2003). When the MARKO-D SA was translated to English, Afrikaans, isiZulu and Sesotho I experienced the headaches of a translation firsthand in the Afrikaans translation (Bezuidenhout, 2018; De Villiers, 2015). During the first round of translation, the instrument was translated from German to English by a teacher at a German school in Johannesburg, who was fluent in both English and German. One of the South African test developers were also fluent in German, English and Afrikaans and completed a back translation to compare the English translation with the original German test. After an initial pilot (n=224) the test results and tester feedback were used to adapt the English test. The adoptions mostly consisted of adoptions to the script to change question initiation and items to a less formal language (the German language uses more formalized ways to address children and ask questions).

The new version of the English test was used to translate the test to Afrikaans, isiZulu and Sesotho. To test the functional- and metric equivalence of the translated MARKO-D SA tests, pilot studies were conducted in the various languages (de Villiers, 2015; Fritz et al., 2013). Functional equivalence ensures that the same construct is tested in the various languages, while metric equivalence refers to equivalence in the difficulty of each item (Peña, 2007). After many rounds of translation and back
A Rasch analysis of the Afrikaans and English tests showed that the items, individually, and the test as a whole, were functionally and metrically equivalent (Bezuidenhout, 2018), but the isiZulu and Sesotho tests showed discrepancies in linguistic equivalence (Henning et al., 2018). Insight from various teachers, linguists, and test administrators, who were native speakers of isiZulu and Sesotho and an assessment expert revealed that the isiZulu and Sesotho tests did not represent the colloquial version of isiZulu and Sesotho. After five iterations and pilot studies, the South African team developed a ‘user-friendly’ test in a familiar register of isiZulu and Sesotho.

Cultural considerations were also taken into account to ensure that children interpret the underlying meaning of the various items similarly across all languages (Canino & Guarnaccia 1997; Hendrickson, 2003). The German test included illustrations of squirrels who played with nuts and berries by preparing for the winter snow. During test administration, children become part of a conversation between animals and provide answers to some of the characters’ questions. However, this particular cultural setting was unfamiliar to the South African children. Therefore, a local art teacher drew meerkats in a South African setting and a new storyline, with meerkats, rabbits and a fox, was incorporated to the MARKO-D SA.

A theoretical model for number concept development, which had been empirically validated, informed the development of the test in Germany (see Section 2.5). It shows how children’s numerical knowledge develops hierarchically (Fritz et al., 2013). The idea of developing a theoretical hierarchical model is to deconstruct a complex skill, like number concept development, into smaller components, to arrange the components in a specific order on a continuum, and to develop specific items which test each skill component. For each level, the test developers compiled an item pool which assesses the specific level’s concepts. Many items were designed to test the concepts of each level and during several pilot studies the best items for each level were selected by means of a Rasch analysis. Lastly, the model was psychometrically assessed by means of Rasch modeling (Rasch, 1960) to determine how well the items and component skills represented by the items build on each other (Fritz et al., 2012). The interview format of the test, which is conducted with individuals, requires experienced test administrators.
3.4.2. A custom-designed vocabulary test

The MMLT is custom-designed measure of children’s early mathematical content language and forms part of a range of materials which aim to describe how young children learn mathematics. Language plays a vital role in the understanding and learning of mathematics. Basic concepts such as classification, differentiation, identifying patterns, serialization, comparison and one-to-one correspondence are prerequisites to numerical learning but are all dependent on language (Fritz, 2016). To understand these concepts, a child needs specific vocabulary (and syntactical sensitivity) to comprehend oral communication about mathematics, specifically numeracy in the instance of this research. If a child performs poorly on the MARKO-D SA test one could assume that the child has not yet acquired the skills needed to accomplish such tasks. However, if a child does not understand the language in which mathematical tasks are formulated, language might itself be the barrier to responding to the questions.

This measure consists of 24 items, assessing numerical language qualifiers (more, many, just as many, fewer, few), comparative language (same size, bigger, tallest, biggest, big/large, tall, shortest, small, smaller, short) and spatial language (in between, first, last, on top of, behind, above, under, after, in front of). Each item required the children to point to the picture describing the concept. None of the items required exact quantitative knowledge, but only an understanding of the word which represents a certain concept. Testing reliability and validity of the test is discussed in Chapter 4.

The language test forms part of a larger set of instruments with a meerkat theme. The instruments include the MARKO-D SA and an early number concept development program, namely the Meerkat Maths (MM) intervention program for teachers in the same school. I decided to develop a language test with the current theme – which explains the use of Meerkat Maths Language Test (MMLT). The pictures used in the MMLT were sketched by the same artist who drew the sketches for the MARKO-D SA and the MM intervention program.

Because the MMLT was designed to test the vocabulary presented in the MARKO-D SA and MM intervention program (adapted from the German Calculia program [Fritz-Stratmann, Ehlert & Klüsener, 2014]), the vocabulary of the MMLT
The idea is that children must know the vocabulary for them to understand the questions and instructions given in the MARKO-D SA. First, one must determine whether the children understand the language that is used to assess their number concepts. Do they know the vocabulary needed to answer the question? This is the foundational question underlying the design of the MMLT. If the test indicates that children do not understand the language, it could mean that poor performance on the MARKO-D SA is not only due to a lack of mathematical knowledge, but also because they don’t understand the words and sentences in the question itself. Table 3.1 provides examples of how the MMLT relates to the concepts in the MARKO-D SA and MM intervention program:

Table 3.1: Parallels between the MMLT vocabulary and concepts in the MARKO-D SA and Meerkat Maths intervention program

<table>
<thead>
<tr>
<th>Concepts in MM intervention program</th>
<th>Vocabulary assessed in MMLT</th>
<th>Concepts assessed in MARKO-D SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classification and differentiation</td>
<td>Big</td>
<td>Classification (selecting items based on common characteristics) and differentiation (excluding items based on different characteristics) are prerequisites for the five levels of concepts the MARKO-D SA tests. These concepts are strengthened in the Calculia program.</td>
</tr>
<tr>
<td></td>
<td>Small</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tall</td>
<td></td>
</tr>
<tr>
<td>Patterns</td>
<td>More</td>
<td>Items 13 to 15 test the child’s ability to complete a pattern by either adding one stone or removing one stone from the number of stones in previous picture:</td>
</tr>
<tr>
<td></td>
<td>Less/Fewer</td>
<td>Item 13, 14 and 15: “How many counters have to go in the empty block? Put the correct number of counters in the empty block.”</td>
</tr>
<tr>
<td></td>
<td>Bigger</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Smaller</td>
<td>Item 34: “We can count backwards like this: 10 – 8 – 6. How does it go on?”</td>
</tr>
<tr>
<td></td>
<td>Before</td>
<td>Item 2: “What comes just before 5?”</td>
</tr>
<tr>
<td></td>
<td>After</td>
<td>Example item: “What comes just before 3?”</td>
</tr>
<tr>
<td></td>
<td>Between</td>
<td>Item 3: “What comes between 2 and 4?”</td>
</tr>
</tbody>
</table>
None of the 24 items in the MMLT require exact number knowledge. The questions were designed in such a way that children were not required to answer any questions by using oral language expressively, but they were required to only point to their selected response. This means that they need to understand the orally spoken request and show receptive language proficiency by their answer. One of the advantages of this test is that, although it is an individual test, it takes only two minutes to complete.

The following pictures provide examples of the test items. For each item the tester
asked the question orally (and therefore the respondent had to understand the instruction given through language receptively) and the child pointed to the picture he or she thought was correct.

Figure 3.5: Examples of MMLT items: “Point to picture with more bugs”; “Put your finger on the tallest animal.”

After the test had been designed in English, with the vocabulary in the MARKO-D SA and the MM intervention program as basis, one of the MARKO-D SA test designers checked each item to ensure that the vocabulary correlates with that of the MARKO-D SA. Then, the test had to be translated to isiZulu and Sesotho. The two testers who assisted with the data collection were isiZulu and Sesotho speakers and are also fluent in English. They translated each item from English to isiZulu and Sesotho. One of the South African MARKO-D SA team members, who is not only fluent in isiZulu, Sesotho and English, but also knows the MARKO-D SA well, translated the original English test separately from the two testers. The two versions of the isiZulu and Sesotho tests were compared, and the differences were discussed in order to find the best possible translation of each item. During data collection, all the Grade R children were tested in their home language, namely isiZulu and Sesotho.

3.4.2.1. Reliability of the MMLT

The MMLT is an instrument which was specifically developed for use during this study and has not been used before. Because it is a newly developed measure, I calculated internal reliability by conducting a split half reliability test and estimated Cronbach’s alpha. During the split half reliability test, correlations between odd numbered items’ averages and even numbered items’ averages were determined. The average for odd
items was .64 and for even items .85. The correlation between the two averages was .25 with .06 statistical significance. A reliability test yielded a Cronbach’s Alpha score of .39. This test did not yield internal validity, which could be expected since the usability of the test had not yet been piloted. The test correlated with the other instruments used in the study (See Table 4.2).

Chronbach’s alpha is only one measure of reliability and assesses the extent to which items in the test are similar to each other. Other measurements include test-retest which was not possible to examine in this study but which could be validated with future studies that include a follow-up assessment. Furthermore, the small alpha for this measure may also reflect the test being too short, or there not being enough items. Another possibility is that the test is composed of several dimensions, in which case alpha’s would be more meaningful if calculated separately for each dimension (see attached reference).

Although a poor Chronbach’s alpha does suggest less precision with the measurement tool, it is unlikely to have introduced spurious effects. Therefore, it is not likely that the associations observed are false, although they may be weakened by the high amount of measurement error indicated by the alpha. Despite these issues with reliability, the validity of this measure was supported by the correlations between this scale and other measures (classroom engagement and MARKO-D). In order to strengthen this measure as an eventual professional tool, it would be helpful conduct additional analyses on more diverse samples of children, perhaps using IRT”.

**Teacher’s inventory of classroom engagement**

The conventional approach to assess executive functions is to measure child performance in a controlled environment – typically by means of individual assessments (Obradović, Sulik, Finch & Tirado-Strayer, 2018). However, while such assessments may have their advantages, they do not inform us about children’s behavior outside of controlled environments (Fitzpatrick & Pagani, 2012; Fitzpatrick & Pagani, 2013). Classroom engagement, much like laboratory-based assessments of executive functions, has been shown to predict both academic and also social competence (Fitzpatrick & Pagani, 2012). These authors argue that classroom engagement reflects executive functions. For example, being able to follow instructions reflects working memory, being able to adjust to classroom routines
reflects flexibility, and playing and working cooperatively reflects inhibitory control. The classroom engagement report does not require an assessment of executive functions outside a child’s natural environment but describes children’s engagement in their natural setting. Therefore, I chose to use teacher ratings of EF-driven productive work behavior and self-regulated learning in the form of classroom engagement (Gioia, Isquith, Guy & Kenworthy, 2012). Teacher reports presented methodological advantages for this study. Foundation phase teachers had ample time to observe the children in their class in a variety of contexts over a long period of time.

The questionnaire which I utilized, comprised teacher responses to seven items that aimed to determine whether the child follows rules and instructions, follows directions, listens attentively, completes work on time, works autonomously, works and plays cooperatively with other children and works neatly and carefully. These items are: Would you say that the child 1) plays and works cooperatively with other children at the level appropriate for her/his age, 2) follows rules and instructions, 3) listens attentively, 4) follows directions, 5) completes work on time, 6) works independently, and 7) works neatly and carefully? Each item was rated between 1 (never) and 3 (always) for each child and a total score indicated the child’s general classroom engagement.

Cronbach’s Alpha for the classroom engagement indicated reliability of .83 for the current study, compared to a high .94 in a study in Canada (Fitzpatrick & Pagani, 2013). A slightly lower reliability could be expected in a small sample and could possibly include teacher effects since it is a subjective teacher observation. The classroom engagement report scores correlated with the other instruments used in this study (See Table 4.2).

3.4.3. The CFT test for logical reasoning

In the 1950s Raymond Cattell designed the CFT to test general intelligence and argued that it would be a ‘culture fair’ test. He distinguished between crystallized intelligence, which represents the knowledge acquired through experience (Cattell & Cattell, 1959) and fluid intelligence, which represents the biological ability to acquire knowledge and to solve problems, independent of instruction, practice or schooling (Cattell & Cattell, 1959). The aim of the CFT is to measure an individual’s fluid intelligence by reducing the influence of verbal fluency, culture and education as much
as possible. The test was designed by using only shapes and figures but no language. It tests abilities such as inductive and deductive reasoning and the ability to identify patterns and relationships.

The CFT 1-R is a revised version of the CFT which was adapted from Cattell’s CFT (Cattell & Cattell, 1959), which assesses fluid intelligence. The CFT1-R consists of six subtests\textsuperscript{34}. The first three subtests assess perceptual skills, attention, and visual motor processing speed and the last three subtests assess the child’s inductive reasoning skills by making use of tasks that require the child to identify the rules of relationship between elements and then use these rules to complete structure (Weiβ, & Osterland, 2013). Because the test is non-verbal and does not largely require pre-existing knowledge, the CFT1-R is considered to be culturally fair since the results can be considered as “independent from culturally-determined experiences” (http://www.en.practest.com.pl/cft1-r-cattell-culture-fair-intelligence-test-version-1) drawing the conclusion that the test could arguably be suitable for the participants in this study.

Only Subtests 3 (similarities), 4 (complete the rows) and 5 (classification) were used for this study. Each subset consists of 15 items. A total score out of 45 was used to describe children’s overall logical reasoning.

Subtest 3 tests children’s ability to identify similarities. In each question, children are presented with a picture with which they must compare five other pictures. Four out the five pictures are slightly different from the original picture, while one is the same. The children must select the picture which is similar to the original picture. In Subtest 4 children are presented with a sequence of three pictures. They must complete the sequence by selecting a picture that follows on the pictures presented to them. As with the ‘Matrix Reasoning’ subtest in the WISC-IV, this subtest accesses information processing, visual abstract and sequential reasoning (Wechsler, 2003). Subtest 5 is designed to assess a child’s categorical, abstract reasoning. In other words, how the child categorizes images and views the relationship between them. The child is presented with five pictures, four of which are similar in some way and must identify the odd one out. Subtest 5 is comparable with the ‘Picture Concepts’

\textsuperscript{34} The APA at some point posed questions about the ‘fairness’ of the test. These questions remain, and in the study I was cognizant of that. Coppard (under review) reviewed the instrument in her study with South African children.
subtest in the WISC-IV, which assesses abstract, categorical reasoning (Wechsler, 2003).

3.4.4. Discursively-oriented semi-structured interviews

In many disciplines, such as child development, cognition, mathematics or linguistics a ‘research to practice’- or ‘translational’ approach is used (Snow, 2015) where research carried out by discipline-based researchers is applied or translated to practice.

[A] disastrous drawback of the traditional basic/applied distinction was the unquestioned assumption that if the basic science was sound, the application process was simple, requiring only interpretation or translation. Thus, enormous energy was invested in applications (e.g., teaching foreign languages using contrastive analysis methods, withholding literacy and numeracy activities from early childhood classrooms) that emerged from basic science but that were themselves at worst severely flawed and at best in need of careful study and systematic evaluation (Snow, 2015:461).

As an alternative, Snow (2015) suggests a partnership between researcher and practicing teacher, addressing the urgent problems in practice as a starting point for research topics. As a practicing teacher and a researcher myself, I identified cognitive skills that could possibly relate to mathematical learning during early school years. But, I also wanted to include the perspectives and ideas of other practicing teachers and therefore I conducted the aforementioned interviews.

By structuring the questions in a specific sequence, I was able to capture rich data from each interview. To find out which skills and other factors in Grade R contribute to mathematical learning in Grade 1, I asked the teachers: “What influences a child’s ability to learn mathematics?”; and: “Describe what children need to learn in Grade R to prepare them for mathematics in Grade 1.” To characterize the research environment, I asked them to describe the teaching school and how school differs from other schools. As part of this study I wanted to determine whether there is a need for diagnostic instruments to identify children who might be at risk to struggle with early number concept development, but also if a need exists to identify contributing cognitive skills which might not necessarily be thought of as mathematics-specific skills, such
as executive functions, language and logical reasoning. For this purpose I asked whether they think there is a need for instruments that can identify children with possible learning difficulties and why they think so. Next, I turned to specific skills I identified as contributors for mathematical learning and asked: “What role does language play in learning mathematics”; and “How do executive functions influence number concept development in a Grade R or Grade 1 class?” Given their important contribution to engagement in the classroom, teachers were asked about their understanding of EF.

From experience, I know that not many teachers are familiar with the concept of executive functions. Therefore, I prepared cards which explain the three different executive functions (Figure 3.6) and gave it to the teachers during the interview so that they could refer to the cards, if needed. In the findings (Chapter 4) I will discuss that even with a short explanation of what the three different executive functions are, the teachers could not answer the question: “How do executive functions influence number concept development in a Grade R or Grade 1 class?”

**Figure 3.6: Explanatory cards about executive functions used during interviews**
3.5. Data collection

After identification and development of suitable instruments, I began to gather the data. The MARKO-D SA, MMLT, and CFT1-R was administered to the Grade R cohort at the beginning of 2017. The two Grade R class teachers completed the classroom engagement report later during 2017. Because the teacher report relied on the teachers’ knowledge of the children, the report was only completed in the middle of the year when the teachers had come to know the children. The MARKO-D SA test was administered to the same cohort in Grade 1 at the beginning of 2018. Together with two multilingual assistants, I administered the tests to the participating Grade R and Grade 1 children. I conducted interviews with the two Grade R and two Grade 1 class teachers during the middle of 2017 and transcribed the interviews myself within a couple of weeks after completing the interviews.

The MARKO-D SA and MMLT were administered individually during the mornings. Each child was accompanied from class by the tester and taken back to the class after having completed the tests. The two assessments took approximately 35 minutes. Close to midday, the children who had completed the MARKO-D SA and MMLT assessments during the morning came back for a group administration of the CFT1-R which was completed within approximately 30 minutes. Each child was out of the class for a total of just more than one hour.

