

# EVALUATION OF THE CREEP COEFFICIENTS OF INTERNATIONAL CONCRETE CREEP PREDICTION MODELS

George C Fanourakis

Associate Professor, Department of Civil Engineering Technology, University of Johannesburg, Johannesburg, South Africa

## ABSTRACT

Creep of concrete is an important design consideration. National design codes therefore provide empirically based models for the estimation of creep deformation. Such models estimate a creep coefficient ( $\phi$ ) and an elastic modulus (E) of the concrete, both of which are used to predict the creep strain at any age.

This paper assesses the accuracy of the creep coefficients ( $\phi$ ) predicted by fourteen "design code-type" models, with a view to ascertain whether the estimated  $\phi$  or E is responsible for the inaccuracy of some of the models. The models considered are those contained in SANS 10100 (2000)/BS 8110 (1985), SANS 10100 (2000) Modified, ACI 209 (1992), AS 3600 (2001 & 2009), CEB-FIP (1970, 1978 & 1990), the Eurocode EC (2004), Gardener and Lockman (2000 & 2004), Gardener and Zhao (1993) and the RILEM B3 (1995) methods.

Laboratory creep tests were conducted on concrete prisms covering a range of mixes. The measured  $\phi$  values were statistically compared to those predicted by the models considered.

The results indicated that, for the range of concretes tested, the CEB-FIP (1990) method yielded the most accurate predictions of creep coefficient, giving the lowest overall coefficient of variation ( $\omega_{all}$ ) of 27,7 %. The least accurate method was the CEB-FIP (1978) which yielded an overall coefficient of variation ( $\omega_{all}$ ) of 112,5 %. Furthermore, the accuracy of the predicted  $\phi$  values correlated highly significantly ( $P = 0,001$  %) with the accuracy of the predicted creep magnitudes.

The results of this investigation led to recommending the SANS 10100 (2000)/ BS 8110 (1985) model for predicting creep coefficients for South African conditions.

**Keywords:** Concrete, creep, models, elastic modulus.

## 1. Introduction

### 1.1 Significance of creep

Creep magnitude is an important design consideration for the durability, long-term serviceability and the load carrying capacity of structures.

The magnitude of creep can be determined by laboratory testing or estimated by means of empirically based models of various complexities. In general, the more deformation sensitive the structure, the more justifiable the cost and time of laboratory testing or complexity of the estimation method employed. In cases where only a rough estimate of the creep is required, design code-type models are ideal for predicting the creep. Such models use a few parameters, which would be known at the design stage, as input to the models.

### 1.2 Accuracy of creep models

Previous work by Fanourakis (1998), Fanourakis and Ballim (2006) and Fanourakis (2011) collectively assessed the accuracy of fourteen code-type creep prediction models when applied to South African concretes. These assessments were based on six different concretes, incorporating combinations of three aggregate types and two w/c ratios, by means of a 168 day laboratory test programme. The predictions were carried out for concrete of the same geometry, temperature and humidity as the laboratory test specimens. Details of the mixes used in the research are given in Table 1.

Further details on the materials and experimental methods are given in the work of Fanourakis (1998).

**Table 1.** Details of the mixes and laboratory test results (after Fanourakis, 2011)

Aggregate Type	Quartzite		Granite		Andesite	
Mix Number	Q1	Q2	G1	G2	A1	A2
Water (l/m <sup>3</sup> )	195	195	195	195	195	195
CEM I 42,5N (kg/m <sup>3</sup> )	348	488	348	488	348	488
19 mm Stone (kg/m <sup>3</sup> )	1015	1015	965	965	1135	1135
Crusher Sand (kg/m <sup>3</sup> )	810	695	880	765	860	732
w/c Ratio	0,56	0,4	0,56	0,4	0,56	0,4
a/c Ratio	5,24	3,50	5,30	3,55	5,73	3,83
Slump (mm)	90	50	115	70	95	55
Cube Compressive Strength (MPa)	37	65	38	65	48	74
Cylinder Compressive Strength (MPa) <sup>a</sup>	30	53,5	30,7	53,5	38	59
Characteristic Cube Strength (MPa)	30	50	30	50	30	50
Characteristic Cylinder Strength (MPa) <sup>a</sup>	25	40	25	40	25	40
Concrete Density (kg/m <sup>3</sup> )	2371	2410	2385	2432	2596	2585
Average Elastic Modulus of included Aggregate (GPa)	73		70		89	
<sup>a</sup> Inferred from cube strength using the conversions from EC 2 (2004)						

