INSTRUCTIONS TO CANDIDATES:

- Answer ALL questions.
- Question paper must be handed in.
- This is a closed book assessment.
- Leave margins and spaces between the questions.
- Show all your calculations.
- Unless otherwise indicated, express your answers correct to two (2) decimal places.
- Where appropriate, indicate the units of your answer. (e.g. Hour, R )
- Write neatly and legibly in **ink not pencil**.
- NOTE: Marks will be awarded for theoretical knowledge, application of the theory and use of relevant examples.
- The general University of Johannesburg policies, procedures and rules pertaining to written assessments apply to this examination.
Question 1  (LP Simplex Method)

Consider the following linear program:

Maximize  \[ Z = 3X_1 + 2X_2 - X_3 \]
subject to:
\[ X_1 + X_2 + 2X_3 \leq 10 \]
\[ 2X_1 - X_2 + X_3 \leq 20 \]
\[ 3X_1 + X_2 \leq 15 \]
\[ X_1, X_2, X_3 \geq 0 \]

1.1 Convert the above constraints to equalities by adding the appropriate slack variables. (5)

1.2 Determine the optimal solution, leave all fractional values in the simplest form, no decimals. (30)

1.3 What are the values of the decision variables in the third tableau? (3)

1.4 In the final simplex tableau, what does the value "- 4" in the "X3" column of the Cj - Zj row mean? (2)

Question 2 (LP Sensitivity Analysis)

The Overnight Food Processing Company prepares sandwiches (among other processed food items) for vending machines, markets, and business canteens around the Johannesburg city. The sandwiches are made at night and delivered early the following morning. Any sandwiches not purchased during the previous day are thrown away. Three kinds of sandwiches are made each night, a basic cheese sandwich \( (x_1) \), a ham salad sandwich \( (x_2) \), and a pimento cheese sandwich \( (x_3) \). The profits are R1.25, R2.00, and R1.75, respectively. It takes 0.5 minutes to make a cheese sandwich, 1.2 minutes to make a ham salad sandwich, and 0.8 minutes to make a pimento cheese sandwich. The company, has 20 hours of labour available to produce the sandwiches each night. The demand for ham salad sandwiches is at least as great as the demand for the two types of cheese sandwiches combined. However, the company has only enough ham salad to produce 500 sandwiches per night.

2.1 Formulate a linear programming model in order to determine how many of each type of sandwich to make to maximize profit (ie the primal) (5)

The optimal simplex tableau follows:
1.25  2.00  1.75  0  0  0

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{c}_j & \text{Basic Variables} & \text{Quantity} & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \\
\hline
0 & s_1 & 200 & -0.3 & 0 & 0 & 1 & -0.8 & -2 \\
1.75 & x_3 & 500 & 1 & 0 & 1 & 0 & 1 & 1 \\
2.00 & x_2 & 500 & 0 & 1 & 0 & 0 & 0 & 1 \\
\text{z}_j & 1,875 & 1.75 & 2.00 & 1.75 & 0 & 1.75 & 3.75 \\
\text{c}_j \text{z}_j & -0.5 & 0 & 0 & 0 & -1.75 & -3.75 \\
\hline
\end{array}
\]

2.2 Formulate the dual for this problem.  
(5)

2.3 Define the dual variables and state their values  
(6)

2.4 Determine the optimal ranges for \(C_1, C_2, \) and \(C_3\).  
(6)

2.5 Overnight Foods is considering advertising its cheese sandwiches to increase demand. The company estimates that spending R100 on some leaflets that would be packaged with all other sandwiches would increase the demand for both kinds of cheese sandwiches by 200. Should it make this expenditure?  
(6)

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SECTION B

Choose and shade the right option in the scanner sheet provided.

INTEGER PROGRAMMING

1) Types of integer programming models are __________.

A) total 
B) 0 - 1 
C) mixed 
D) all of the above 

2) In a __________ integer model, some solution values for decision variables are integer and others can be non-integer.