3.5.1. Capturing MARKO-D SA data

Before administering the MARKO-D SA, the tester captured the descriptive information of each child on the recording sheet, which consists of the child’s name and surname, gender, date of birth, date of testing, home language, language of instruction, language of testing, test supervisor and the name of the institution. A personal identification number (TPNR) was assigned to each child according to their registration number at the school. The general conditions (e.g. whether there was an interruption) and further observations (e.g. anxiety, comprehension problems or restlessness) were indicated. The child’s response to each of the 48 questions was recorded on the recording sheet (example items’ recording sheet is shown in Appendix B). Because the data was handled as dichotomous data, the tester encircled a 1 for a correct response and a 0 for an incorrect response. There was also space for additional notes at each item. A total score was derived by adding all the correct answers together.
To administer the MARKO-D SA each tester used a booklet which integrates the instructions (on the side which faces the tester) and the pictures (on the side which faces the child). The tester simply flipped the pages to turn to the following question and presented the child with the next picture. In Figure 3.6, the photo on the left shows that the child could see a picture in which the meerkats were introduced. The tester could see the “script” which consists of the questions relating to the specific question. In the picture on the right, Jobo (one of the meerkats) and his mother are looking for food. While the child could see the picture and had to answer questions relating to the picture, the tester could see the questions she had to ask.

Figure 3.7: The booklet which was designed to stand up comprises of the pictures which faces the child and the instructions facing the test administrator

In addition to the picture booklet, the children were provided with ten red counters and ten blue counters which they used as manipulative objects for some items. Figure 3.8 shows an example of a question where the child had to complete the pattern and put the correct number of counters in the box (Item 15). This item tested the child’s understanding of the ordinality principle. In this example, the child put down four counters instead of two. This was a typical error and indicates that the child possibly didn’t understand the concept of ordinality yet. In the photo on the right, the child was asked to put down eight counters. There had to be two more blue ones than red ones (Item 42). This was the most difficult question on the test and required an understanding of relationality of numbers: that eight is the same as five and three and that five is two more than three. There should have been three red counters and five blue ones.
During the norming and standardization of the MARKO-D in South Africa, the MARKO-D items were matched with a specific level of difficulty. Difficult- and easy items were mixed in the test sequencing so that the children did not get frustrated with increasingly difficult items. Table 3.2 is a summary of the items according to the five levels described by the theoretical model.

Table 3.2: Various items according to the five levels of the conceptual model

<table>
<thead>
<tr>
<th>Item numbers and level of performance</th>
<th>Examples of item</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level 1: Counting</strong></td>
<td></td>
</tr>
<tr>
<td>1, 4, 6, 7, 17, 18, 19, 20, 22, 24, 37, 38, 45, 46</td>
<td>Count the stones! How many are there? Can you give me 4 counters?</td>
</tr>
<tr>
<td><strong>Level 2: Ordinality</strong></td>
<td></td>
</tr>
<tr>
<td>2, 3, 8, 9, 10, 12, 43</td>
<td>What comes just before 5? My sister has 2 apples. Grandmother gives her two more apples. How many apples does my sister have now?</td>
</tr>
<tr>
<td><strong>Level 3: Cardinality</strong></td>
<td></td>
</tr>
<tr>
<td>11, 13, 14, 15, 27, 33, 34, 44, 47, 48</td>
<td>[9, 11 …] Can you show me how to do this with the counters? How many counters have to go in the empty block? Put the correct number of counters in the empty block. 5 - 4 - [...] – 2</td>
</tr>
<tr>
<td><strong>Level 4: Part-part-whole</strong></td>
<td></td>
</tr>
<tr>
<td>26, 28, 39, 40, 41</td>
<td>Give me 9 apples, 4 of them must be blue! Give me 9 counters, 4 of them must be blue.</td>
</tr>
</tbody>
</table>
Give me 6 apples, there must be more blue ones than red ones! Give me 6 counters; there must be more blue ones than red ones.

**Level 5: Relationality**

We can count backwards like this: 10 – 8 – 6. How does it go on?

Give me 8 apples, there must be two more blue ones than red ones. Give me 8 counters. There must be 2 more blue ones than red ones.

### 3.5.2. Capturing MMLT data

Before administering the MMLT, the test administrator completed the personal information on the score sheet. For each item the tester showed a picture to the child and asked the child to point to the correct answer. Each page of the test booklet comprises a picture and the question. The child saw the picture and the tester read the question. The child simply pointed to the correct answer. On a score sheet (as shown in Appendix C), the tester used a ✓ to indicate a correct response and a ✗ to indicate an incorrect response. The total correct scores were written down at the bottom of the score sheet. The following photos show examples of the test items:

![Example MMLT items](image)

Figure 3.9: Examples of MMLT items: "Point to the picture with more bugs"; "Put your finger on the tallest animal."

Table 3.3 indicates the content of the 26 items.

Table 3.3: MMLT items
1. This is Jobo. Show me the meerkat just as big as Jobo.
2. Put your finger on the picture with more bugs.
3. In which picture is the meerkat between the rocks?
   Lona wants to eat the bug closest to her.
4. Show me which bug she will eat first.
5. Show me which bug she will eat last.
6. Point to the meerkat on top of the rock.
7. Point to the meerkat behind the rock.
8. Put your finger on the bigger meerkat.
9. Show me which bird is above the tree.
11. Jobo wants to eat the biggest bug. Which bug will he eat?
12. Show me the big monkey.
13. In which picture are there many bugs?
15. Show me the shortest animal in the row.
17. Which picture has fewer animals?
18. Which picture has just as many birds as bugs?
19. Show me the bird under the tree.
20. This is Jobo and his sister. Who is smaller?
21. Which picture has fewer meerkats?
22. Show me the short tree.
23. These animals are in a row. Put your finger on the animal in front of the zebra.
24. These animals want to sit under the tree with Lona. Which animal will get to the tree after the bird?
25. In which picture are there no meerkats?
26. In which picture does the bird have half as many bugs as the meerkat?
3.5.3. Capturing CFT1-R data

The CFT1-R was administered in groups no larger than four. Each child received a questionnaire comprising of three subtests (as shown in Appendix D). Each subtest consisted of 15 items. The name of the child and TPNR number were indicated on the front page. The rest of the personal information was the same as the MARKO-D SA and MMLT and was therefore already captured. Subtests 3, 4 and 5 were used to assess children’s logical reasoning as described in Section 2.6.3. Each subtest began with an example item during which the tester may have helped the children if they didn’t know the answer, as indicated in the example below.

Figure 3.10: Test administrators provide help during CFT1-R example items

After completing the example item with the help of the tester, the first subtest (Subtest 3) was completed by circling the correct answers. After completing the 15 questions, there was a page with a stop sign to indicate that the child had to stop and wait for the test administrator to complete the examples of the next subtest and give the instructions to complete the next 15 items. After all the children completed the CFT1-R tests, I marked them and scored each subtest. The total score was derived by adding the three subtest’s scores to get a score out of 45.

Figure 3.11: Examples of the CFT test administration
3.5.4. Capturing classroom engagement report data

Classroom engagement reports were completed by each class teacher. The report consisted of seven statements and the teacher had to encircle 1 if the statement was always true, 2 if the statement only sometimes applied to the child and 3 if the statement was not true. The data was captured and all responses were recorded on an Excel spreadsheet. A total score was derived by adding the responses. In this case a higher score indicated lower engagement and a lower score indicated higher classroom engagement. Table 3.4 shows an example of the recording sheet. Section 4.2 explains how the data was analyzed.

Table 3.4: Recording sheet example for capturing classroom engagement

<table>
<thead>
<tr>
<th>Would you say that the child...</th>
<th>plays and works cooperatively with other children at the level appropriate for his/her age</th>
<th>follows rules and instructions</th>
<th>listens attentively</th>
<th>follows directions</th>
<th>completes work on time</th>
<th>works independently</th>
<th>works neatly and carefully</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child 1</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td>Child 2</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td>Child 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
</tr>
</tbody>
</table>

3.5.5. Capturing interview data

Data was captured by interviewing two Grade R and two Grade 1 class teachers. All the participants were female and their experience ranged from 0 to 18 years. Three teachers were experienced teachers who have been teaching for at least eight years. Because the Grade 1 isiZulu teacher was on maternity leave, I interviewed the part time teacher who was a B.Ed. honours student at the University of Johannesburg at the time. The ‘student teacher’ gave an additional perspective – one of a student who has been trained at the University of Johannesburg.

All participating teachers, the two heads of department and headmaster were informed about the research for which the interview data is used. Although I video
recorded the interviews, the recordings were only used for transcription- and analysis purposes and the teachers' privacy was protected. The interviews took place during break times in June 2017. I interviewed the teachers in a research classroom where there were no disturbances. I made electronic notes during the interview and transcribed the data as soon as possible. Figure 3.12 shows an example of the interview setting.

![Interview Setting](image)

**Figure 3.12:** Interviewing teachers and a student teacher while the executive function cards and questions are visible to the participants

### 3.6. Analytic strategy

In Section 3.2 I mentioned that the study is designed to integrate quantitative- and qualitative data to answer the research question. The analytic strategy involved three phases: quantitative analysis, qualitative analysis and integrating all the findings. The aim of the data analysis was to capture the data in such a way that the reader is left with a clear understanding of the data (Henning et al., 2004). In other words, the quantitative data analysis aimed to capture the concurrent and predictive associations between the variables, while the themes of the qualitative data were derived in such a way that it could support the quantitative findings.

#### 3.6.1. Quantitative analysis

Quantitative data is usually analyzed in three steps, namely data preparation, descriptive analysis and inferential analysis (Maree, 2009; Trochim, 2006). To prepare the data for statistical analysis, the raw scores were captured on an Excel spreadsheet by integrating the various measures on a single database. The data was checked for accuracy and possible missing scores (Trochim, 2006). During this phase all data in string format (non-numerical data, for example male or female or different languages)
was coded as numerical data (https://www.spss-tutorials.com/spss-variable-types-and-formats/). For example, all females can be coded as 1 and males as 2.

Next, the data was described by means of descriptive statistics. The purpose of descriptive statistics is to summarize and analyze the data in a meaningful way to provide the analyst with a good understanding of the data (Maree, 2009). Descriptive statistics present what the data show (Trochim, 2006) and can be presented graphically or numerically. The examination of descriptive indicators allowed me to assess if the data is normal. Both the participants and the measures are described in Sections 3.3 and 3.4.

Prior to conducting the analyses, I examined the distribution of each variable to assess normality (Maree, 2009). Distributions summarize the frequency of individual values for a variable (Trochim, 2006). A normal distribution of the dependent variable was a precondition to conduct inferential statistics. It was also possible to identify potential outliers by examining the distribution.

I conducted inferential analyses to evaluate the hypothesis that there are positive concurrent and predictive associations between number concept development, mathematics-specific language, classroom engagement and logical reasoning. I first used a paired sample t-test to compare change in means of the MARKO-D SA scores in Grade R (tested in home language) and the MARKO-D SA scores in Grade 1 when children were tested in their home language and in English. Independent sample t-tests were used to find out if there was a difference in the mean scores of number concept measures, mathematics-specific language, classroom engagement or logical reasoning between the boys and girls and the Sesotho and isiZulu classes.

Next, for a correlation analysis a Pearson correlation coefficient (always between 0 and 1) indicated the strength and direction of the correlations between the variables. To conclude inferential statistics, I conducted a multiple regression analysis to examine the relationship between the number concept development in Grade 1 and the independent variables. The aims were 1) to find out how well the distinct independent variables, namely Grade R number concept attainment, mathematics-specific vocabulary, classroom engagement and logical reasoning, predict number concept development, and 2) to examine the relative contribution of each independent
variable to explain the outcome of number concept development by controlling for language of instruction. I examined these associations while controlling for language of instruction.

### 3.6.2. Qualitative analysis

The interview data, comprising teachers’ utterances in response to a specific set of questions in the protocol, meant that the coding could not be done entirely inductively as is the convention in grounded theory analysis (Charmaz, 2014; Strauss & Corbin, 1998). I thus coded and thereafter categorized the codes within the specific framework of the protocol, meaning that the coding was largely deductive even though I invoked a discursive position (Henning et al., 2004:57) with an element of discourse analysis, going beyond only ‘content’ analysis; I picked up on word choice and moments of reference to power relations (Fairclough, 2003). Henning et al. (2004) critique the use of content analysis alone. They argue that this method may lead to superficial findings because the data is not ‘interrogated’. Instead, they propose interpretive data analysis where the researcher accesses the data by reasoning about the data in a systematic and organized way. In this regard, the classic work of Miles, Huberman, Huberman and Huberman (1994) has been helpful, emphasizing ‘data displays’ or exhibits of the analysis process. In showing how one works with the data, the researcher shows her understanding of the coherence of the design of the study, analysis- and interpretation of the data.

To analyze the data by means of content- and discourse analysis, I followed the advice of Henning et al. (2004) to code and categorize the data, to find themes from the categories and lastly to look for a general pattern in the data. Figure 3.13 shows this process.

![Figure 3.13: Grouping content during qualitative research](image-url)
In this way of analyzing the data, the researcher starts with a verbatim transcription of each interview. The transcription is typed in double spacing with a large margin on the right where the codes can be written down. First, the researcher reads through the data to get a general impression of the interview. If the researcher conducted and transcribed the interviews, she may already have a general feeling of each interview. After the first reading of the data, the analyst identifies and marks units of meaning at short sentence or phrase level which is written down. It is important that each code relates to the research question. This process is highly interpretive, and the researcher has to rely on her knowledge of the field of inquiry.

Once each interview’s transcription has been coded, the codes are grouped or categorized. Again, the researcher’s extensive knowledge of the theory that informed the inquiry, enables her to systemize knowledge in her discipline by categorizing the codes (Henning et al., 2004). The codes are grouped together as “research chunks” of reality. These clusters then form categories. Henning et al. (2004) suggest asking questions like, “What do they say together?”; “What is missing?”; “What are the relationships between the codes?”

Next, the researcher finds themes which originate in the various categories. The researcher has to ask herself how the categories answer the research question and based on commonalities between categories, she can formulate themes. The themes must link to what the researcher already knows about the topic, what the analysis foregrounds and to the research question. Each theme is formulated in such a way that it makes a statement and contributes to the argument of the study. Each argument and sub-argument is evidence for the researcher’s point of view or emerging claims (Henning et al., 2004).

Lastly, a general- or global pattern is established from the themes. A global pattern provides an integrated view of the data in such a way that the themes, categories and codes cohere with a holistic view of the data. Often a mind map or graphical mind map or diagram helps the researcher to find meaningful patterns between the themes. A diagram allows the researcher to see how certain concepts connect (Henning et al., 2004).
3.6.3. Integrating quantitative- and qualitative findings

The purpose of ‘mixing’ quantitative- and qualitative data is to develop an integrated model grounded in two types of data (Ivankova et al., 2006). In this study I rely on both epistemologies by mixing the findings of the two types of data during the stage of data interpretation. Although there have been many studies that ‘mix methods’, this study is unique in the way that an ex post facto study is combined with interviews to draw conclusions from both types of data. By tabulating the quantitative findings and indicating how the findings from the qualitative data clarify the quantitative findings, I contribute my own intellectual integration of the findings. The one main finding of this study is that children struggle to develop number concepts in Grade 1 if they have to work with these concepts in English rather than their home language, which was also their language of instruction in Grade R. I elaborate on the findings in Chapter 4 and 5. By tabulating the various findings in such a coherent way, the researcher integrates quantitative- and qualitative data to best understand the phenomenon of interest (Maree, 2009). The other main finding is that mathematics-specific vocabulary showed a clear association with learning early number concepts.

3.7. Reliability, validity and research ethics

Research ethics

The school where the research was conducted serves as a ‘teaching’ school, which also serves as a ‘pedagogical laboratory’ (Ramsaroop, 2016). This means that when the parents choose to enter their children at this school, they agree that their children will participate in research conducted at the school. Appendix E is an example of a parent letter outlining their consent. To protect the children and teachers from unethical or unfair research, the school and the Faculty of Education Research Ethics committee ensure the protection of the research participants. Another committee oversees and monitors the research projects in the school itself. I applied for permission with both committees. To inform the parents about the research, I provided them with a letter that explains the research procedures (Appendix F). I also applied for an ethical clearance number (See Appendix G for the certificate; number: 2017-053). When teachers are appointed at the school, they also give permission to take part in research approved by the committee. ‘Pre-approved’ consent to partake in research avoids the long and tedious process of getting consent to do research at a
public primary school. The findings have been discussed with the teachers and the principal of the school. Although the children and their parents, as well as the teachers, have given consent to take part in the research, the results were handled with great confidentiality to protect the participants of the study.

Reliability

The MARKO-D SA has been standardized and has South African norms (Henning et al., 2018). Although the CFT1-R has not been validated in South Africa, the test has been standardized in Germany. It has been used in three studies, one of which has been published (Aunio, Mononen, Ragpot & Törmänen, 2016). The test designers argue that the instrument is language- and culture-free and therefore does not need South African norms. Because the mathematics-specific language test has not been standardized or normed I conducted a split half reliability test which tests internal reliability. Although the test did not yield internal reliability, the MMLT scores did correlate with the other test instrument’s scores.

Validity

In terms of validity I argue for construct validity and conclusion validity (Borsboom et al., 2004; Trochim, 2006). Since internal validity refers to causal relationships and external validity refers to the ability of generalizing the findings, I did not argue for internal or external validity (although the findings will add value to the general understanding of how children learn maths).