In the abovementioned assessments, the predicted and measured creep results were presented in the form of specific creep ( $C_c$ ), which is the creep strain per unit stress, as defined by Equations 1 and 2.

$$C_c = \frac{\varepsilon_c(t)}{\sigma} \quad (1)$$

Which can also be expressed as:

$$C_c = \frac{\varphi(t)}{E} \quad (2)$$

Where:

$\varphi(t)$  is the creep coefficient at time  $t$ .

$E$  is the elastic modulus of the concrete.

Fanourakis (2011) investigated the correlation between the predicted specific creep ( $C_c$ ) and the estimated elastic ( $E$ ). Table 2 shows the accuracy of the creep predictions and elastic modulus estimations. The coefficient of variation of errors ( $\omega_j$ ) after Bazant and Panula (1979) was used to assess the accuracy. The more accurate the estimation, the lower the value of  $\omega_j$ .

**Table 2.** Accuracy of creep predictions and elastic moduli estimations for various models (after Fanourakis, 2011)

Prediction Method	$\omega$ (%)	
	$C_c$ ( $\omega_{all}$ %)	E ( $\omega_j$ %)
SANS 10100 (2000)	31,3	9,3
SANS 10100 (2000) Modified	34,7	14,9
BS 8110 (1985)	23,6	15
ACI 209 (1992)	50,5	12,8
AS 3600 (2001)	38,6	12,8
AS 3600 (2009)	74,7	9,3
CEB-FIP (1970)	18,1	23,8
CEB-FIP (1978)	96,1	13,7
CEB-FIP (1990)	32,2	15,5
EC 2 (2004)	33,4	16,4
GL (2000)	31,9	14,4
GL (2004)	35,4	14,4
GZ (1993)	49,5	14,4
RILEM Model B3 (1995)	35,6	14,6
Green = Most accurate; Red = Least accurate		

The overall coefficient of variation ( $\omega_{all}$ ) was used to estimate the average (pooled) coefficient of variation of the six independent coefficients of variation ( $\omega_j$ ), pertaining to the six mixes, compared against the same prediction model, as defined by Equation 3.

$$\omega_{all} = \sqrt{\frac{\sum_j \omega_j}{N}} \quad (3)$$

where,

N = number sets considered

Referring to Table 2, Fanourakis (2011) established that most accurate creep prediction model, the CEB-FIP (1970), which yielded a  $\omega_{all}$  of 18 % (for the  $C_c$ ) was the least accurate in estimating E. Furthermore, the models that yielded the most accurate estimation of E (SANS 10100, 2000 and AS 3600, 2009) did not yield the most accurate estimation of  $C_c$ . In fact, the AS 3600 (2009) model yielded the second least accurate prediction of  $C_c$ . The most and least accurate predictions and estimations are indicated in green and red, respectively.

These observations form the basis for the justification of this investigation, which aimed to establish whether, in the case of inaccurate models, the inaccuracy is rooted in the creep coefficient ( $\phi$ ) component of the model.

### 1.3 Objectives of this paper

This paper assess the accuracy of the creep coefficients predicted (with time) by the code-type models by comparing the actual creep coefficients, measured on a range of concretes under laboratory control conditions, at various ages, with the predicted creep coefficients of the concrete at those ages. This investigation was conducted with a view to distinctly ascertain whether the expressions used to estimate  $\phi$  or those used to estimate  $E$  are responsible for the inaccuracy of some of the models.

The models considered were the following.