A) total 
B) 0 - 1 
C) mixed 
D) all of the above 

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3) In a ________ integer model, all decision variables have integer solution values.
   A) total
   B) 0 - 1
   C) mixed
   D) all of the above

4) In a ________ integer model, the solution values of the decision variables are 0 or 1.
   A) total
   B) 0 - 1
   C) mixed
   D) all of the above

5) Which of the following is not an integer linear programming problem?
   A) pure integer
   B) mixed integer
   C) 0-1 integer
   D) continuous

6) In using rounding of a linear programming model to obtain an integer solution, the solution is
   A) always optimal and feasible
   B) sometimes optimal and feasible
   C) always optimal
   D) always feasible
   E) never optimal and feasible

7) The branch and bound method of solving linear integer programming problems is ________.
A) an integer method
B) a relaxation method
C) a graphical solution
D) an enumeration method

8) If a maximization linear programming problem consist of all less-than-or-equal-to constraints with all positive coefficients and the objective function consists of all positive objective function coefficients, then rounding down the linear programming optimal solution values of the decision variables will __________ result in a(n) __________ solution to the integer linear programming problem.
A) always, optimal
B) always, non-optimal
C) never, non-optimal
D) sometimes, optimal
E) never, optimal

9) If a maximization linear programming problem consist of all less-than-or-equal-to constraints with all positive coefficients and the objective function consists of all positive objective function coefficients, then rounding down the linear programming optimal solution values of the decision variables will __________ result in a feasible solution to the integer linear programming problem.
A) always
B) sometimes
C) never

10) If a maximization linear programming problem consist of all less-than-or-equal-to constraints with all positive coefficients and the objective function consists of all positive objective function coefficients, then rounding down the linear programming optimal solution values of the decision variables will __________ result in an optimal solution to the integer linear programming problem.
A) always
B) sometimes
C) never
11) If we are solving a 0-1 integer programming problem, the constraint $x_1 + x_2 \leq 1$ is a__________ constraint.

A) multiple choice
B) mutually exclusive
C) conditional
D) corequisite
E) none of the above

12) If we are solving a 0-1 integer programming problem, the constraint $x_1 + x_2 = 1$ is a__________ constraint.

A) multiple choice
B) mutually exclusive
C) conditional
D) corequisite
E) none of the above

13) If we are solving a 0-1 integer programming problem, the constraint $x_1 \leq x_2$ is a__________ constraint.

A) multiple choice
B) mutually exclusive
C) conditional
D) corequisite
E) none of the above

14) For a maximization integer linear programming problem, feasible solution is ensured by rounding ________ non-integer solution values if all of the constraints are less-than—or equal-to type.

A) up and down
B) up
C) down
D) up or down

15) The linear programming relaxation contains the objective function and the original __________ of the integer programming problem, but drops all integer restrictions.
A) different variables
B) slack values
C) constraints
D) decision variables
E) surplus variables

16) The linear programming relaxation contains the objective function and the original constraints of the integer programming problem, but drops all __________.
A) different variables
B) slack values
C) integer restrictions
D) decision variables
E) nonnegativity constraints

17) The __________ contains the objective function and the original constraints of the integer programming problem, but drops all integer restrictions.
A) linear programming maximization
B) linear programming minimization
C) linear programming relaxation
D) linear programming problem

18) The solution to the linear programming relaxation of a minimization problem will always be __________ the value of the integer programming minimization problem.
A) greater than or equal to
B) less than or equal to
C) equal to
D) different than

19) If the optimal solution to the linear programming relaxation problem is integer, it is_________ to the integer linear programming problem.
   A) a real solution
   B) a degenerate solution
   C) an infeasible solution
   D) the optimal solution
   E) a feasible solution

20) In a capital budgeting problem, if either project 1 or project 2 is selected, then project 5 cannot be selected. Which of the alternatives listed below correctly models this situation?
   A) x1 + x2 + x5 \leq 1
   B) x1 + x2 + x5 \geq 1
   C) x1 + x5 \leq 1, x2 + x5 \leq 1
   D) x1 - x5 \leq 1, x2 - x5 \leq 1
   E) x1 - x5 = 0, x2 - x5 = 0

LP FORMULATION

21) Using linear programming to maximize audience exposure in an advertising campaign is an example of the type of linear programming application known as
   (A) media selection.
   (B) marketing research.
   (C) portfolio assessment.
   (D) media budgeting.
   (E) all of the above

22) A type of linear programming problem that is used in marketing is called the
   (A) media selection problem.
   (B) Madison Avenue problem.
   (C) marketing allocation problem.
(D) All of the above are examples of marketing linear programming problems.
(E) None of the above are examples of marketing linear programming problems.