“Construct validation concerns the simultaneous process of measure and theory validation” (Strauss & Smith, 2009:2). In the particular case of the MARKO-D SA, children’s mathematical competence was measured by the test which is informed by theory of how children develop early number concepts. The same theory was used as a lens to analyze the data so as the ones who perform at level five competence were able to answer most questions because they understood the concepts which are described in the theoretical model. Other children (for instance those who performed on level one competence) were only able to complete counting tasks and were therefore only able to do the tasks described on the first level of the theoretical model. The manual of the test explains these (Henning et al., 2018).
In Cronbach and Meehl’s classic article, “Construct validity in psychological tests” (1995) they claim that to argue for construct validity one should first and foremost define what a construct is:

A construct is some postulated attribute of people, assumed to be reflected in test performance. In test validation the attribute about which we make statements in interpreting a test is a construct. We expect a person at any time to possess or not possess a qualitative attribute or structure, or to possess some degree of a quantitative attribute. A construct has certain associated meanings carried in statements of this general character: Persons who possess this attribute will, in situation X, act in manner Y (with a stated probability) (Cronbach & Meehl, 1995:283-284).

In this study the attribute under investigation was the level of development of numerical concepts of young children as described by the literature. If construct validity refers to “the degree to which inferences can legitimately be made from the operationalizations in a study to the theoretical constructs on which those operationalizations were based” (Trochim, 2006), I argue that I had made inferences about numerical development through the lens of theory – the same theory that underlies the test, much as Herholdt (2017) did in her dissertation study. The concepts which were measured are counting, ordinality, cardinality, part-part-whole and relationality of numbers and their additive relations up to 10.

Lastly, in terms of ecological validity I aimed to test performance of behavior in a ‘real world’, or everyday setting. By using a classroom engagement report and by including teachers’ perspectives, I aimed to ‘mirror’ real life (Grade R). I believe that the test scores are true representations of children’s ‘real-world’ academic functioning and that the test scores resemble their abilities.

3.8. Summary

This chapter outlined the design of the study. I described how I have integrated qualitative data with quantitative data to answer the research question. I described the participants, as well as the instruments. In the next chapter I ‘show’ how I have analyzed the data and in the last chapter I interpret the findings.
CHAPTER 4:  
DATA ANALYSIS

4.1. Introduction

This chapter narrates the process of data collection and analysis. After I had collected and captured the data, I first analyzed the quantitative data emanating from the assessments, using statistical procedures to show general trends in the concurrent and predictive associations between numerical competence and contributing cognitive skills (MSV, CE and LR). I then analyzed the interview data to identify themes in the teachers’ statements. I searched for linkages between the results of the qualitative and quantitative data sets. Figure 4.1 represents the chronological process of the collection, the analysis and the interpretation of the data. The process started in August 2016 and ended in April 2018. Coalesced with the design of the study (Chapter 3) it adds the time line of when the empirical work was done, such as the administering of the tests and conducting of the interviews.
4.2. Data analysis

After administering the MARKO-D SA, MMLT, classroom engagement report and CFT1-R, all test scores were entered in Excel 2016. Interviews were transcribed in...
Word 2016 using a protocol of questions. Statistical analyses were conducted in IBM\textsuperscript{35} SPSS Statistics 25. The data analysis process consisted of three inductive phases, namely quantitative analyses, qualitative analyses and integration of all the data. For the quantitative analyses I describe how data was checked for missing values and present descriptive and inferential statistics. Then, I show how I analysed the interview data. Lastly, I include an example of how qualitative data was used to explain the quantitative findings by integrating the quantitative- and qualitative data in table format.

4.2.1. Missing data
Data was checked for accuracy by screening for missing data and by making sure all the score sheets were completed correctly. There were no missing values, however six children were retained in Grade R and as a result did not have Grade 1 outcome data. Eight children were retained in Grade 1 and did not have Grade R data. In order to keep such children in the sample for regression analysis, I conducted multiple imputations. I explained this procedure in Section 3.3.1.

4.2.2. Descriptive statistics
Descriptive statistics are reported in Table 4.1. Independent samples t-test revealed no significant gender differences on the assessments of NCD, MSV, CE or LR. Other studies have found gender differences (Aldermann, Behrman, Ross, & Sabot, 1996; SACMEQ, 2010). When tested in their home language, children in the isiZulu classes outperformed children in the Sesotho classes on NCD in Grade R (means $= 20.55$ vs 17.11, $p < .01$) and Grade 1 (means $= 25.71$ vs 21.93, $p < .01$). There was no significant difference when the children’s NCD was tested in English. Finally, there were no significant tester effects.

NCD scores increased from Grade R to Grade 1 when tested in the child’s home language (means $= 18.93$ vs 23.86) and decreased when they were tested in English in Grade 1\textsuperscript{36} (means $= 18.93$ vs 17.73). There was a statistically significant difference between Grade 1 NCD when tested in home language and Grade 1 NCD when tested in English (means $= 23.86$ vs 17.73).

\textsuperscript{35} International Business Machines

\textsuperscript{36} Unfortunately Grade R children’s number concept development was not assessed in English, because the need to assess them in two languages only arose during the second test period when the children were in Grade 1.
Table 4.1: Descriptive statistics for independent-, dependent- and control variables

<table>
<thead>
<tr>
<th>N=59</th>
<th>Possible total score</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
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</table>

Dependent variables (Grade 1)

<p>| | | | | | |</p>
<table>
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<tr>
<th></th>
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<td>Number concept development _ English</td>
<td>47</td>
<td>17.73</td>
<td>5.9</td>
<td>7</td>
<td>41</td>
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<tr>
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<td>23.86</td>
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Independent variables (Grade R)

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<td>Number concept development</td>
<td>47</td>
<td>18.93</td>
<td>4.6</td>
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<td>32</td>
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<td>Mathematics-specific vocabulary</td>
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<td>19.41</td>
<td>2.35</td>
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<td>23</td>
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<td>Logical reasoning</td>
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Independent categorical variable (Grade R)

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<table>
<thead>
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<tbody>
<tr>
<td>Classroom engagement</td>
<td>Highly engaged:</td>
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</tbody>
</table>

Most previous studies in which the MARKO-D SA was used, assessed the usability of the instrument itself. The only study where the MARKO-D SA was used as a test instrument, was in an evaluation study in the Western Cape where an intervention program was evaluated. The MARKO-D SA was administered in two urban districts, each consisting of an intervention- and control group to whom the MARKO-D SA was administered at the beginning and end of their Grade R year to assess the impact of the ‘R-maths’ intervention program (KELLELO, JET Educational services, 2018). In this study, the results varied. In the first district the control group’s NCD scores (means=17.8 at baseline\textsuperscript{37} and 27.0 at endline\textsuperscript{38}) improved more than the intervention group’s NCD (means=16.5 at baseline and 25.9 at endline). In the second district the intervention group’s NCD scores (means=19.7 at baseline and 27.4 at endline) improved more than the control group’s NCD scores (means=18.8 at baseline and 25.4 at endline).

\textsuperscript{37} At the beginning of Grade R.
\textsuperscript{38} At the end of Grade R.
The mean scores of the current study’s assessments at the beginning of Grade R compares well with that of the R-maths evaluation study’s assessments at the beginning of the year (means=18.9 compared to 17.8; 16.5; 19.7 and 27.4). The mean scores of the current study’s home language assessment at the beginning of Grade 1 is poorer than that of the evaluation study’s mean scores (means=23.9 compared to 27.0; 25.9; 27.4 and 25.38).

Figure 4.2 shows the distributions of the outcome variables. The outcome measures were normally distributed. The difference in spread between the English and home language outcomes is noteworthy. The English test was more difficult for the children, since the spread is negatively skewed. Three independent variables (NCD, MSV and LR) were also normally distributed, but CE was skewed (see Appendix H). Because CE scores showed a high negative skew, I dichotomized this variable to reflect a natural break in the distribution. Scores between 0 and 10 were categorized as highly engaged (1) and scores between 11 and 21 as low engagement (0).

![Figure 4.2: Outcome variable distributions](image)

**4.2.3. Associations between predictors and NCD**

Table 4.2 shows bivariate correlations between predictors and outcome variables. Except for NCD in Grade R, MSV in Grade R had the strongest correlation with Grade 1 NCD tested in English (.35, p< .01). Except for Grade R NCD scores, (.4, p < .01), MSV, LR and CE were not significantly correlated with Grade 1 results of the MARKO-D SA tests administered in isiZulu and Sesotho. Finally, English NCD scores in Grade 1 were correlated with Grade 1 home language NCD scores (.28, p < .05).
Table 4.2: Correlations between predictor- and outcome variables

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number concept development (NCD)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Home language Grade 1</td>
<td>--</td>
<td>.28*</td>
<td>.4**</td>
<td>.12</td>
<td>.14</td>
<td>.13</td>
</tr>
<tr>
<td>2. English Grade 1</td>
<td>--</td>
<td>.27**</td>
<td>.35**</td>
<td>.28**</td>
<td>.31**</td>
<td></td>
</tr>
<tr>
<td>3. Home language Grade R</td>
<td>--</td>
<td>.2*</td>
<td>.24*</td>
<td>.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade R predictors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Mathematics-specific vocabulary</td>
<td>--</td>
<td>.27**</td>
<td>.36**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Logical reasoning</td>
<td>--</td>
<td>.35**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Classroom engagement</td>
<td>--</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. HL=home language. *<.05, **<.01. Correlations involving classroom engagement were conducted using Kendall’s tau-B correlation coefficient.

Multiple regression analyses were calculated to examine associations between each predictor variable in Grade R and outcome variable in Grade 1, while controlling for language of instruction. Because the Grade R predictors correlated with the Grade 1 English NCD test results, but not with the Grade 1 home language NCD, a regression model was only estimated for the English assessment. In total 6 regressions were run on imputed data. Each of these regressions examined associations between each of the predictors while controlling for language of instruction. In terms of model fit, all regressions accounted for a significant proportion of the variance in the Grade 1 NCD scores.

I ran two regression models to determine how each independent variable contributed to the variance in NCD. In the first regression model I examined each predictor separately controlling for language of instruction, given that the predictors are statistically and theoretical closely related. Of the individual predictors, NCD in Grade R was the strongest predictor of Grade 1 NCD, $\beta=.42, p<.05$. MSV was the second strongest predictor ($\beta=.16, p<.05$).

In a second regression analysis, the Grade R NCD score was omitted as predictor, while still controlling for home language. In the second model I determined concurrent associations between Grade R MSV, CE and LR, and Grade R NCD. I also
determined prospective associations between Grade R MSV, CE and Grade 1 NCD. The second model indicated that MSV, CE and LR significantly contributed to variance in the model. Table 4.3 indicates concurrent- and prospective associations between Grade R cognitive skills (NCD, MSV and LR) and NCD in Grade R and Grade 1, with Grade R NCD omitted as predictor, while controlling for language of instruction.

Table 4.3: Standardized regression coefficients depicting associations between cognitive skills and NCD for Grade R and Grade 1

<table>
<thead>
<tr>
<th>Number concept development</th>
<th>Grade R</th>
<th>Grade 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predictors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Logical Reasoning</td>
<td>.57***</td>
<td>.34*</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.47</td>
<td>.11</td>
</tr>
<tr>
<td>Mathematics-Specific Vocabulary</td>
<td>.39**</td>
<td>.38**</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.32</td>
<td>.14</td>
</tr>
<tr>
<td>Classroom engagement</td>
<td>.37**</td>
<td>.35**</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.3</td>
<td>.12</td>
</tr>
</tbody>
</table>

Notes: Models are adjusted for children’s classroom (IsiZulu vs Sesotho). *** $p<.001$; ** $p<.01$; and * $p<.05$.

MSV, CE and LR were all significant predictors for NCD in grade R and 1. The concurrent cognitive skills (MSV, CE and LR) explained 30% to 47% of the variance in Grade R NCD with LR explaining the most variance and CE the least. In terms of prospective associations, Grade R predicting skills explained 11% to 14% of the variance in Grade 1 NCD with MSV explaining the most variance and LR the least.

4.2.4. Analyzing interview data

The interview data builds on the quantitative data by explaining and elaborating on the quantitative data (Invankova et al., 2006). I conducted and transcribed the interviews myself, working closely with the data even before the process of analysis had started. The transcriptions are verbatim and typed in double lined spacing so that there was ample space for notes. Before analyzing the data, I watched the video recordings again and reread the transcribed data a few times during which I made notes to ease the next phase. In the first round of coding I worked inductively, although I remained mindful of the interview specific questions I had posed and also of the research question.
I coded the data by assigning just over 100 codes to utterances that formed single units of meaning attributed by myself. I followed the example of Henning et al. (2004) closely, aiming not only to classify or sort data as much as awarding meaning in the context of the specific question. I realized that I would be biased – also because I was by that time so deeply involved in the inquiry. I composed codes, comprising phrases and even short sentences to ensure that there would be precision (Henning et al., 2005). This was my mindset when I coded the interview data, wanting to find out how teachers think about the learning mind of the child, when they teach them how to become numerate.

After codes had been awarded to segments/units of meaning, the codes were grouped, or semantically ‘collapsed’ into 15 categories, also through an inductive process of shared meaning and my conceptualization and epistemological mindset as analyst. This is, in grounded theory analysis, a tool known as ‘axial coding’ (Strauss & Corbin, 1998). This has become a generic tool in different genres of qualitative research (Merriam, 2009). Groups of ‘collapsed’ codes were then labeled as ‘categories’ of the interview data. I categorized the codes by tabulating them (Table 4.4) to ‘see’ similarities and differences between participants’ responses to the seven interview questions:
Table 4.4: Teacher interview responses

<table>
<thead>
<tr>
<th>Teacher 1</th>
<th>Teacher 2</th>
<th>Student teacher</th>
<th>Teacher 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What influences a child’s ability to learn mathematics?</strong></td>
<td>Natural ability they are born with - many/few</td>
<td>Cognitive skills</td>
<td>Environment</td>
</tr>
<tr>
<td>Environment</td>
<td>Environment (exposure)</td>
<td>Environment Exposure to help them form math understanding and knowledge. If the environment is rich the children will learn</td>
<td>Environment</td>
</tr>
<tr>
<td>- outdoor</td>
<td>- home</td>
<td>- home</td>
<td>- home</td>
</tr>
<tr>
<td>- indoor</td>
<td></td>
<td>- classroom</td>
<td>- school (e.g. teacher’s relationship with children)</td>
</tr>
<tr>
<td>- school</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- home (prepare food - number of slices bread/ poor drink - full/half glass)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- shops</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- everywhere</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Incidental learning (notice patterns, sizes, differences in buildings)</strong></td>
<td><strong>Incidental learning</strong></td>
<td>Instruction (if the teacher knows number sense and mathematical development she can instruct learners better)</td>
<td></td>
</tr>
</tbody>
</table>


Please describe what children need to learn in Grade R to prepare them for mathematics in Grade 1

<table>
<thead>
<tr>
<th>Practical things</th>
<th>Incidental (through playing)</th>
<th>Formal and structured play</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in space</td>
<td>Shapes</td>
<td>Shape</td>
</tr>
<tr>
<td>Measurement</td>
<td>Quantity</td>
<td>Quantity</td>
</tr>
<tr>
<td>Quantity</td>
<td>Quantity</td>
<td>Quantity</td>
</tr>
<tr>
<td></td>
<td>Many</td>
<td>Many</td>
</tr>
<tr>
<td></td>
<td>Few</td>
<td>Few</td>
</tr>
<tr>
<td></td>
<td>Less</td>
<td>Less</td>
</tr>
<tr>
<td></td>
<td>More</td>
<td>More</td>
</tr>
<tr>
<td>Patterns</td>
<td>Counting with meaning (not rote)</td>
<td>Counting (rote counting): getting used to counting numbers</td>
</tr>
<tr>
<td>Counting</td>
<td>One-to-one correspondence</td>
<td>One-to-one correspondence</td>
</tr>
</tbody>
</table>

Please describe this teaching school. How does this particular teaching school differ from other schools?

<table>
<thead>
<tr>
<th>School Quality education</th>
<th>Curriculum</th>
<th>School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude to be flexible with curriculum: CPAS + DoE policies but rearrange how concepts are taught</td>
<td>First of its kind Collaboration between university (research - never static) &amp; DHET (policies &amp; research) is a good combination</td>
<td></td>
</tr>
</tbody>
</table>

39 Department of Education
### Teachers
- Welcome students in classrooms
- Mentors
- Discuss how children learn a certain concept
- Kept on their toes: be prepared at all times

### Teachers
- Mentors to education students
- Engage students while they observe lessons
- Ask reflective questions while students teach
- Keep students interested

### Teachers
- Experienced, professional, determined
- Exposed to relevant research so they implement and extend curriculum
- Workshops/development happens every day in the school
- In contact with lecturers/many groundbreaking work

### Teachers
- Prepared at all times
- Mentor: Reflect on what you are teaching because the knowledge is NB to those who observe.
- Accommodate both children and students (without neglecting one or the other)
- Get exposed to children and students

### Students
- Often (3-4 times a week) see teaching in practice as opposed to one week practical time
- Based at this school
- Be mentored

### Student teacher
- Observe (watch us teach") Tues, Wed, Thurs (engage students by asking them questions")
- Thurs students have opportunity to teach, teachers evaluate
- Feedback and "mentoring happens"

### Novice/student teachers
- Practice/implement what they learnt in class

### Children
More adults in class means:
- Get a lot of attention
- Safety and attention ensured
### Do you think that there is a need for instruments that can identify children with possible learning difficulties? Why do you think so?