- SANS 10100 (2000) (formerly SABS 0100, 1992). This model is based on the British Standard method (BS 8110, 1985), with a small modification to the equation used to calculate the  $E$  of the concrete.
- Modified SANS 10100 (2000) model. This model is essentially the SANS 10100 (2000) model with additional aggregate specific modifications to the equation used to estimate the  $E$  of the concrete.
- British Standards Institution - Structural Use of Concrete, BS 8110 - Part 2 - (1985).
- Standards Association of Australia - Australian Standard for Concrete Structures - AS 3600 (2001).
- AS 3600 (2009).
- American Concrete Institute (ACI) Committee 209 (1992), reapproved by ACI Committee 209 in 2008.
- Comité Euro-International Du Béton - Federation Internationale De La Précontrainte (CEB-FIP) Model Code (1970).
- CEB-FIP Model Code (1978).
- CEB-FIP Model Code (1990).
- EUROCODE (EC 2) – BS EN 1992-1-1:2004, which will be referred to as EC 2 (2004). This model, which supersedes the BS 8110 (1985) model, is the same as the CEB-FIP (1999) model.
- Gardner and Lockman 2000 and 2004 versions which will be referred to as GL (2000) and GL (2004), respectively. The GL (2000) model was published in 2001.
- Gardner and Zhao (GZ, 1993).
- International Union of Testing and Research Laboratories for Materials and Structures (RILEM) Model B3 (1995), after Bazant and Baweja (1995).

## 2. Creep models

### 2.1 Structure

The models considered are all empirically based and vary widely in their approach and methodologies. With the exception of 28 day compressive strength, no other results from laboratory tests are required as input. However, certain intrinsic and/or extrinsic variables, such as mix proportions, material properties and age of loading are required as input to these models.

With the exception of the RILEM Model B3 (1995) all the models considered express creep strain in terms of the creep coefficient,  $\phi(t)$ , where:

$$\varepsilon_c(t, \tau) = \phi(t) \varepsilon_{e,\tau} \quad (4)$$

In Equation 4,  $\varepsilon_c(t, \tau)$  is the creep strain at any concrete age  $t$  for a concrete loaded at age  $\tau$ , where  $t > \tau$  and  $\varepsilon_{e,\tau}$  is the elastic strain of the concrete at age  $\tau$ . The creep coefficient is empirically determined by considering one or more intrinsic and/or extrinsic variables such as concrete stiffness and age at first loading. The elastic modulus used to estimate the elastic strain is estimated using an empirical equation prescribed by that method.

The RILEM Model B3 (1995) is, by comparison, more complex than the design code models and takes a more fundamental materials approach to creep prediction. In the case of this model, an elastic modulus

is estimated, which is used in the calculation of the compliance function for additional creep due to drying and may be used to calculate the creep coefficient ( $\phi_{(t)}$ ) from the relevant compliance function equations. However, in the case of all the other creep prediction models considered in this paper, the predicted creep strain is directly dependent on the value of the estimated elastic modulus. Hence, the reliability of estimation of the creep coefficient significantly influences the reliability of the prediction of creep.

## **2.2 Factors considered by creep coefficients**

The following is evident with regards to the factors considered in the prediction of the creep coefficient component of each model.

- Age of first loading, duration of load, effective thickness and relative humidity are considered by all of the methods.
- The SANS 10100 (2000) is identical to the BS8110 (1985) method. This is the only method that considers aggregate type.
- The ACI 209 (1992) method is the only method that considers the ratio of fine to total aggregate (by mass) as well as the slump of the wet concrete.
- The CEB-FIP (1978) is the only method that considers elastic strain.
- The RILEM Model B3 is the most complex method and is the only method that considers the aggregate to cement ratio (by mass), elastic modulus at loading and shrinkage.

## **3. Materials**

CEM I 42,5 cement, from the Dudfield factory of Alpha Cement (now AfriSam), was used for all the tests carried out in this investigation.

Quartzite (Q) from the Ferro quarry in Pretoria, granite (G) from the Jukskei quarry in Midrand and andesite (A) from the Eikenhof quarry in Johannesburg were used as both the coarse and fine aggregates for the concrete. The stone was 19 mm nominal size and the fine aggregate was crusher sand.

## **4. EXPERIMENTAL METHODS**

### **4.1 Preparation of prisms**

For each of the concretes listed in Table 1, six prisms were prepared, measuring 100 x 100 x 200 mm and cast with the 200 mm dimension vertical. After de-moulding, these prisms were continuously water cured up to an age of 28 days.

After curing, three of the six prisms of each mix were used for creep tests and the remaining three were used for shrinkage measurements.

### **4.2 Elastic Modulus Measurements**

The creep test prisms were stacked into creep loading frames and subjected to elastic strain measurements, within 10 minutes of application of the loads, which were used to determine the secant moduli of the concretes.