23) The selection of specific media from among a wide variety of alternatives is the type of LP problem known as

(A) the product mix problem.
(B) the investment banker problem.
(C) the Wall Street problem.
(D) the portfolio selection problem.
(E) none of the above

24) The following does not represent a factor a manager might typically consider when employing linear programming for a production scheduling:

(A) labor capacity
(B) space limitations
(C) product demand
(D) risk assessment
(E) inventory costs

The following exhibit pertains to questions 25 to 30.

<table>
<thead>
<tr>
<th>EXHIBIT 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A small furniture manufacturer produces tables and chairs. Each product must go through three stages of the manufacturing process—assembly, finishing, and inspection. Each table requires 3 hours of assembly, 2 hours of finishing, and 1 hour of inspection. Each chair requires 2 hours of assembly, 2 hours of finishing, and 1 hour of inspection. The profit per table is R120 while the profit per chair is R80. Currently, each week there are 200 hours of assembly time available, 180 hours of finishing time, and 40 hours of inspection time. Linear programming is to be used to develop a production schedule. Define the variables as follows:</td>
</tr>
<tr>
<td>T = number of tables produced each week</td>
</tr>
<tr>
<td>C = number of chairs produced each week</td>
</tr>
</tbody>
</table>

25) According to Exhibit 1, which describes a production problem, what would the objective function be?

(A) Maximize $T + C$
(B) Maximize $120T + 80C$
(C) Maximize $200T + 200C$
(D) Minimize $6T + 5C$
(E) none of the above
26) According to Exhibit 1, which describes a production problem, which of the following would be a necessary constraint in the problem?

(A) \( T + C \leq 40 \)
(B) \( T + C \leq 200 \)
(C) \( T + C \leq 180 \)
(D) \( 120T + 80C \geq 1000 \)
(E) none of the above

27) According to Exhibit 1, which describes a production problem, which of the following would be a necessary constraint in the problem?

(A) \( T + C \geq 40 \)
(B) \( 3T + 2C \leq 200 \)
(C) \( 2T + 2C \leq 40 \)
(D) \( 120T + 80C \geq 1000 \)
(E) none of the above

28) According to Exhibit 1, which describes a production problem, suppose it is decided that there must be 4 chairs produced for every table. How would this constraint be written?

(A) \( T \geq C \)
(B) \( T \leq C \)
(C) \( 4T = C \)
(D) \( T = 4C \)

29) According to Exhibit 1, which describes a production problem, suppose it is decided that the number of hours used in the assembly process must be at least 80 percent of the time available. How would this constraint be written?

(A) \( 3T + 2C \geq 160 \)
(B) \( 3T + 2C \geq 200 \)
(C) \( 3T + 2C \leq 200 \)
(D) \( 3T + 2C \leq 160 \)
(E) none of the above

30) According to Exhibit 1, which describes a production problem, suppose it is decided that the number of hours used in the assembly process must be at least 90 percent of the number of hours used in the finishing department. How would this constraint be written?
(A) $3T + 2C \geq 162$
(B) $3T + 2C \geq 0.9(2T + 2C)$
(C) $3T + 2C \leq 162$
(D) $3T + 2C \leq 0.9(2T + 2C)$
(E) none of the above

31) Which of the following does not represent a factor a manager might consider when employing linear programming for a production scheduling?

(A) labour capacity
(B) employee skill levels
(C) warehouse limitations
(D) shipping limitations
(E) none of the above

32) Media selection problems are typically approached with LP by either ________________.

(A) maximizing audience exposure or maximizing number of ads per time period
(B) maximizing the number of different media or minimizing advertising costs
(C) minimizing the number of different media or minimizing advertising costs
(D) maximizing audience exposure or minimizing advertising costs
(E) minimizing audience exposure or minimizing advertising costs

END OF ASSESSMENT

TOTAL MARKS [100]