<table>
<thead>
<tr>
<th>Important. There is a need.</th>
<th>Very important</th>
<th>There is a need</th>
<th>What would such an instrument be?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children are unique – abilities and disabilities/bars.</td>
<td>I assume when children enter Grade 1, they all know colours, shapes. Only when you teach new concepts you realize that you have to reteach a basic concept because they struggle with a prior concept.</td>
<td>It should be a continuous assessment because of changing circumstances (assess over a period of three months) for a correct diagnosis.</td>
<td>Planning and lessons aim to identify children with MLD in order to give support and intervention.</td>
</tr>
<tr>
<td>Too big load for a teacher to identify if there is for example 30 children.</td>
<td>It wastes time to reteach.</td>
<td>Note: This is for Grade R. Would it help for Grade 1?</td>
<td></td>
</tr>
<tr>
<td>Instrument will help identify children with MLD so they can get support.</td>
<td>Intervention/support could take place in Grade R/earlier.</td>
<td>Unintentional assessment of concepts (while teaching).</td>
<td></td>
</tr>
</tbody>
</table>

### What role does language play in learning mathematics?

<table>
<thead>
<tr>
<th>Helps with understanding a concept.</th>
<th>Important role</th>
<th>Language is a form of expression.</th>
<th>Grade R: home language is user friendly for children. They can relate because they know it from young age.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Important to start properly (vocabulary).</td>
<td>Most math concepts are in English</td>
<td>How will children or teachers communicate without language?</td>
<td></td>
</tr>
<tr>
<td>Sesotho is not always the equivalent of English.</td>
<td>No communication = no understanding.</td>
<td>No communication = no understanding.</td>
<td></td>
</tr>
<tr>
<td>Language is a form of expression.</td>
<td>First few years should include mother tongue and English.</td>
<td>First few years should include mother tongue and English.</td>
<td></td>
</tr>
</tbody>
</table>

Introduce them to a new language: teach a concept in HL and then in English: in preparation for Grade 1.
<table>
<thead>
<tr>
<th>Correct math language is important.</th>
</tr>
</thead>
</table>

**How do executive functions influence number concept development in a Grade R class?**

<table>
<thead>
<tr>
<th>Have an influence</th>
<th>Working memory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Entering Grade 1 children should be able to replace old info with new ones.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Working memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math is hierarchical.</td>
</tr>
<tr>
<td>Prior knowledge is important.</td>
</tr>
<tr>
<td>Integrate prior knowledge and new concepts.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Working memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher enhances by giving a few tasks in order and they must remember the tasks. Ability to remember a few tasks indicates ability to do maths.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cognitive flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning how to sequence.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cognitive flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switch between + and – strategies (perseveration).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cognitive flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switch between different learning areas.</td>
</tr>
<tr>
<td>Switch between contexts.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cognitive flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher should help with integrating different areas.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cognitive flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaches order(nality), sequencing.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inhibitory control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Especially important at young age (short attention span).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inhibitory control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher must recollect children’s minds.</td>
</tr>
<tr>
<td>Have strategies to collect their minds and shift their minds when necessary.</td>
</tr>
<tr>
<td>Develop weaker learners’ EF and</td>
</tr>
</tbody>
</table>
Table 4.5 is an example of how codes were attributed to a part of Teacher 3’s answer to the question: “What role does language play in learning mathematics?”

Table 4.5: Example of coding

<table>
<thead>
<tr>
<th>Text</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language plays a very important role but in our case and in this, in our situation what I’ve observed as a teacher is that most mathematical concepts, whether we like it or not, are in English. In our own language the mathematical concepts are intense. There’s lot of intense and very long. Sometimes they do not exactly… it’s not a correct equivalent of English. So sometimes it causes a lot of clashes.</td>
<td>T3: isiZulu and Sesotho use English mathematical vocabulary so it is familiar when children use them in Grade 1. T3: Sesotho is not always equivalent to English. T3: Translation from Sesotho to English causes clashes.</td>
</tr>
<tr>
<td>So for me, coming from my experience as a teacher it is much easier to teach maths in English because I last attended a course in Singapore maths when I was teaching in orange farm. There were school who were doing maths in isiZulu and the others we</td>
<td></td>
</tr>
</tbody>
</table>
opted for English. Come the time when we had to present – we were doing place value - it was complicated, the teacher was just so confused - we could not understand what the teacher was saying. Imagine adults, what is going to happen to children? So I think language plays an important role but in mathematics I believe that English – it’s much better when we teach maths in English. As long as we start properly.

T3: Adults get confused with isiZulu number names because they prefer English number names.

T3: Teach math concepts in English, but use correct language.

After having coded the corpus of interview data, I created categories by grouping the codes. Table 4.6 shows the process.

Table 4.6 : Categories derived from codes

<table>
<thead>
<tr>
<th>Codes</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1T2T3: Environment influences maths learning</td>
<td>Teachers are aware of maths concepts in everyday life</td>
</tr>
<tr>
<td>T1: Environment includes outdoor, indoor, school, home, shops, everywhere</td>
<td></td>
</tr>
<tr>
<td>T2: School environment is friendly</td>
<td></td>
</tr>
<tr>
<td>ST: Exposure in maths learning – environment (school and home)</td>
<td></td>
</tr>
<tr>
<td>ST: Exposure in maths learning – environment (school and home)</td>
<td></td>
</tr>
<tr>
<td>T2: Teachers have a relationship with the children</td>
<td></td>
</tr>
<tr>
<td>T1ST: Children learn maths through incidental learning</td>
<td></td>
</tr>
<tr>
<td>T2: Children learn by applying concepts in everyday life</td>
<td></td>
</tr>
<tr>
<td>ST: Cognitive skills influence maths learning</td>
<td>Contextual- and individual factors influence early number concept development</td>
</tr>
<tr>
<td>ST: Disorders influence maths learning</td>
<td></td>
</tr>
<tr>
<td>ST: Genetics influence maths learning</td>
<td></td>
</tr>
<tr>
<td>T3: Natural maths ability refers to many and few</td>
<td></td>
</tr>
</tbody>
</table>
ST: Grade R children learn shapes
ST: Grade R children learn by e.g. climbing on a jungle gym
T3: Grade R children learn colours, shapes, quantities, many, few, less, more
T1: Grade R children learn practical things
ST: Grade R children learn to count in an ordinally way
T3ST: Grade R children learn one-to-one-correspondence – not rote counting
T1: Grade R children learn to measure
T1ST: Grade R children learn to count
T1: Grade R children learn quantity and patterns
T2: Children learn maths through free- & structured play
T2: Toys (resources) are important during play/learning

ST: Teacher knowledge of number sense- and maths development influences maths learning
T1: Children learn maths by noticing patterns, sizes and differences
ST: Teachers’ knowledge enable them to instruct children better
T3: Learning materials (from Grade R) influence maths learning
T2: A well-organized Grade R class is important for learning

T1T2: Use of practitioner's-, not academic language
T1: School specific discourse
T3: Developmental cognitive discourse
T1: Teacher training/course discourse

ST: The teaching school is one of its kind
ST: The teaching school is a ‘collaboration’ between university and DoE
ST: The university adds research to practice

Teachers’ knowledge of cognitive development contributes to pedagogy for number concept development

Variety of discourses play a role

Age appropriate maths concept learning (and teaching)

Good teacher training is the result of collaboration between teaching school, university and the DoE
ST: The DoE adds research and policy to practice

ST: The teaching school is progressive

ST: Teachers are exposed to research at the teaching school

T1: The teaching school provides quality education

| T2: Teachers must be experienced and qualified to know what and how to teach children | Good quality teacher professional development and experience at the school |
| ST: Teacher at the teaching school are able to implement and extend the curriculum |
| ST: Teachers at the teaching school are experienced and professional |
| ST: Teachers at the teaching school are exposed to many workshops |
| T1: Teachers are well qualified |
| T1T2: Teachers provide a practitioner's view with examples |
| T3: Teaching at the teaching school are guided by CAPS\textsuperscript{40} and DoE policies but teachers rearrange how concepts are taught |
| T1T2T3: Teachers are mentors for students |
| T1: Teachers discuss how children learn concepts with the students |

| T1T2: Teachers must be prepared at all times because students observe them |
| T3: The teaching school has flexibility within curriculum |
| T1: Teachers welcome students into class |
| T1: Teachers take ownership of students |
| T2: Teachers' knowledge is important |
| T1: Teachers reflect on student lessons and provide feedback |
| T2: Teachers must reflect on their own teaching and knowledge so that students can really learn |

\textsuperscript{40} Curriculum and Assessment Policy Statement
| T1: Students are often in classes (3-4 x per week) as opposed to one week practical time | Students at the teaching school observe, apply theory, teach, reflect |
| T1: Students see teaching in practice |
| T1: Students are ‘based at’ the school |
| T2: Students observe teachers (and learn) |
| T3: Students watch teachers three times a week |
| T3: Students have an opportunity to teach one a week |
| ST: Students/novice teachers try, practice and implement what they learn in class |

| T3: Teachers evaluate the students and give feedback |
| T3: Teachers are actively involved in the students’ lessons |
| T3: Teachers ask reflective questions to the students during the teachers’ lessons to engage the students |
| T1: Teachers have dual roles – trainer |
| T2: Teachers accommodate both children and students without neglecting one |

| T1T3ST: There is a need for math diagnostic instruments |
| ST: The instrument should be a ‘continuous’ instrument |
| ST: Test results may vary due to circumstances |
| T3: Diagnostic instruments provide baseline assessments so that teachers know what children know from Grade R |
| T3: Without diagnostic instruments teachers only realize later which children struggle and have to reteach concepts |
| T1: Children are unique in terms of abilities, disabilities and barriers. Therefore a diagnostic test is important |
| T1: It is a too big load for teachers to identify children with MLD if there are 30+ children in a class |

| Teachers accept responsibility for children’s learning and student training |

| Variance of opinions about diagnostic instruments |
T1: A diagnostic instrument will help identify children so that they can get support

T2: A teacher questioned the use of diagnostic instruments because her planning, teaching and assessment focus on the identification of MLD

T2T3: Identification of MLD must lead to support and intervention

T3: Intervention can take place in Grade R already and therefore early identification of MLD is important

T1: Teachers are unsure about the role of language for maths learning

ST: Language is pivotal to mathematics

ST: Language is a form of expression

ST: Include mother tongue in maths teaching so that children may have a proper grasp of a concept

T3: Teach maths concepts in English, but start properly

T1: Language helps children understand a concept

T2: Teach maths concepts first in the children’s home language and then translate it to English in preparation for Grade 1

T3: Adults get confused with isiZulu number names because they prefer English number names

T3: isiZulu and Sesotho speakers use English mathematical vocabulary, so it is familiar when children use them in Grade 1

ST: If children are not ‘grounded’ in their language, it is difficult to learn, communicate or understand

T3: Translation from Sesotho to English causes clashes

T3: Sesotho is not always equivalent to English

T2: Children can relate to their home language in school because it is familiar

T2: In Grade R the home language is not the language of instruction

ST: Correct maths language is important to maths learning

Diagnosis of MLD always aims to intervene and support children

Varying views on the role of language during early number concept development

Language limitations contribute to (in)ability to develop number concepts
Teachers don’t know what EF are
Teachers’ inadequate/old school’ knowledge of cognitive skills

Children learn through repetition – both maths and language

Children must be able to move between tasks (switching)

Cognitive flexibility is like sequencing tasks and ordinality

Children’s ability to remember a few tasks (sequencing) indicate their ability to do maths (step by step)

Teachers must often ‘recollect children’s minds’

Teachers must use appropriate strategies to shift children’s minds

EF are important for weaker and stronger children

EF teach discipline which is important for maths

Routine is the master of learning

WM enables children to apply old information to new information (replace the old with the new)

Children must be able to switch from plus to minus

Some children struggle to move from plus to minus (perseveration)

Children must be able to switch between different contexts and learning areas

Teachers must explicitly help children switch and integrate their knowledge

Young children’s attention span is short

Inhibitory control is important for maths learning

You cannot learn if your brain is ‘on the other side of town’

WM enables children to use prior knowledge to develop new number concepts
After coding and categorizing the raw data, I established six themes that could be ‘woven’ though the data. These themes form the basis for the qualitative data analysis.

Table 4.7: An example of formulating themes

<table>
<thead>
<tr>
<th>Categories</th>
<th>Themes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Teachers’ knowledge of cognitive development contributes to pedagogy for NCD</td>
<td>1. Teachers’ knowledge of Grade R children’s cognitive skills is beneficial for their teaching</td>
</tr>
<tr>
<td>2. Teachers’ inadequate/‘old school’ knowledge of cognitive skills</td>
<td></td>
</tr>
<tr>
<td>3. Age appropriate maths concept learning and teaching</td>
<td></td>
</tr>
<tr>
<td>4. Varying views on the role of language during early NCD</td>
<td>2. Language acquisition and cognitive skills as intersects for the development of number concepts</td>
</tr>
<tr>
<td>5. Language limitations contribute to (in)ability to develop number concepts</td>
<td></td>
</tr>
<tr>
<td>6. Teachers are aware of maths concepts in everyday life</td>
<td>3. Teachers recognize social context and individual differences</td>
</tr>
<tr>
<td>7. Contextual- and individual factors influence early NCD</td>
<td></td>
</tr>
<tr>
<td>8. Good teacher training is the result of collaboration between teaching school, university and DoE</td>
<td>4. Emulating a Finnish ‘teaching school’-model</td>
</tr>
<tr>
<td>9. Good quality teacher professional development and experience at the school</td>
<td></td>
</tr>
<tr>
<td>10. Ambiguous views of the teaching school, student training and teachers’ roles</td>
<td></td>
</tr>
</tbody>
</table>
11. Teachers accept responsibility for children’s learning and student training

12. Students at the teaching school observe, apply theory, teach, reflect

13. Variance of opinions about diagnostic instruments

14. Diagnosis of MLD always aims to intervene and support children

15. Variety of discourses play a role

Henning et al. (2005) suggest concept mapping as a tool with which the researcher can find coherent links between the themes. Such a process of cohering the themes to make sense and to form a pattern can be regarded as a way of firming up the veracity of the process. Figure 4.3 is a flow diagram that shows how the various themes fit the whole – which would be the pattern in the ‘puzzle’ of what could be messy and non-cohesive or even incoherent loose pieces of data. The pattern I derived from the interview data is that: Teachers’ technical epistemology of Grade R children’s NCD shows some emergent critical reflective capability. There is some fidelity in their discourse, but not, as yet, integrity. They have learned the craft of reflection (techne), have experience to philosophize on practice (phronesis), but have not yet developed their technical skill into a view of children’s learning and NCD that has a strong epistemic character (episteme). In the discussion of the themes, I will explain how each theme connects with the other themes and also with the central pattern. I use the traditional Greek terms for types of knowledge to show that the teachers have developed ‘craft’ knowledge quite well and that they are able to reflect in practice, but that they have not yet developed an epistemological position about young children’s learning of numeracy. They do not see it (yet) as a conceptual phenomenon, more than a procedural or a factual form of knowledge.
The reliability of the process of analysis depends, in part, on how the overall pattern, the themes, categories, codes and raw data (text) link with each other. The following diagram shows an example of the coherence of the ‘raw’ text, the codes, the categories, the themes and the central pattern.
4.2.5. Outcome of the thematic analysis

Theme 1: Teachers’ knowledge of Grade R children’s cognitive skills is beneficial for their teaching.

Teachers know that Grade R children learn about colours, shapes, measurement, patterns and counting (some argue for rote counting, while others opt for the use of one-to-one correspondence and counting principles) and concepts such as many, few, less and more. Teacher 2 said that Grade R children learn both through formal- and unstructured play and use toys (what we know as recourses).

Teachers did not understand the construct of EF. Some gave examples from their experience to try to make sense of the construct. Others recognized the word ‘memory’ and linked their answers to what they know about memory. Teacher 2 explained that routine is the “master of learning” and children learn and remember new
concepts through repetition. She combined working memory and inhibitory control by saying:

So if you keep repeating it it’s easy for them to start to learn, because their memory will start to keep what is important and to release what is not important.

Teacher 3 tried to explain something about the development of newly formed conceptual systems by linking it to her understanding of ‘memory’:

Children in Grade R, when they are ready to come to Grade 1, they should be able to understand, to be able to replace old information with new ones. ... We ask them a question they are able to refer from the old, but they know that you are asking a question which is relevant to what you have just taught them.

The student teacher also combined her knowledge of hierarchical development of number concepts to her understanding of memory:

So the child should have a very good working memory in order to use whatever prior knowledge they have into this new concept or this new stage of NCD they are growing into.

Most teachers linked inhibitory control to what they know about ‘attention span’:

Remember, children’s minds are everywhere and anywhere and sometimes you have to recollect it every time (Teacher 2).

Inhibitory function... it’s very important. I’m not sure how it’s important to NCD. I know it’s important. Because you cannot learn if your brain is on the other side of town. So I think it’s very important (student teacher).

Teacher 3 gave an example of how poor cognitive flexibility could influence NCD: Many children struggle to learn the concept of subtraction, because they struggle to “block the addition part, now I can switch over to subtraction.” The student teacher spoke more in general about cognitive flexibility by saying that children must be able to switch between contexts and learning areas. Similar to Teacher 1, Teacher 2 linked cognitive flexibility to the concepts of sequencing and ordinality by saying that children must be able to switch between tasks and remember the order in which to complete tasks.
Theme 2: Language acquisition and cognitive skills as intersects for the development of number concepts.

The teachers said that language helps with understanding (Teacher 1), it is a form of expression (student teacher), language enables communication (student teacher) and an inability to communicate may lead to an inability to learn. “If you cannot communicate something, it means you cannot understand it” (student teacher).

Three of the four teachers argued about the timing of the introduction of English to Sesotho and isiZulu children. Teacher 2 said that children should first learn basic mathematic concepts in their home language. Only later should the concepts be explained to them in English. Also, the introduction of English vocabulary prepares the Grade R children for Grade 1. Teacher 2 said:

It is easy for them to relate to what the teacher is saying because they know it from a young age. As they are growing, the language that they use at home is the language that we use here at school. We only promote it by interpreting and exposing them to new language – the additional language.

The student teacher felt that English terms should be included from the beginning:

I think we should include mother tongue language in our mathematics teaching so that the child may have a proper grasp of this concept.

Teacher 3, on the other hand, argued that most Sesotho and isiZulu speakers use English number names rather than their mother tongue number names and that children are therefore familiar with the English terms. She claimed that some isiZulu speaking adults can even get confused when isiZulu number names are used rather than English number names:

What I’ve observed as a teacher, is that most mathematical concepts, whether we like it or not, are in English. In our own language the mathematical concepts are intense.

Teacher 3 also highlighted the headaches of translation. A translation of concepts from one language to another may not always be an exact translation (as discussed in Section 2.4.1). With reference to translating mathematical concepts from isiZulu or Sesotho to English, Teacher 3 said:
Sometimes they do not exactly... it is not a correct equivalent of English. So sometimes it causes clashes.