### **4.3 Creep and shrinkage measurements**

The creep tests commenced immediately after the elastic modulus measurements were taken. These tests entailed subjecting the prisms in each frame to an applied load of approximately 25 % of the 28-day compressive strength, for the 168 day period, in a room controlled at  $22 \pm 3$  °C and RH of  $65 \pm 5$  %.

The shrinkage (companion) prisms were placed on a rack in the same room as the creep samples and, in order to ensure a drying surface area equivalent to the creep samples, the two 100 mm square ends were dipped in warm wax to prevent drying from these surfaces.

Creep and shrinkage measurements were recorded daily for the first week, thereafter, weekly for the remainder of that month and then monthly until the culmination of the approximately six-month total

loading period. The strain of each group of prisms, that is the three creep prisms or the three companion shrinkage prisms of a particular mix, was taken as the average of the strains of the prisms in that group. The results of shrinkage measurements were subtracted from the total time-dependant strain of the loaded specimens to determine the total creep strain.

## 5. Results and discussion

### 5.1 Coefficients of variation

The coefficient of variation of errors ( $\omega_j$ ), was used to quantify the extent to which predicted creep coefficients values at different ages after loading (determined by applying a particular model) deviated from the values measured at the relevant ages on the specimens of a particular concrete mix. The more accurate the prediction, the lower the value of  $\omega_j$ .

The overall coefficient of variation ( $\omega_{all}$ ) was used to estimate the average (pooled) coefficient of variation of a number of independent coefficients of variation ( $\omega_j$ ), as defined by Equation 3.

The calculated values of  $\omega_j$  and  $\omega_{all}$  for the different models assessed are shown in Table 3. The most and least accurate predictions and estimations are indicated in green and red, respectively.

**Table 3.** Coefficients of variation for creep coefficients ( $\phi$ )

Prediction Method	Coefficients of Variation ( $\omega_j$ %)						$\omega_{all}$ (%)
	Mix Q1	Mix Q2	Mix G1	Mix G2	Mix A1	Mix A2	
SANS 10100 (2000)/ BS 8110 (1985)	19,9	27,7	31,4	14,4	48,8	21,2	29,4
SANS10100 (2000) Modified	23,6	21,0	53,3	23,1	35,4	15,8	31,3
ACI 209 (1992)	42,7	24,7	48,4	26,9	66,8	52,0	45,9
AS 3600 (2001)	88,9	32,2	42,6	16,4	16,6	21,2	44,3
AS 3600 (2009)	129,0	96,7	75,5	72,5	50,1	40,2	82,7
CEB-FIP (1970)	64,0	82,4	29,2	62,0	9,0	26,5	52,5
CEB-FIP (1978)	112,0	174,6	57,3	140,2	33,5	94,1	112,5
CEB-FIP (1990)	30,5	22,7	12,3	6,4	42,3	34,4	27,7
EC 2 (2004)	30,5	17,3	11,8	16,9	44,8	48,6	31,7
GL (2000)	31,0	66,9	10,9	45,2	32,8	24,3	39,3
GL (2004)	34,7	72,7	9,7	50,3	31,1	27,6	42,5
GZ (1993)	55,3	49,1	49,8	34,7	67,3	52,5	52,3
RILEM Model B3 (1995)	43,5	33,4	39,4	17,6	61,1	37,4	40,8
Green = Most accurate; Red = Least accurate							

The CEB-FIP (1990) yielded the most accurate predictions of creep coefficient, giving an overall coefficient of variation ( $\omega_{all}$ ) of 27,7 %.

When considering specific mixes, the CEB-FIP (1978) model yielded by the least accurate predictions of creep coefficient for all the high strength mixes (Q1, G1 and A1). This model also yielded the least accurate results, with the highest overall coefficient of variation ( $\omega_{all}$  of 112,5 %).

The SANS 10100 (2000)/ BS 8110 (1985) model, which considers the least number of factors (four) in predicting creep coefficient, yielded the second most accurate results ( $\omega_{all} = 29,4 \%$ ). Furthermore, the RILEM Model B3, which considers the most factors in the prediction of creep (15 off), yielded the sixth most accurate predictions ( $\omega_{all} = 40,8 \%$ ). Hence, it is evident that there is no correlation between the accuracy of the creep coefficients predicted by a model and the number of factors considered in the prediction.