Theme 3: Teachers recognize social context and individual differences.

The student teacher in particular made reference to children’s cognitive skills, cognitive disorders and ‘genetic makeup’. Teacher 1 said that “their natural ability that they are born with to understand there’s many, there’s few” influence their ability to learn maths. Teacher 1 has attended workshops on core math cognition and it is evident in her reference to the AMS. However, the two Grade R teachers did not make any reference to individual differences, but only referred to the influences of different social contexts on NCD.

Three teachers mentioned that learning not only takes place during formal education, but that children learn incidentally by “noticing things” (Teacher 1), for instance by recognizing patterns, comparing sizes or by noticing different shapes in their surroundings. Teacher 2 gave an example: “Maybe I’m walking home and I count the things on my way and see there is one or there is four”.

Theme 4: Emulating a Finnish ‘teaching school’-model.

Teachers at the school (see Addendum A) are experienced and appropriately qualified which contribute to the school’s high standard of education. All four teachers explained that they have a dual role: A teacher and a mentor for the novice teachers. Teacher 2 explained that the teaching school model should be beneficial for the children, teachers and students: “I wouldn’t neglect my learners because of the adults and I wouldn’t neglect the teacher students because of my learners.” Because of such a dual role, teachers ought to be prepared at all times and often reflect on the quality of their lessons (although this should be the case at all schools). All four teachers described their roles as mentors: Mentors teach while students observe, ask their students to reflect on the lessons they observed, give students the opportunity to practice what they have learnt by allowing the them to teach, help the students to reflect on their own lessons and discuss how children learn certain concepts.

The teachers believe that the collaboration between the school and university results in good teacher training. Teacher 2 argued that the children receive more attention and that their safety is ensured due to the larger number of adults in the
class. Teacher 3 said that the relationship between the school, university and DoE allows them “latitude to be flexible when working with the curriculum. We follow the CAPS curriculum and the department of education policies, but working in the latitude the flexibility to rearrange how concepts are taught.”

Theme 5: Diagnostic- and intervention programs support early math learning.

Teachers had different views about the use of diagnostic instruments. When asked if there is a need for instruments that can identify children with possible MLD, teachers agreed that the aim of such a diagnostic instrument and the use of intervention programs should always be to provide children with support they need to optimally learn.

However, teachers differed in opinion regarding the use of a diagnostic instrument. Teacher 1 argued that such an instrument can take the load off the teacher to identify struggling children when there are approximately 30 children in her class. Contrary to Teacher 1’s opinion, Teacher 2 said:

I wonder what such an instrument could be. Because what we do we always work towards identifying those learners who can possibly have difficulties in learning maths. Whatever the lessons we prepare it will always determine who can and who cannot.

The student teacher seemed to agree with Teacher 2 to some extent. She argued that a diagnosis at one given moment cannot reflect the child’s abilities and that a child’s results will differ from one day to the next due to circumstances. Therefore, a diagnostic instrument should be a ‘continuous instrument’ over perhaps three months. Teacher 3 is a Grade 1 teacher. She argued that the diagnostic instrument could be utilized during the Grade R year to provide Grade 1 teachers with valuable information about children’s cognitive abilities when the children enter Grade 1. Furthermore, she argues that intervention should already start in Grade R so that the “basics of certain concepts” are already established when children enter Grade 1.

4.3. Integrating quantitative- and qualitative findings

In this section I include a ‘data display’ (Miles et al., 1994) table (Table 4.8) to show how I integrated the results. I include a column that contains my reflection to complete
the ‘triangulation’ as explained in Section 3.2. The table is a compact rendering of findings. In this way I could ‘map out’ the data and include some of my interpretation of the associations between NCD, MSV, CE and LR.

For example, the quantitative results indicated that there was no statistical significant increase in NCD in Grade R and Grade 1 when children were tested in English in Grade 1; and that there was a significant difference between the Grade 1 home language NCD scores and English Grade 1 NCD scores. These results aligned with the second theme from the qualitative analysis: Language acquisition and cognitive skills as intersects for the development of number concepts. Together with the quantitative finding that showed that there was a statistically significant increase in NCD from Grade R to Grade 1 when children were tested in their home language in Grade 1, I conclude that language plays a substantial role in the different scores. Therefore, I added my own reflection by applying theoretical knowledge to the findings (see Section 3.2), what this means in terms of what I know about language and mathematical cognition, and specifically early numeracy. Some theories explain likely reasons for the association between NCD and language input (e.g. Purpura et al., 2011). Then, if the thesis of my study holds, this would make sense – language is input and MSV is data for NCD (see Section 2.3); if there is not sufficient (English) ‘number talk’ as input prior to Grade 1, some children will learn less, or at a slower pace, until they have had sufficient (English) numerical language input in Grade 1 (see Section 2.3). I surmise that they ‘translanguage’ during Grade 1, but that this action of forming an idiolect may come at a cost – they would need increased working memory activity, which may affect EF.

Table 4.8 : An example of integration of data

<table>
<thead>
<tr>
<th>Quantitative data</th>
<th>Qualitative data</th>
<th>Researcher’s reflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qualitative data which are used to enrich and explain quantitative findings</td>
<td>There was a statistical significant increase in NCD in Grade R (18.93) and Grade 1 (23.86) when children were tested in their home language in Grade 1.</td>
<td>Teachers’ knowledge of Grade R children’s cognitive skills is beneficial for their teaching.</td>
</tr>
<tr>
<td></td>
<td>Teachers need tools for reflection about children’s NCD.</td>
<td></td>
</tr>
</tbody>
</table>

There was no statistical significant increase in NCD in Grade R (18.93) and Grade 1 (17.73) when children were tested in English in Grade 1. In fact, the mean decreased.

Language acquisition and cognitive skills as intersects for the development of number concepts.

MSV is data for NCD. If there is not sufficient number talk in English, children cannot learn maths optimally in English in Grade 1.

Elaborated-/ restricted linguistic codes.

Idiolect of individuals goes across natural languages - translanguaging explains this.

There was a significant difference between the Grade 1 home language NCD scores (23.86) and English Grade 1 NCD scores (17.73).

The isiZulu children outperformed the Sesotho children in NCD when tested in their home language in Grade R (means = 20.55 vs 17.11, p < .01) and Grade 1 (means = 25.71 vs 21.93, p < .01).

Teacher 3’s translator perspective on number names.

I hypothesize that longer number names in isiZulu might influence NCD. Language acquisition in Grade R influences Grade 1 NCD.

There was a significant correlation between the MARKO-D SA Grade R scores and MARKO-D SA Grade 1\textsubscript{HL} scores (.4, p <.01); MARKO-D SA Grade R scores and MARKO-D SA Grade 1\textsubscript{eng} scores (.27, p < 01); and MARKO-D SA Grade 1\textsubscript{HL} and MARKO-D SA Grade 1\textsubscript{eng} scores (.28, p <.05).

Diagnostic instruments and intervention programs support NCD.

The MARKO-D SA is a reliable, diagnostic instrument.
30% of the variance in Grade 1 NCD was explained by the regression model. Mathematic specific vocabulary, classroom engagement and logical reasoning were used as predictors while controlling for language of instruction.

Language acquisition and other cognitive skills in Grade R explain variance in Grade 1 NCD. Language acquisition and cognitive skills thus intersect with NCD.

When diagnosing possible MLD in Grade R children, not only NCD should be taken into account, but also MSV, CE, EF and LR.

Foundation phase teachers’ PCK must include the development of all cognitive skills for NCD.

Emulating a Finnish ‘teaching school’- model

Teachers’ discourse about NCD is disintegrated

Although the student teacher could not integrate theory and practice yet, it was clear that she had theoretical knowledge of cognitive development. The novice teachers’ training may be sufficient.

The three experienced teachers could not integrate their experience with theory about cognitive development. In this regard, the Finnish model might not have been implemented effectively.

Practice and research are starting to come closer together, but there is a long road ahead.

There was no significant difference between boys’ and girls’ NCD in Grade R or Grade 1.

This is an interesting finding and is a potential topic for future research.

4.4. Conclusion

In this chapter I presented the analysis of the data obtained from four instruments that reflected Grade R and Grade 1 children’s knowledge and cognitive skills. I examined
concurrent and predictive association between MSV, CE and LR and Grade R and Grade 1 NCD (in English and HL). To add practitioners’ views, I also analyzed the data of four interviews. In the last chapter I interpret the findings and draw conclusions which aim to respond to the research question. I close this paragraph with a statement I made in an earlier paragraph: If the thesis of my study holds, ‘number talk’ is important input – and, hence, MSV can be regarded as data for NCD and the ‘work’ of the number input analyzer.
CHAPTER 5:
DISCUSSION AND CONCLUSION

5.1. Introduction

In Chapter 1 I discussed the poor performance of many young South African children on national and international mathematics assessments (DBE, 2013, 2014; SACMEQ, 2010; TIMSS, 2015). It is clear from these assessments that South African children are not ready to learn early arithmetic and have not developed the crucial principle of cardinality of number yet. The implication is that these children are likely to struggle with mathematics learning throughout their school career (Spaull, 2017). Yet, mathematics competence at the end of a school career, remains the gateway for career opportunities (Adelman, 1999; Evan, Gray, & Olchesfke, 2006; McGregor, 1994). In this study I have investigated possible reasons for why many children in South Africa struggle to develop early number competence. I have examined how cognitive skills (MSV, CE and LR) are concurrently and predictively associated with number concept development. The aim was to find out how assessment outcomes of contributing skills can inform teachers’ pedagogy – not only how teachers teach, but also especially how they assess.

In Chapter 4 I presented the results of the inquiry, which showed that (existing) numeracy achievement upon the children’s entry into Grade R was the strongest predictor for achievement on a number concept development test upon entry into Grade 1. This is not surprising, as children tend to show developmental continuity in achievement. As hypothesized, the second strongest predictor for Grade 1 number concept development (NCD) was mathematics-specific vocabulary. It was to be expected that children who do not understand the language of instruction in mathematics classes well – and specifically the mathematics vocabulary – will perform at a lower level on number concept assessments than children who understand the language and know the vocabulary used for instruction (LeFevre, 2018). In this study, I conclude that the existing level of number concepts in Grade R and mathematics-specific vocabulary (MSV) are the most valuable input for number concept development. In other words, what children come with when entering Grade R is a
foundation that will predict their progress. The data also indicates that classroom engagement (CE) and logical reasoning were important contributors for number concept development. I conclude that classroom engagement (which might be an indicator of EF) and logical reasoning (LR) also serve as input and facilitate input analysis.

The claim of this study is that if teachers are informed about NCD, MSV, LR and CE, this knowledge could contribute positively to their pedagogy and to learning outcomes. Pedagogy is not concerned only with how teachers teach, but also with how they assess children’s cognitive skills and support children who need additional attention. To advance Grade 1 children’s numerical competence, I argue that teachers should take serious note of age-appropriate number concepts teaching, expand Grade R children’s mathematics-specific vocabulary, enhance their executive functions throughout the Grade R year to strengthen classroom engagement, and develop their logical reasoning skills during Grade R. All of these put expectations on a teacher to be highly informed practitioner and, as shown by recent studies such as Radebe (2018) and Henning (2013b), this is not easily achieved, because most teachers have been trained to focus on strictly methods, procedures and ‘curriculum coverage’ rather than understanding the cognitive phenomena studied in this thesis. In this study it was evident that the four teachers’ pedagogy was not yet sufficiently informed by theoretical knowledge of number concept development.

All four teachers, including the student teacher, were part of the ‘teaching school community’ and used the discourse of that specific school in the interviews. Because the teachers were positioned at Funda UJabule where they are often exposed to research and training, I had expected them to use a more integrated discourse which displayed knowledge of cognitive skills. However, although their discourse reflected some emergent knowledge of early NCD and diagnostic instruments, they struggled to consolidate the constructs of MSV, EF and LR and how these influence NCD.

Three of the four teachers were experienced and had used the language of experienced practitioners by providing examples to clarify what they meant, but their use of language did not reflect a specific ‘epistemology of awareness’ yet. Teacher 1’s answers were brief and it was clear from her language use that she ‘talked the talk’ of
the teaching school. She was quick to answer and used the discourse, not only of a teacher, but also that of a teacher educator. She accepted responsibility for both the children and the students. Teacher 2 spoke with a voice of authority and gave an honest, true representation of Grade R number concept teaching. She answered slowly to make sure her answers were thorough. She argued that she had a role of a teacher and teacher educator, but she is also responsible for her own professional development and therefore often reflects on her own teaching. When she was unsure about the meaning of executive functions, she tried to understand the question properly before attempting to answer. Teacher 3’s answers were brief, crystalized and to the point. She spoke with the authority of an experienced teacher and her answers reflected a more integrated understanding of NCD than the other three teachers, however she did not know what EF were and struggled to integrate LR and NCD.

The student teacher locum, on the other hand, made sense of her reality from a more theoretical perspective, rather than from her limited experience. Her discourse reflected that of an ambitious academic and researcher rather than an experienced teacher. She had a good theoretical foundation and was well informed about the curriculum. Her answers reflected a sound knowledge of psychological development, developmental neurology and early NCD. However, her answers weren’t well integrated with practice and were at times somewhat segmented. She could mimic the knowledge, but not necessarily apply her knowledge to practice.

5.2. Interpretation of the data

The findings of this study showed that the assessment of each cognitive skill (NCD, MSV, CE and LR) assessed at the beginning of Grade R explained variance in children’s achievement in Grade 1 NCD assessments. In the regression models, each one of the variables (NCD, MSV, CE and LR in Model 1; and MSV, CE and LR in Model 2) were used as predictors, while controlling for language of instruction. This leads me to conclude that each of the combination of instruments used in this study can possibly be used to identify ‘deficits’ in specific cognitive skills that may lead to MLD. However, replications of the study with larger, more representative samples must be conducted to confirm this hypothesis.
From experience as a foundation phase teacher, I know that knowledge of these skills allows a teacher to take all possibilities into account when assisting children who struggle to develop number concepts. The combination of, or individual use of the four instruments used in this study, and knowledge about the constructs that the instruments measure, can contribute to early grade teachers’ PCK. This type of knowledge is likely to inform the way teachers teach, assess number concept development and support children who struggle.

In this section I present an interpretative rendition of the findings in five broad categories. I begin with a discussion about the association between results of NCD Grade R and NCD Grade 1 since NCD Grade R was the strongest predictor for NCD in Grade 1. Then, I discuss language as input for NCD because the data showed an association between MSV and NCD. Thirdly, I discuss individual differences in children with a focus on varying CE and LR, which influenced the variation in NCD scores in Grade 1. Next, I discuss early grades diversity and lastly, how knowledge of NCD, MSV, CE and LR could collectively contribute to early grade teachers’ pedagogy – for in-service and pre-service teachers.

5.2.1. The MARKO-D shows developmental continuity in early NCD

I have argued, with authors such as Brannon (2000), Fritz et al. (2012, 2013), Henning et al. (2018) and Resnick (1989), that number concepts develop hierarchically, and that children’s number concepts are likely to develop over a period of one year (from one grade to the next in school). It is not surprising that the level of their number concept development in Grade R was the strongest predictor for number concept attainment in Grade 1 (tested in their home language). To test the construct of NCD, the MARKO-D SA was used as the instrument. This test has previously been proven to be a reliable diagnostic instrument (de Villiers, 2015, Herholdt, 2017; KELLELO & JET Educational services, 2018) and in this study the instrument also proved to be reliable. The Grade R NCD achievement score not only correlated with numerical competence score in Grade 1 (HL and English), but the Grade 1 NCD score also correlated with the Grade 1 English NCD score.

In 2017 KELELLO, an evaluation consultancy, and JET Educational Services conducted an evaluation of the ‘R-Maths’ intervention program in the Western Cape
(KELLELO & JET Educational Services, 2018). They used the MARKO-D SA to assess children’s number concepts in a pre- and post-test with a random sample (N=622\textsuperscript{41}) and a control group of Grade R learners in the Western Cape Province in three South African languages. Apart from the ‘R-Maths’ evaluation research project, no other study in South Africa has used the MARKO-D SA on such a large scale. Previous studies with the MARKO-D SA (de Villiers, 2015; Fritz et al., 2012, 2013) were only used to assess the instrument itself and not to use the MARKO-D SA to determine a correlation between Grade R and Grade 1 NCD.

Qualitative data analysis indicated that teachers agreed that the aim of a diagnostic instrument, or set of instruments, should always aim to support children in the areas where they may struggle. Therefore, it is helpful to know that the MARKO-D SA has the potential to predict poor numerical competence in Grade 1 as early as the beginning of Grade R. Grade R children’s number concept development can either be improved as part of the daily teaching which takes place over a long period of time or via an intervention program which may only be used to strengthen one skill for a short period of time.

5.2.2. Language as input for early NCD

An important finding of this study is that, although children’s performance on the early number concept assessment increased from Grade R to Grade 1 when tested in their home language, it decreased when tested in English\textsuperscript{42} in Grade 1. The difference between the English Grade 1 MARKO-D SA scores and Grade 1 home language MARKO-D SA scores was statistically significant. Also, although NCD in Grade R was the strongest predictor for NCD in Grade 1 (which was to be expected), MSV in Grade R was the second strongest predictor of NCD in Grade 1. This suggests that MSV is a key tool for early number concept development.

This finding answers the secondary research question, “Does language of teaching play a role in children’s number concept development” and can be explained, to some extent, by the analysis of qualitative data. Teacher 1 said that language helps

\textsuperscript{41} Intervention group sample =310; control group sample = 312.

\textsuperscript{42} Although the hypothesis did not address the difference in MARKO-D scores when tested in the children’s HL and English, it indicated that language is likely to contribute to children’s NCD. Therefore, this is still an important finding.
children to understand a concept. The student teacher confirmed this idea by saying that: If children are not ‘grounded’ in their language it is difficult to learn, communicate or understand. The student teacher also noted that instruction influences how children learn mathematics. She said that it is pivotal that teachers use the correct mathematical language when they teach. Although teachers had some idea about how language and conceptual development intersect, they were not sure precisely how and did not address the question: “How is the development of words or other linguistic structures that encode certain types of concepts related to the development of the concepts themselves?” (Gopnik & Meltzoff, 1997:189).

To discuss the idea that language influences learning and to explain the association between mathematics-specific vocabulary in Grade R and numerical competence in Grade 1, I integrate 1) García’s theory of translanguaging (García & Hesson, 2015; García & Seltzer, 2016 & García & Lin, 2017) and Bernsteins’ theory of restricted/elaborated linguistic codes (1971) with 2) Levine and Baillargeon’s theory of number talk as data input (Levine & Baillargeon, 2016); 3) Vygotsky’s theory of interaction between pre-linguistic concepts and pre-conceptual language (Kozulin, 1990; Vygotsky, 1962, 1978, 1986); and 4) Carey’s theory of conceptual change and the role of language in this theory (Carey, 1988, 2009).

In Chapter 1 I discussed the ‘multilingual maze’ (Henning, 2012a) many children in South Africa encounter and referred to ‘code-specific’ environments in South African classrooms. At Funda UJabule, children are instructed in their home language in Grade R and then change from one ‘code specific’ environment to an English ‘code’ as the language of instruction in Grade 1 in mathematics lessons. Although English is the language with strong local currency, it is not a code many suburban children are exposed to before formal education. Children who come from some suburban and ‘township’ communities have, what Bernstein (1971) refers to as, a ‘restricted code’, compared to children in middle class households with an ‘elaborated code’ because of their exposure to several ‘restricted codes’. Children with elaborated codes can cross between different linguistic registers and speech genres and can engage in number talk in various codes. Children with a ‘restricted code’ use the rich code of their direct community, but do not cross many code barriers before they enter school – even, I would argue, in the era of digital technology.
The findings of this study indicate that children in this sample, with limited knowledge of number talk (and a limited exposure to linguistic mathematical codes) in the language of instruction in Grade 1, face many linguistic challenges. García and Lin (2017) argue that the cognitive tool of translanguaging can assist young children to identify the meaning of a message in an unknown language. Otheguy et al. (2015:283) define translanguaging as “the deployment of a speaker’s full linguistic repertoire without regard for watchful adherence to the socially and politically defined boundaries of named (and usually national and state) languages.”

Often, children in South African classrooms learn English mathematics-specific vocabulary incidentally as they are being instructed in English as well as their African home language. When translanguaging, children may at first not explicitly understand concepts. Gradually, by means of an inductive process of conceptual change, children begin to make sense of the words in a ‘new more elaborated code’, which crosses code barriers. When the new, slightly unfamiliar, English mathematics-specific vocabulary is connected with their familiar word knowledge, they might be more able to understand mathematics-specific English terms and be able to map the English vocabulary onto their existing number concepts. Lev Vygotsky described the mapping of words onto existing concepts as a “sense that becomes objectivized in words” (Kozulin, 1990:8). When English vocabulary has been mapped onto existing number concepts, children can also express numerical understanding by representing their number concepts through language.

In Section 2.4.1 I referred to Lev Vygotsky’s view of the interaction between the development of language as a place-holding structure for mathematical understanding (pre-intellectual language) and conceptual development that precedes the development of mathematics-specific vocabulary (pre-linguistic concepts) (Kozulin, 1990; Vygotsky, 1986). In this study it is evident that children find it hard to integrate Grade R number concepts (concepts developed prior to learning English) and English mathematics vocabulary. I propose translanguaging (García & Lin, 2017; Otheguy et al., 2015; Vogel & García, 2017) as tool to assist them in the integration of their vocabulary for concepts (that they may have acquired via their mother tongue) with English vocabulary.
During the interviews, Teacher 3 gave an example of translanguaging. She explained that most Sesotho and isiZulu speakers prefer to use English number names (word numerals) and that most children are familiar with the English terms, rather than their home language number words. She explained that the number words in isiZulu and Sesotho “are intense” and that “most mathematical concepts, whether we like it or not, are in English”. English number names are evidently part of children’s idiolect, or linguistic code when they enter Grade 1.

If children have a restricted ‘input of linguistic code’, poor numerical competence is likely to be observed in Grade 1. In Levine et al.’s study (2010), discussed in Section 2.4, the authors concluded that 1 200 number word utterances per year compared to 100 000 is not a level playing field for number concept development. Children who do not have sufficient experience of ‘number talk’ are likely to struggle more than those who have optimal linguistic input. In the same vein I would argue that a ‘restricted code’ and ‘elaborated code’ of mathematics-specific language is not a level playing ground. Levine et al. (2010) found that the quality of number talk input also mattered. Levine and Baillargeon (2016) argue that language itself can be seen as data input for developing numerical concepts. The more ‘number talk input’ children receive (or exposure to elaborated codes) and the better the quality of number talk, the better their chances are of developing an understanding of number concepts. That is why teachers’ PCK should include a critical view of the role of language with all its levels (phonological, lexical, semantic, syntactic and conceptual – as suggested by Dowker & Nuerk, 2016).

During the interviews, Teacher 2 said that in Grade R “it is easy for [the children] to relate to what the teacher is saying because they know it from a young age”. Their home language is the ‘restricted code’ or number talk they are exposed to, but it already has code-mixes in urban township life and the beginnings of an idiolect. But changing from the ‘known code’ to an ‘elaborated code’ which is unfamiliar, has significant implications if teachers do not understand the notion of an idiolect.

Furthermore, the data of this study show that the implications of limited number talk (in any configuration of code) as input are negative for children who have to find their way in various codes in a ‘maze’. The student teacher proposed that “we should include mother tongue language in our mathematics teaching [in Grade 1] so that a
child may have a proper grasp of this concept” while still “using the correct mathematical language” in both English and children’s home language. By adopting translanguaging as a pedagogic tool to teach number concepts, teachers allow for children to access their whole idiolect, which may consist of various linguistic codes or forms of number talk.

The theory of translanguaging can also be applied to Carey’s model of conceptual change and conceptual development (Carey, 2009). In Section 2.4.2 I argued that children’s number concepts also have ‘conceptual vocabulary’. According to Carey, the challenge is to understand the transition from one vocabulary to the next – the change from one set of terms with elaborations (conceptual change indicators, I would argue). While developing new vocabularies, children learn new concepts from adults “through making sense of adult language... and as the result of explicit teaching” (Carey, 2009:415). However, if teachers do not know how to ‘explicitly teach’ and to assess number concept development, they might not be able to transition to new knowledge, in other words, conceptual change.

Creese and Blackledge (2010:108) say that the information conveyed by a teacher will not be complete without using the recourses of both languages that children use, complementarily in the case of bilingual education (which is the model of the school where I did the research). Using a monolingual code, like English, may not be as helpful as ‘immersion’ theorists, such as Krashen, believe (Krashen, 1985). This was affirmed by the results of this study: When using mother tongue as language of assessment of number concepts with the MARKO-D SA, the mean increased from Grade R to Grade 1. But when using an English test only, the mean decreased from Grade R to Grade 1.

5.2.3. Individual differences in young children’s NCD

Not all children struggle with the same cognitive skills that contribute to NCD. Some children may experience difficulty, due to a ‘restricted’ linguistic code that prevents them from making sense of number concepts explained in a language they do not understand sufficiently, while others may struggle with inhibitory control or poor working memory. One child may need intervention to develop an understanding of the decomposability of numbers, while another may experience mathematical difficulties due to poor logical reasoning skills.
This study’s results show that some children have difficulty in forming number concepts in Grade R, but are actively engaged in class and can reason logically. Yet, they seem to still perform weakly in Grade 1. This can possibly be explained by the hierarchical structure of number concept development. Some children simply take longer to build concepts on their ‘wall of understanding’. Other children performed poorly on Grade R MSV and CE assessments and Grade 1 NCD assessments. In this instance I conclude that language and classroom engagement could possibly explain their poor performance on NCD assessments in Grade 1. Many children with low MSV scores were less engaged in class and performed poorly on the number concept assessment in Grade 1. My interpretation of these cases is that children who do not understand the language of instruction seem to become bored and less engaged in class. Poor classroom engagement and lack of understanding the language of instruction are likely to deprive children of learning opportunities which allows for number concepts to develop.

In Section 2.6.2 I referred to the correlation between classroom engagement and executive functions. I argued that a child who is self-regulated and who can pay attention in class – which leads to more active class engagement – is more likely to grasp number concepts. Children with good executive functions tend to be more engaged in class (Fitzpatrick & Pagani, 2012). There is also an ever-increasing view from scholars that mathematical competence and executive functions are closely related (Blair, 2016). When new strategies are learnt, children need to suppress their desire to return to a familiar, less efficient strategy. For instance, when calculating 34 + 25, children should rather make use of their knowledge of place value rather than reverting to counting strategies. Or, when comparing a set of four large objects to a set of seven small objects which takes up more space, inhibitory control is needed to pay attention to the number of items rather than the size of items. In the interview data, Teacher 4 explained the association between executive functions and number concept development by mentioning that Grade 1 children often struggle to shift from addition to subtraction once a new operation is learnt.

Furthermore, working memory is likely to be important in a multi-step calculation, where children have to keep different parts of information in mind concurrently to complete the calculation. For children to understand a number concept,
they must keep in mind what that concept is, use their working memory to integrate information to calculate and very often they employ logical reasoning to think about the solution to the problem. For example, when children must find out what half of 35 is, they must first remember what the word ‘half’ means, then they must be able to break 35 up into 30 and 5. Next, applying logical reasoning and factual knowledge they break 30 up into 20 and 10 and find out that the half of 20 is 10 and the half of 10 is 5. So, relying on their knowledge of place value they are able to see that half of 30 is 15. Next, they must know and understand that the half of 5 is 2½, implementing their understanding of fractions. Working memory and cognitive flexibility then operate to switch between all these steps, while they integrate 15 and 2½ and arrive at the answer of 17½. Children’s cognitive flexibility and logical reasoning may also allow them to think of another way of finding half of 35. Cragg and Gillmore (2014) argue that executive function is a dependent predictor for academic performance in general rather than just in mathematics.

Logical reasoning is also associated with number concept development and other cognitive skills. Logical reasoning and language is both dependent on working memory (Ashcraft & Kirk, 2001) and inhibitory processes (e.g., Bull & Scerif, 2001; Simoneau & Markovits, 2003). Both logical reasoning and mathematics require the child to retrieve and apply normative rules and draw conclusions (Morsanyi & Szüz, 2015). Based on these similarities, Morsanyi and Szüz (2015) hypothesized that number concept development and logical reasoning must be correlated. The findings of this study have proven that this hypothesis was true for the current sample.

Thus, the concurrent and predictive associations between Grade R and Grade 1 NCD assessments and MSV, CE and LR, indicate that children with an ample mathematics-specific vocabulary, who are engaged in classrooms, and who can reason logically, are likely to develop early numerical competence with more ease than children who have inadequate skills. Also, children who are highly engaged in the Grade R classroom, and can exercise self-control, are more likely to develop an elaborated linguistic code that will allow them to make meaning of number concepts. Children who are cognitively more flexible than their peers, are more likely to be able

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Although this study did not test associations between EF and logical reasoning, the link between these skills is interesting, and therefore I mention it.
to reason logically and therefore perform better on number concept assessments. In turn, number concept development itself provides children with skills and knowledge to reason more logically. These are reciprocal cognitive development skills and concepts. This interconnectedness possibly explains individual differences in Grade 1 children’s numerical competence achievement.

Dowker (2008) argues that there are many individual differences in children and because of children’s individuality it is important to know which specific skill (that contributes to number concept development) needs to be supported to strengthen in Grade R for each child. For this purpose, I propose a combination of diagnostic instruments for assessment at the beginning of Grade R.

5.2.4. Early grades learner diversity

The qualitative data of the study showed that teachers recognized diversity in children that explains differences in early grades academic achievement (Theme 3). All four teachers mentioned that the social environment plays a role in cognitive development. As children “notice things” (Teacher 1) in their unique environment, they develop their own idiolect and theories about reality and number concepts. Also, the student teacher argued that children’s genetic makeup – or core conceptual knowledge – contributes to individual differences in the early grades.

Another difference in the number concept development was observed in the difference between isiZulu and Sesotho speaking children. T-tests showed that isiZulu children outperformed the Sesotho children in Grade R and Grade 1 in the home language versions of the assessments. Dehaene (2011:90) argues that poorer performance on number concept tests in one language, compared to another language, can possibly be attributed to longer number names and the lexical and grammatical composition of number names (see also LeFevre, 2018). Regarding isiZulu and Sesotho differences in number words, the difference in the two language groups different performance on the NCD test is a topic that must still be investigated, but I propose that Dehaene’s theory may apply in this context. The Sesotho number names are “very long and intense” (Teacher 1). The hypothesis that longer number names may overload the working memory and in that way influence concept
development, must still be explored. However, the isiZulu number words are also long, because it is an agglutinative language.

Coupled with linguistic- and environmental differences, some studies suggest that boys outperform girls on tests in early mathematics (Leder & Taylor, 2010), while other studies found that there are only negligible differences in number concept development between males and females and that females then outperform males (Hyde, Fennema & Lamon, 1990). According to Kersey, Braham, Csumitta, Libertus and Cantlon (2018), discussions suggest that under-representation of females in mathematics careers are due to differences in early in childhood, but if these claims are true, gender differences should be found in early number concept assessments. Kersey et al. (2018) assessed children’s numerosity perception, culturally trained counting and elementary mathematics concepts. Their findings revealed that across all stages of development, there are no gender differences between number concept development in boys and girls and that all children are thus equally able to reason about mathematics in early childhood (Kersy et al., 2018). My analyses revealed no mean difference between boys’ and girls’ number concept development in Grade R or in Grade 1. The reason for this finding is yet to be explored. Van Broekhuizen and Spaull (2017) showed that the situation in South African learners’ performance is different and that girls perform better than boys (see also Saito, 2011).

5.2.5. Pedagogy for early NCD in a teaching school

The school environment of this study is unique in South Africa. Because of the relationship between the school, the university and the provincial education department, teachers have many development opportunities. Teachers are exposed to research – both from the university and the provincial department of education – attend workshops and are in contact with the lecturers at the university. Student teachers have the opportunity to observe teaching in practice. Teachers at Funda UJabule have a good knowledge of the curriculum and know what the curriculum requires to prepare Grade R children for Grade 1. Students spend time in class and learn how to develop young children’s concepts.

One would think that a relationship between the school, university and department of education, as discussed in Section 1.3, provides teachers access to knowledge and support to identify children with possible mathematical difficulties.
However, I conclude that the quality and frequency of the interaction between the school and university is insufficient. During the interviews I realized that the teachers are experienced to teach with confidence and to mentor novice teachers by reflecting both on their own lessons and on the lessons of the students. However, when I listened to their reference to knowledge of children’s cognitive skills, I realized, like Henning (2013b), that they had not integrated their knowledge to their practice. As Premack and Premack (2003:227) say: “A theory of education could only be derived from understanding the mind that is to be educated” (as mentioned in Section 2.7.1). Teachers are yet to develop their own ‘theory of education’ by understanding how various cognitive skills contribute to the development of number concepts. For example, when asked how other cognitive skills, such as executive functions, contribute to children’s math learning, teachers had no idea. Teacher 1 asked that I repeat the question and read the informational cards, which explained the three executive functions, again. Teacher 3 expressed her uncertainty by saying: “If I’m answering this correctly...” and again later: “I’m not sure if I understand this question clearly” and reread the question regarding executive functions. The student teacher also begun her answer with: “Not really sure, but...” Teacher 1 only mentioned that executive functions must have something to do with sequencing and that it is therefore linked to maths, but didn’t elaborate because she did not understand the question.

5.3. Contributions of the study

This study has implications for research, practice and policy, but I suggest that the value of the contribution of this study (and other studies) can only reach its potential if there is a greater amalgamation of research and practice. Currently there are some joint practice, such as explored by Petker (2018), between the university and the school, but it has not yet succeeded in advancing the teachers’ knowledge of early mathematics pedagogy.

Within the current environment at Funda UJabule, teachers, researchers and teacher educators should be working from a shared knowledge base about how young children’s number concepts develop and which cognitive skills contribute to their mathematical development. Although such a structural organizational base has been developed (Gravett & Ramsaroop, 2015; Petker, 2018), in practice it does not seem as if teachers and teacher educators communicate sufficiently to ensure coherence
between theoretical and practical teacher training. Such a shared knowledge base should serve as the basis for teacher’s PCK, test design, curriculum implementation, teacher training and the design of new research projects so that all the components of the system work together towards a coherent set of goals. In this section I discuss the implications for practice, policy and research of this study.

5.3.1. Practice

The most important contribution of this study is that I can suggest instruments with a solid theoretical foundation that teachers can incorporate in their PCK. However, the sample of the study was small and not representative of the population beyond this school. Therefore, replications of the study are needed to confirm the use of this combination of instruments. Based on the findings of this study I propose that Grade R NCD, MSV, CE and LR collectively contribute to Grade 1 NCD. With that I propose that a combination of the MARKO-D SA, MMLT, CE and CFT1-R can be used to assess children’s contributing skills to identify children who may need additional support in developing numerical competence.

As a teacher, one of my greatest concerns is that children struggle to ‘show’ their knowledge of number concepts because they are tested in a language they do not understand – here I specifically refer to English, the language in which the children need to build their mathematics lexicon to use for the rest of their school career. That is why I have developed an assessment instrument that could assess children’s mathematics-specific vocabulary – the MMLT. In addition to the suggestions about instrument use, this study also contributes this newly developed test to assess mathematics-specific vocabulary. During this study, the test was developed in English and was translated to isiZulu and Sesotho. Although a pilot study should still be conducted to review and adapt the test, this instrument has the potential to assist teachers and other role players to determine whether vocabulary knowledge may be the reason for poor number concept development. It has the potential to be standardized and normed for use in South Africa in different languages.

Although I found that teachers’ knowledge of Grade R children’s cognitive skills is beneficial for their teaching and that they recognize social context and individual differences, teachers’ discourse about early number concept development is still fractured. This means that they have not yet developed an integrated understanding
and theoretical view about children’s number concept development. For example, the teachers’ discourse during the interviews revealed that they are unfamiliar with executive functions. Therefore, the literature study of this thesis can contribute to teacher education and development programs. Instruction in child cognition, and in the context of this study, specifically number concept development, can be successfully included in such programs, along with mathematics-specific vocabulary, classroom engagement (and executive functions) and logical reasoning skills.

In 2017, teachers at Funda UJabule and four other partner schools in the area participated in number concept development programs such as the ‘Meerkat Maths’ training program, which was adapted from the Calculia program of Fritz-Stratmann et al. (2014). To my knowledge, there are no training programs for teachers which focus on developing their knowledge of executive functions, mathematics-specific language and logical reasoning and how these constructs contribute to the development of numerical proficiency. Therefore, such programs can be developed to train both in-service and pre-service teachers.

In terms of mathematical language, I suggest the use of translinguaging as a code-elaboration technique to include the children’s mother tongue, as well as English. For this purpose, teachers should be bilingual and be allowed to use their own idiolect to help develop children’s mathematics-specific vocabulary so that they are prepared to learn mathematics in English once they reach Grade 1.

In terms of quality of number talk for the specific school where I conducted the study, I suggest a closer relationship between the university and Funda UJabule. Such a relationship could be embodied in short regular meetings (perhaps once a month) between teachers (not only head of departments) and early childhood lecturers. The meetings should consist of discussions about how to streamline teacher training so that student teachers can observe what they learn (at that specific time) in their academic classes. Conversely, teachers can also adapt their pedagogy in class so that they demonstrate the theory which lecturers discuss in academic classes.

5.3.2. Policy

I will present a brief note with a recommendation to the DBE to contribute to the ongoing investigation of the feasibility of the current foundation phase mathematics
curriculum – specifically also regarding language policy. This recommendation from a small study will also be forwarded to the DHET – as a contribution for initial teacher education in the PrimTEd program.

5.3.3. Research

The children who participated in this study were selected due to unique learning environment of South Africa’s only teaching school which is based on a Finnish model. This made the selection of the sample particularly relevant to the current study. In terms of research contribution, this is the first study where Grade R and Grade 1 children were studied in South Africa at a school at the beginning of their school career at a specific type of school.

There is a growing body of literature on children’s mathematical conceptual development. In this study I criss-crossed between theories of cognitive development, developmental psychology, developmental neuroscience, socio-cultural theories of conceptual development. From these theories I constructed a proposition for a view of number concept development which represents the intersect of language, classroom engagement and logical reasoning and of the development of numerical competence, the dependent variable of the study.

I approached this study with a focus on one issue that is situated at the confluence of research and practice. I argue that four cognitive skills are part of the (cognitive) input for numerical competence in Grade 1. In this section I organize the implications of this study for research in four broad categories, namely 1) synthesis of existing literature through an extensive literature review; 2) research that adds to an existing knowledge base about the development of young children’s number concepts; and 3) identification of new research themes. This study also included methodological contributions which will be discussed in the following section.

Synthesis of existing literature through a literature review.

Accumulated knowledge of early number concept development and associated cognitive skills has been synthesized in this study and is available for use in multiple educational constituents, namely teachers, educational researchers, curriculum developers, test developers, teacher trainers and policy makers. I recognize that there
is an ongoing need to accumulate, synthesize and distribute new knowledge and to continuously construct a knowledge base which could be shared by all role players.

Research that adds to an existing knowledge base about the development of young children’s number concepts

Funding was provided by the NRF for the South Africa Research Chair: ‘Integrated studies of Mathematics, Language and Science in Primary Schools’ which is a large program of research that is guided by the idea that children should empirically be studied, rather than focusing on teachers’ ideas and what the literature says alone. By focusing on a design that integrates statistical findings with the opinions of practicing teachers, this study added to an existing knowledge base by 1) Defining associations between mathematics-specific vocabulary, number concept development, classroom engagement, logical reasoning and numerical competence in Grade 1; 2) Developing a new assessment instrument to assess children’s knowledge of mathematics-specific vocabulary; 3) Confirming the hypothesis that instruction in a different language as mother tongue (or known languages) negatively impacts learning; 4) Concluding that teachers’ discourse about children’s cognitive development is disintegrated; and by 5) Confirming that the MARKO-D SA is a reliable instrument for identifying children at risk of MLD.

Identification of new research themes.

I put forward a few suggestions: 1) translanguaging as a technique, 2) ways to increase collaboration between research and practice, 3) the MMLT should be reviewed, adapted and put to the test in a pilot study and large-scale contexts, 4) why isiZulu children outperformed the Sesotho children on the number concept assessments, and 5) why there was no gender difference on number concept assessments.

5.4. Methodological contribution

This study offers an innovative methodological and analytical approach in looking at associations between preschool cognitive skills and Grade 1 children’s numerical competence upon entering formal schooling. It combines the simultaneous examination of four interview scripts and the statistical analysis of four sets of
measures of predicting cognitive skills and two sets\(^\text{44}\) of measures of the dependent variable, namely achievement on number concept development tests. The thesis focuses on a particular group of learners who learn number concepts in isiZulu or Sesotho in Grade R and in English in Grade 1.

Currently, relatively little empirical data is available on cognitive skills that influence Grade 1 children’s numerical competence in a multi-language South African setting. To describe the impact of preschool number concept development, mathematics-specific vocabulary, classroom engagement and logical reasoning on Grade 1 numerical competence, this PhD thesis applies an adapted design of integrating data to describe possible associations. The integration of quantitative- and qualitative data is not new. However, the particular design is valuable to do research in the field of education and offers 1) an ex post facto design for the quantitative data component, which assesses Grade 1 children’s numerical competence in two languages at the same point in time to compare how the ‘fact’ of Grade R has influenced numerical competence in Grade 1; 2) an integration of the qualitative data in an explanatory way to offer four practitioners’ insights to why a single group of children’s attainment on the same measure in two different languages is radically different; 3) a strong emphasis on the collaboration of researchers and practitioners within the only teaching school in South Africa; and 4) the unique combination of instruments.

Correlational studies that assess the association between predicting cognitive skills and an outcome skill usually use one language as language of instruction when assessing the outcome skill. Because I argue that language is one of the most significant contributors of number concept development, this design allows the researcher to assess the same construct (NCD) at the same point in time in two different languages in order to compare the attainment on home language instruments and English instruments. This strengthens the finding that language is an important factor in learning – not only for mathematical learning. This method can be applied to assess various skills and not only mathematical development.

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\(^{44}\) Numerical competence in Grade 1 was assessed in the children’s mother tongue and English which yielded two very different sets of data.
In this study I chose to use one test which has been adapted and translated from a European instrument. The MARKO-D SA has also been normed in South Africa and has been used in various studies in South Africa. The main focus of these studies, until thus far, has been to test the workability of the test itself rather than to use the test to assess children’s number concept development. I also used an instrument which has been developed and used in Canada (Fitzpatrick & Pagani, 2012). The classroom engagement report did not need any adaption or translation and could be used immediately in South Africa. The third instrument, the CFT1-R is currently also being adapted for use in South Africa. Lastly, because there was no instrument to assess children’s mathematic specific language, I designed such an instrument to fit the needs of this study. This instrument has been translated into isiZulu and Sesotho. The combination of a translated instrument (the MARKO-D SA), a ‘ready-to-use’ Canadian instrument for classroom engagement, an instrument which has been developed according to the needs of the study (MMLT) and an instrument which has been used in other SA studies, but not yet validated in SA (CFT1-R) has functioned well together and can therefore be used in future research projects.

Meta-methodologically, this research was conducted from an interpretivist perspective with the assumption that social constructs, such as mathematics and language, can be understood by trying to understand what the meanings are that people assign to them (Maree, 2009). The interpretive perspective was based on the assumptions (Maree, 2009) that 1) number concept development can only fully be understood from ‘within’ and therefore teachers’ perspectives were fundamental, 2) the human mind is a purposive source of meaning and therefore perspective of the researcher (who is also a teacher) was important during the integration of quantitative- and qualitative data, 3) the social world, with artifacts such as language and number concept development, does not ‘exist’ independently from human knowledge and therefore I included knowledge generated from quantitative data analysis.

5.5. Limitations

The design of this study required a single researcher to apply both statistical knowledge and knowledge of qualitative analysis. Therefore, I had to be dexterous in different modalities. I cannot claim to have optimized the possibilities of the ‘raw’ data, but have produced some synthesis. This study had a small sample and there is little
chance of it being replicated on a large scale, due to the time it requires. Larger scales of this design can be very time consuming and expensive (Johnson and Onwuegbuzie, 2004). During the first data collection, all four measurements were not done at the same time, because the teachers did not know the children well enough during February to complete the classroom engagement report. Therefore, the Grade R number concept development, mathematics-specific vocabulary and logical reasoning were assessed in February, while the classroom engagement reports were done in June. The ideal would have been to collect the data at the same time so that all the skills were assessed at the same age and within the same time frame. Furthermore, although I capture measurements of different variables that may influence number concept development, one of the limitations of a study with this design is that there are numerous influences on number concept development. Poor performance on a test which measures number concept development may be due to poor conditions at home, hunger, anxiety and so on.

This design does not allow the researcher to make causal inferences but can shed light on potentially important factors of which the importance can be confirmed through strong correlational designs and experimental intervention research. Causality means that if the value of one independent variable is changed, the value of the dependent variable will change accordingly. However, when there are many influences on number concept development, which the researcher cannot control, one cannot conclude that a change in numerical competence can definitely be ascribed to a change in one of the independent variables.

In terms of the tests, the MMLT that measures mathematics-specific vocabulary must still be reviewed by means of a pilot study with a larger sample. Although the MMLT correlated with the MARKO-D SA, CFT1-R and classroom engagement report, a split half reliability test indicated that the test does not have internal reliability for this sample. To compare this CFT1-R scores with another was difficult since there are no South African norms for the CFT1-R. The only other study (Coppard, under review) which currently uses the CFT1-R, uses a different version of the test.

The quality of the items of the test will be tested in a pilot study. Despite it being a custom-designed instrument, fitted to the children in a specific school, the MMLT can be seen as an everyday classroom test, which was inserted with the eventual aim
of validating the test with a much larger and more diverse sample. For purposes of diagnostic assessment and to be used widely by psychology practitioners, the test will be standardized and normed during future studies and will ultimately registered with the HPCSA. It will also be available in open access.

5.6. Future directions

Like others previously (LaRusso et al., 2016; Snow, 2015), I recognize that there is a gap between research and practice and that it takes time to bridge such a gap so that research and practice can truly interact. Although this thesis calls for interaction between research and practice and uses the collaboration between research and practice as basis for the research design, it is unlikely that the insights gained from this thesis will contribute to such an interaction. As Snow (2015) points out, research and practice must interact on more concrete levels by collectively building knowledge that benefits both parties.

In this study, one specific point of collaboration, which is evident, is the need to build a uniform knowledge base which is accessible for teachers, students and researchers of how young children’s number concepts are formed and how its pedagogy can insert this. Such a knowledge base should be constructed on the principle that research must investigate urgent problems in practice to formulate research questions and objectives rather than only ‘gaps in the literature’. Future research should thus focus on how to improve the collaboration between research, teacher training and practice by including all parties in constructing such a knowledge base. Funda UJabule provides an excellent environment where such a collaboration is possible. However, researchers and teachers should work more closely together. There could possibly be an individual who goes back and forth between classrooms and research centers to ensure optimal communication between the parties and to use input from all parties to create a uniform knowledge base to ensure that topics in teacher training and classrooms correspond. In this way, students can observe the same concepts in practice that they are learning at that particular point in time in the training program. In the same sense, teachers can benefit from research if the research topics are developed from practicing teachers’ ideas.

Some specific examples which could be researched in the near future within such a relationship, by negotiating researchable formulations of the urgent problems
in practice (Snow, 2015) include, 1) how teachers can support young children’s executive functions and classroom engagement, 2) how translanguaging could be used as a tool to teach number concepts in Grade 1, 3) why isiZulu children outperformed the Sesotho children on the number concept assessments, and 4) why there was no difference between the scores of male and female on the number concept assessments. These four topics arose from the current study’s findings.

Unfortunately, participating teachers did not know what executive functions are and could therefore also not provide sufficient information about how classroom engagement and executive functions contribute to number concept development. In the future, teacher training for in-service and teacher students should include training about executive functions and how these skills relate to classroom engagement and early number concept development.

**5.6.1. Where to with future diagnostic assessment?**

I would argue that the purpose of diagnostic assessment should always be to use the results to facilitate high levels of conceptual understanding. The vision for diagnostic assessments and teachers’ PCK is that teachers will ground their practice and assessments in the theory of how young children’s number concepts develop and how mathematics-specific vocabulary, classroom engagement and logical reasoning contribute to number concept development. Teachers and other test users should use this knowledge to design tests which will result in feedback on the level of number concept development and particular cognitive skills children can improve.

The MARKO-D SA has been tested and found to be a reliable instrument. Therefore, researchers and practicing teachers can continue to use this test as a diagnostic instrument. Together with the MARKO-D, the MMLT, classroom engagement report and CFT1-R, can provide useful information about contributing cognitive skills that affect numerical competence in Grade 1 as proven in this study. However, for future use the MMLT must still be reviewed and adapted. The CFT1-R is currently being adapted and implemented by Coppard (under review) and will be available for future use.

In terms of the classroom engagement report, another observation can be included from a different assessor, other than the child’s classroom teacher to improve
reliability. At Funda UJabule, in collaboration with the University of Johannesburg, each student is assigned to observe one individual at the beginning of their studies (which is also the beginning of the Grade R children’s school career). As the child progresses through the four years in the foundation phase (Grade R-3), the same student continues to observe the specific child that was assigned to the student in the first year of teacher training. Such a student can also complete a classroom engagement report at the same time as the teacher so that the two reports can be compared in order to provide a more accurate idea of each child’s classroom engagement.

5.7. Conclusion: Pedagogical content knowledge is evident in teacher’s discourse

This section summarizes the main findings of the thesis and presents conclusions drawn about the research question and objectives. I begin by referring to conclusions about the literature review and then turn to conclusions regarding the empirical findings of this study.

The purpose of the literature review of this study was to view the trends in research about influences on number concept development. Because there are many cognitive skills that collectively contribute to Grade 1 children’s number concept development, an assessment of these skills in early grades can highlight possible difficulties in terms of number concepts, mathematics-specific vocabulary, classroom engagement and logical reasoning that may hinder a child to develop numerical competence (Desoete, 2015).

The literature review confirmed the hierarchical structure of number concepts that young children develop during preschool and Grade 1. Because of this hierarchy, children who have not yet formed the early number concepts – for instance counting, ordinality and cardinality – struggle to grasp the part-part-whole concept and relationality between numbers, which come later. Therefore, it was not surprising to find a strong correlation between Grade R NCD scores and Grade 1 NCD scores. This meant that children who start off with higher levels are inclined to stay ahead. The findings of the study again lead me to the conclusion that teachers should be
knowledgeable about children’s cognitive development to align with their teaching, rather than ‘chase the curriculum’ (a teacher’s comment in the interviews).

The literature is also rich with explications of how language contributes to number concept development. From verbal working memory, phonological awareness, lexical composition of number words, semantic meaning of words, syntactic structure of language and especially conceptual properties of language are all input for number concept development. In this study I focused on the conceptual properties of language and hence the strong correlation between mathematics-specific vocabulary and number concept development. I conclude that an assessment of conceptual properties of mathematics-specific language should be included when assessing children’s cognitive skills in Grade R. This has prediction value for academic achievement in Grade 1 and expands teachers’ PCK.

Positive learning behaviors in the form of task orientation and classroom engagement has been shown to predict academic achievement – including number concept development (McKinney, Mason, Perkerson, Clifford, 1975). Also, preschool children, with a positive attitude towards learning, who reflects a good attention span, persistence and independence show better academic achievement than less engaged children (McWayne, Fantuzzo & McDermott, 2004). Together with the findings of this study – that classroom engagement scores were correlated to the scores of numerical competence – I conclude that an assessment of classroom engagement can contribute to identify possible difficulties with learning. Lastly, there was also a correlation between logical reasoning skills and number concept development. Thus, I conclude that this skill should also be assessed in Grade R to contribute to the diagnosis of possible MLD in Grade 1.

In terms of the study design, I tested children’s number concept knowledge in Grade 1 in two different languages. In particular, the observed differences between English and home language scores allowed for a hypothesis about a possible linguistic teaching tool, namely translanguaging, which is a free use of code-switching and translation, distinguished from purist views of language of instruction. These observed differences also indicates that language of teaching does seem to play a role in young children’s number concept development. Further, the integration of qualitative assisted the researcher to analyze the data from an interpretivist perspective and to explain
relations between predicting cognitive skills and number concept development from a practitioner's point of view. Analysis of the interview data also provided the researcher with information about current teachers' perceptions of cognitive input for number concept development. However, the qualitative analysis led me to the conclusion that, although teachers' technical 'epistemology' of Grade R children's number concept development shows some emergent critical reflective capability, their discourse is fragmented and they lack knowledge of some cognitive skills, such as executive functions. The teachers have developed the skill of reflection and have experience to reason about children's number concept development but are not able to integrate their knowledge into a coherent view of the skills that contribute to children's numerical competence yet.

With regard to the objectives to capture Grade R and Grade 1 children's performance/attainment on the tests of the study and identify a relational pattern between the scores of the various instruments, I conclude the following:

1. Number concept development in Grade R is an important predictor for numerical competence in Grade 1. The MARKO-D SA has also been confirmed, as in previous studies, to be a useful instrument.

2. The difference between the Grade 1 number concept scores in English and their home language, suggest that children cannot make the transition from learning in their home language to learning in English. To assist with the transition I propose translanguaging as discussed in Section 2.4.4.

3. Mathematics-specific vocabulary is important knowledge if children want to learn mathematics. The instrument that was designed for this purpose, as part of the study, was sufficient for this study, but still needs refinement.

4. The lack of gender difference in number concept development and the significant difference between the isiZulu and Sesotho groups must be investigated.

Lastly, the instruments used for assessment of number concept development in Grade R, mathematic specific language, classroom engagement and logical reasoning, collectively contribute to number concept development in Grade 1. Knowledge of assessments and theory that grounds these instruments expands
teachers’ PCK. The MARKO-D SA is sufficient in its current form, however, the MMLT needs refinement and further testing. The CFT1-R is currently being reviewed, but has proved to be a useful instrument. The classroom engagement report can be used, but I would suggest that more than one person scores each child’s engagement to obtain more objective scores.

To summarize, isiZulu and Sesotho speaking Grade R children’s attainment on four assessment instruments can be used to identify difficulties in the hierarchical development of number concepts, a lack of mathematics-specific vocabulary, poor classroom engagement and poor logical reasoning which could individually or collectively contribute to possible mathematical learning difficulties in Grade 1. Together with knowledge of each cognitive skill, which contributes to teachers’ PCK, these instruments allow teachers to describe children’s mathematical development. Unfortunately, teachers seem reluctant to change their PCK and discourse due to continuous pressure from the DBE to teach according to a daily schedule rather than from a foundation of informed teacher knowledge. Teachers are expected to implement the curriculum, rather than to teach numerical competence. I argue that teachers will only change the way they teach once language policy in education and political influences change.

Finally, I suggest that 1) if we want children to develop the necessary number concepts in Grade 1, Grade R teachers must be trained to improve their knowledge of contributing cognitive skills for number concept development in Grade 1; and 2) translanguaging should be considered as an alternative teaching tool to elaborate children’s linguistic codes in Grade 1 to include both mother tongue and English so that they learn the phonemes, lexical composition of number words and conceptual properties of language in both languages.


Fritz, A. (2016, October). A mathematics test to assess conceptual knowledge of number in the early grades: Theoretical basis, evaluation and interpretation of test results. Paper presented at the meeting of how a standardized interview test for numerical concept assessment of 4-8 year-olds developed in four languages, Johannesburg, SA.


Van den Berg, S. (2016). What the ANA’s tell us about socioeconomic learning gaps in South Africa. RESEP conference proceedings held at the University of Stellenbosch, Stellenbosch, April 2016. Stellenbosch: Department of Economics.


APPENDIX A:

THE SETTING:
A PRIMARY SCHOOL WITH A SPECIFIC BRIEF

This study took place in a teaching school at a comprehensive university in Johannesburg. The school, Funda UJabule, was born from a partnership between the Gauteng Department of Education and the University of Johannesburg. In 2010, the school started with two Grade R classes, one isiZulu and one Sesotho class. The school has progressed to now offer education to children from Grade R to 7. The school serves the children from the surrounding suburban/township area, namely Soweto. Funda UJabule also operates as a practice site for student teachers who are registered at the University of Johannesburg and provides support programs for teachers from four partner schools in the surrounding area. The mission of this ‘laboratory’ school is to integrate research-based teaching, and serve as site for primary school teacher preparation. The school, which is located on the campus of the university, is also a research site, with studies on pre-service teachers (Loukomies, Peterson & Lavonen, 2018) and child- and teacher development (Henning & Ragpot, 2015; Radebe, 2018; Ramsaroop, 2016; Petker, 2018).

Funda UJabule is a mainstream school and follows the South African curriculum. Within mainstream education the focus of management, governing bodies and professional staff are aligned with the inclusion model of school education (DoE, 2001). According to this model in-school support structures enable teachers to identify children who might need additional support and early intervention. Children with severe disabilities attend special schools. Although I mention the special schools and acknowledge children with severe disabilities, this particular study’s focus is on children who can attend mainstream schools. According to the Education White Paper 6: Special needs education (DoE, 2001), when children with possible learning difficulties are identified, the school-based support team designs a learning and support plan for each individual child. A district support team provides a co-ordinated professional support service that draws on expertise in further and higher education and local communities. The district support teams are based at full-service schools where support material and experts provide further support (DoE, 2001).
The setting and structure of the Funda UJabule teaching school in Soweto is based on the model of Finnish teaching schools. The philosophy of teacher education reflects the belief by the larger community that the quality of teachers greatly contributes to the quality of the education system (Morgan, 2014). According to Morgan (2014), learners in Finnish schools perform well on international tests because of the way in which education students are selected and educated. At Finnish universities, only 700 of the best performing applicants for teacher training are selected for the teacher training program, which is sponsored by the Ministry of Education and Culture (Morgan, 2014).

The model of a teaching school can be compared to that of an academic hospital where doctors are taught how to apply their knowledge in the setting of a real hospital during their undergraduate years. During teacher training programs, students are prepared to be researchers and teacher practitioners. Their training includes time spent at model schools where they learn how to apply science-based teaching pedagogy by being mentored by practicing teachers (Morgan, 2014:454). 15 to 25% of their training consists of practical training in teaching schools where they observe expert teachers, practice teaching, and receive feedback and coaching from faculty members and supervising teachers (Morgan, 2014). The teachers in Finland annually spend on average 600 hours per year in the classroom, teaching, compared to 1080 hours a year for teachers in the United-States. Teachers in Finland therefore have significantly more time to spend on acquiring and perfecting new teaching skills, interacting with the community, and working on curriculum and assessment. Assessments in the Finish system also tends to be much less formal. They prefer a narrative form to give feedback to students and describe their learning progress (Morgan, 2014; Sahlberg, 2015).

The teaching school in Soweto follows the Finnish model in many respects. The faculty of education, in collaboration with the school, is the only model for this kind of teacher education in South Africa, with one variant at the Siyabuswa campus of the University of Mpumalanga (Gravett & Ramsaroop, 2015). Similar to the teacher education in Finland, education students spend time in the foundation phase classrooms and have opportunities to observe teachers and to practice what they have observed by teaching some of the lessons under the supervision of a mentor teacher (Petker, 2018).
APPENDIX B:
MARKO-D SA SCORE SHEET
EXAMPLE ITEMS

The items provided in this Appendix is only examples and many of the items’ content have been changed due to copyright.

Child's first name ____________________________________________
Child's last name ____________________________________________
Sex      O F       O M

Do not ask learner for date of birth, check the register or ask learner's teacher

Date of birth ________________________________
Date of test __________________________________
Home language ________________________________
Language of instruction _________________________
Language of testing ____________________________
Test supervisor _______________________________
Name of institution ____________________________

GENERAL CONDITIONS (E.G., INTERRUPTIONS):
_________________________________________________________________
_________________________________________________________________

FURTHER OBSERVATIONS (E.G., COMPREHENSION PROBLEMS, ANXIETY, RESTLESSNESS):
_________________________________________________________________
_________________________________________________________________
If child answers incorrectly, please note down the answer in the box on the right side!

### Counting and Identifying Preceding and Succeeding Numbers:

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Counting to 10</td>
<td></td>
<td>1</td>
<td>0</td>
<td>Counted correctly up to: Other numbers named:</td>
</tr>
<tr>
<td>2</td>
<td>Number before 8</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
</tbody>
</table>

### Counting of Sets

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>How many stones?</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
<tr>
<td>5</td>
<td>How many stones?</td>
<td>8</td>
<td></td>
<td></td>
<td>Incorrect answer given:</td>
</tr>
<tr>
<td></td>
<td>Or previously named number</td>
<td></td>
<td></td>
<td></td>
<td>O answers directly</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>O counts again</td>
</tr>
</tbody>
</table>
Counting of Sets in Word Problems:

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Immediately answers correctly.</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
<tr>
<td>8</td>
<td>Task with counters: $2 + 2 = 4$</td>
<td>$2 + 2 = 4$</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given: Quantity laid:</td>
</tr>
</tbody>
</table>

Organising sets:

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>Complete rows by adding 4</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
<tr>
<td>14</td>
<td>Complete rows by adding 2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
</tbody>
</table>
### Recognising Differences between Sets:

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Which row has more (stars)?</td>
<td>Bottom</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
<tr>
<td>24</td>
<td>How many more are there?</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
</tbody>
</table>

### Determining Number Relations:

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>One smaller than 6</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
<tr>
<td>26</td>
<td>One bigger than 7</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
</tbody>
</table>

### Counting in Steps:

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>10..8..6..</td>
<td>4..</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
</tbody>
</table>
## Determining Sub-Sets, when only One Set is Visible

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>7 counters, how many to be taken away so that 5 are left?</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
<tr>
<td>43</td>
<td>4 counters taken away, 5 are left how many were there under my hand before?</td>
<td>9</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
</tbody>
</table>

## Making Equal Sets

<table>
<thead>
<tr>
<th>Item</th>
<th>Requirement</th>
<th>Solution</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>Make sets equals by putting down 5</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
<tr>
<td>47</td>
<td>Make sets equals by putting down 5</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>Incorrect answer given:</td>
</tr>
</tbody>
</table>
APPENDIX C:
MEERKAT MATHS LANGUAGE TEST: GRADE R – 1
SCORE SHEET

Read the introductory paragraph to the child. Make a ✓ if the child gave the correct answer and a ✗ if the child did not know the answer. Some pictures ask two questions.

<table>
<thead>
<tr>
<th>Answer</th>
<th>✓ or ✗</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Same size</td>
<td></td>
</tr>
<tr>
<td>2 More</td>
<td></td>
</tr>
<tr>
<td>3 In between</td>
<td></td>
</tr>
<tr>
<td>4 First</td>
<td></td>
</tr>
<tr>
<td>5 Last</td>
<td></td>
</tr>
<tr>
<td>6 On top of</td>
<td></td>
</tr>
<tr>
<td>7 Behind</td>
<td></td>
</tr>
<tr>
<td>8 Bigger</td>
<td></td>
</tr>
<tr>
<td>9 Above</td>
<td></td>
</tr>
<tr>
<td>10 Tallest</td>
<td></td>
</tr>
<tr>
<td>11 Biggest</td>
<td></td>
</tr>
<tr>
<td>12 Big/large</td>
<td></td>
</tr>
<tr>
<td>13 Many</td>
<td></td>
</tr>
<tr>
<td>14 Tall</td>
<td></td>
</tr>
<tr>
<td>15 Shortest</td>
<td></td>
</tr>
<tr>
<td>16 Small</td>
<td></td>
</tr>
<tr>
<td>17 Less</td>
<td></td>
</tr>
<tr>
<td>18 Just as many</td>
<td></td>
</tr>
<tr>
<td>19 Under</td>
<td></td>
</tr>
<tr>
<td>20 Smaller</td>
<td></td>
</tr>
<tr>
<td>21 Fewer</td>
<td></td>
</tr>
<tr>
<td>22 Short</td>
<td></td>
</tr>
<tr>
<td>23 In front of</td>
<td></td>
</tr>
<tr>
<td>24 After</td>
<td></td>
</tr>
<tr>
<td>25 No</td>
<td></td>
</tr>
<tr>
<td>26 Half</td>
<td></td>
</tr>
</tbody>
</table>

Total correct:

Name of tester: _____________________
Name of child: _____________________
Surname of child: _________________
Date of birth: _________________
Age: _____________________
School: _____________________
Teacher: _____________________
Grade: _____________________
Home language: _________________
Language of instruction: ______
Boy or girl: _____________________
Subtest 3:

Example items (complete with the help of the tester):

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Examples of questions:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image12.png" alt="Image" /></td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
<td><img src="image16.png" alt="Image" /></td>
<td><img src="image17.png" alt="Image" /></td>
</tr>
</tbody>
</table>
**Subtest 4:**

Example items (complete with the help of the tester):

<p>| | | | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image16.png" alt="Image" /></td>
<td><img src="image17.png" alt="Image" /></td>
<td><img src="image18.png" alt="Image" /></td>
<td><img src="image19.png" alt="Image" /></td>
<td><img src="image20.png" alt="Image" /></td>
<td><img src="image21.png" alt="Image" /></td>
<td><img src="image22.png" alt="Image" /></td>
<td><img src="image23.png" alt="Image" /></td>
<td><img src="image24.png" alt="Image" /></td>
<td><img src="image25.png" alt="Image" /></td>
<td><img src="image26.png" alt="Image" /></td>
<td><img src="image27.png" alt="Image" /></td>
<td><img src="image28.png" alt="Image" /></td>
<td><img src="image29.png" alt="Image" /></td>
<td><img src="image30.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image31.png" alt="Image" /></td>
<td><img src="image32.png" alt="Image" /></td>
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**Examples of questions:**

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Subtest 5:

Example items (complete with the help of the tester):

Examples of questions:
Dear Parent/Guardian

We welcome you to the Funda UJabule School and thank you for choosing this school for your child.

The Funda UJabule School is a partnership between the Gauteng Department of Education (GDE) and the University of Johannesburg (UJ). The school is not only a GDE public school, but also a training school (“teaching school”) for student teachers at the University of Johannesburg.

In addition, the MoA with the GDE allows the UJ to conduct research at the school. Research on teacher education is conducted and some of the research will also involve the Funda UJabule learners. When you enrol your child, you will be requested to give permission in writing for your child to be involved in the ongoing research in the school.

This letter serves to confirm what the research may involve. Research will be conducted in the following manner:

1. Classroom visits and observations by staff and students from UJ (researchers) who will study classroom activities

2. Interviews with the teachers, families of the children and the children themselves
3. Studying of the children’s work
4. Assessments of children to track their progress

During the research, video-taping and audio-taping will be used sometimes to study what happens in the class and when interviewing your child. Participation in such video- and audio-recordings will always be voluntary and no pressure will be placed on your child to take part. He/she has the right to not participate in such recordings, and that will be respected.

The video-taping will record classroom interaction. This means that the following will be recorded: interaction between teacher and learners, which includes the recording of facial expressions, full body images in the classroom and the conversation of the teacher and the learners.

The main idea behind the research is to learn from what happens in the school with regard to teaching and learning and child development. Also, the research aims to benefit your child because the information that is collected will be used to give support to your child. All research will always be conducted in an ethical manner. Your child will never be harmed or will feel intimidated in any way. For the purposes of anonymity and confidentiality the name of your child will not be mentioned when findings of the research are reported. Fictional names will always be used when reporting findings.

The school principal and teachers will explain to the children what research is and what the research in the school involves. We request that you also explain this to your child prior to enrolling him/her.

By agreeing that your child takes part in the research you are also giving consent for the free use, duplication and distribution of the recorded research information which involves your child, for training and research purposes.

Should you have any questions about the research, please contact the Dean of Education (UJ): 011 559-5233.

Yours sincerely,

Prof Sarah Gravett
CONSENT FROM PARENT/GUARDIAN

I the undersigned……………………………………………………………………………… (Full name and surname of parent/guardian) have read the preceding information in connection with the research that is conducted at the Funda UJabule School.

I as parents/guardian hereby declare that I give my voluntary consent that my child may take part in the ongoing research at the Funda UJabule school. I confirm that I have explained to my child what the research would involve. I also give my consent for the free use, duplication and distribution of the recorded research information which involves my child, for training and research purposes.

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<tr>
<th>Full name of parent/guardian</th>
<th>Signature of parent/guardian</th>
<th>Date</th>
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Dear Parents,

The Grade 1 pupils of Funda UJabule was selected to participate in a research project of the University of Johannesburg’s Center for Educational Practice Research. The project has been launched to determine how young children develop mathematical and language skills over the period of one year. The first years of schooling is regarded as particularly important and therefore we are interested to find out how they learn.

A German mathematics diagnostic test, the MARKO-D, measures mathematical knowledge and skills in the Foundation Phase. The test has already been used successfully in Germany and in several languages in South Africa. We will use this test to determine which mathematical skills the Grade 1 children have already developed at the beginning of the year.

We are also interested in their math specific language development. Therefore we use a very short test to determine which math specific vocabulary they already know and which concepts are still to be learnt. Reasoning skills also influence math development and therefore we will also use a short reasoning test (CFT) to see how their reasoning skills develop.

We will start the testing on Monday, 29 January 2018. How will the tests be conducted? Each child will be tested individually with the MARKO-D and language test. A lady will fetch your child from class, test him or her with the two tests (which will take approximately 30 minutes) and then take your child back to class. After second brake the same children will be fetched for a group test (CFT) which will also take approximately 30 minutes. Please note that each child will only be tested once.
during this testing period which means that your child will only be effected one day
during the testing period. This will not influence their school assessment since we
discussed the process with all of the teachers.

The results will be treated with great confidentiality. We will give feedback to each
parent individually and the results that will form part of the study will not include any
personal information about the individual children.
APPENDIX G:
ETHICS CERTIFICATE

NHREC Registration Number REC-110613-036

ETHICS CLEARANCE

Dear H Bezuidenhout

Ethical Clearance Number: 2017-053

Development of children’s early numerical competence: Number talk and executive function as input analyzer data

Ethical clearance for this study is granted subject to the following conditions:

- If there are major revisions to the research proposal based on recommendations from the Faculty Higher Degrees Committee, a new application for ethical clearance must be submitted.
- If the research question changes significantly so as to alter the nature of the study, it remains the duty of the student to submit a new application.
- It remains the student’s responsibility to ensure that all ethical forms and documents related to the research are kept in a safe and secure facility and are available on demand.
- Please quote the reference number above in all future communications and documents.

The Faculty of Education Research Ethics Committee has decided to

☑ Grant ethical clearance for the proposed research.
☐ Provisionally grant ethical clearance for the proposed research
☐ Recommend revision and resubmission of the ethical clearance documents

Sincerely,

[Signature]

Prof Geoffrey Lautenbach
Chair: FACULTY OF EDUCATION RESEARCH ETHICS COMMITTEE
22 June 2017
APPENDIX H:
DISTRIBUTIONS OF INDEPENDENT VARIABLES

Normal distribution of the Grade R MARKO-D scores in the beginning of 2017

Normal distribution of the Grade R MMLT scores in the beginning of 2017
Normal distribution of the Grade R CFT scores in the beginning of 2017

Negatively skewed distribution of the Grade R EF scores in 2017