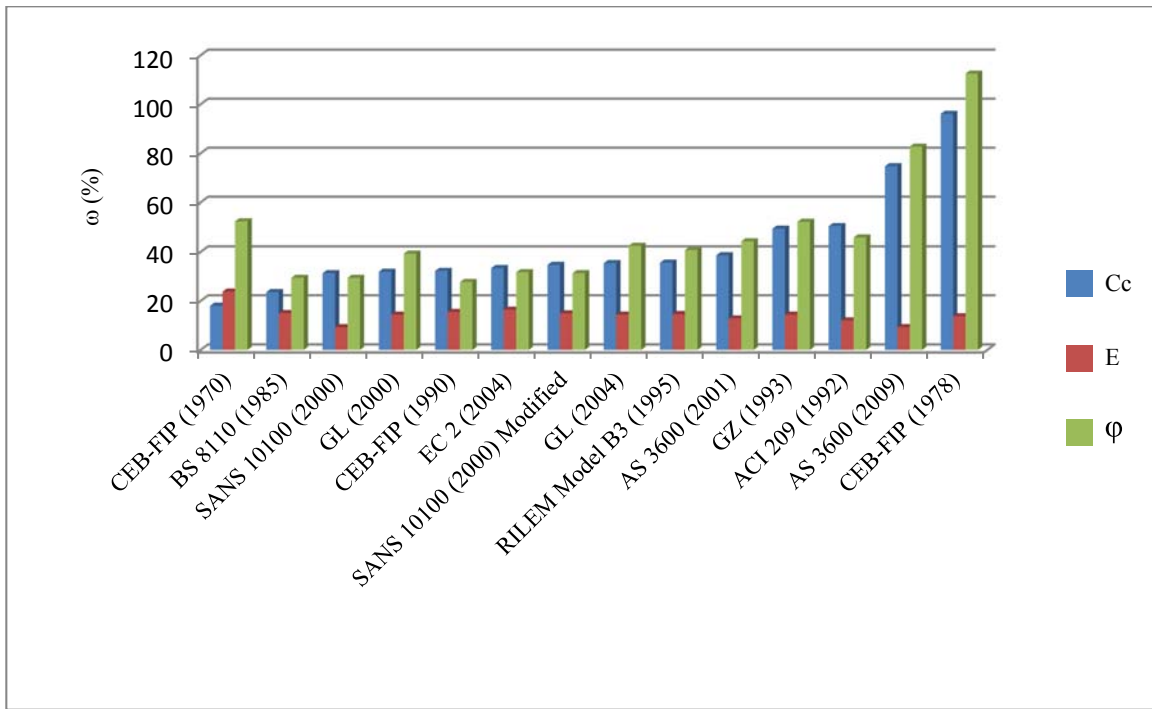
## 5.2 Comparisons of coefficients of variation

Table 4 shows the coefficients of variation for specific creep ( $C_c$ ) and elastic modulus (E), as reflected in Table 2, together with the coefficients of variation for creep coefficients ( $\varphi$ ) that were determined above (from Table 3). The most and least accurate predictions and estimations are indicated in green and red, respectively.

**Table 4.** Comparison of coefficients of variation for  $C_c$ , E and  $\varphi$

Prediction Method	$\omega$ (%)		
	$C_c$ ( $\omega_{all}$ %)	E ( $\omega_j$ %)	$\varphi$ ( $\omega_{all}$ %)
SANS 10100 (2000)	31,3	9,3	29,4
SANS 10100 (2000) Modified	34,7	14,9	31,3
BS 8110 (1985)	23,6	15	29,4
ACI 209 (1992)	50,5	12,8	45,9
AS 3600 (2001)	38,6	12,8	44,3
AS 3600 (2009)	74,7	9,3	82,7
CEB-FIP (1970)	18,1	23,8	52,5
CEB-FIP (1978)	96,1	13,7	112,5
CEB-FIP (1990)	32,2	15,5	27,7
EC 2 (2004)	33,4	16,4	31,7
GL (2000)	31,9	14,4	39,3
GL (2004)	35,4	14,4	42,5
GZ (1993)	49,5	14,4	52,3
RILEM Model B3 (1995)	35,6	14,6	40,8
Green = Most accurate; Red = Least accurate			

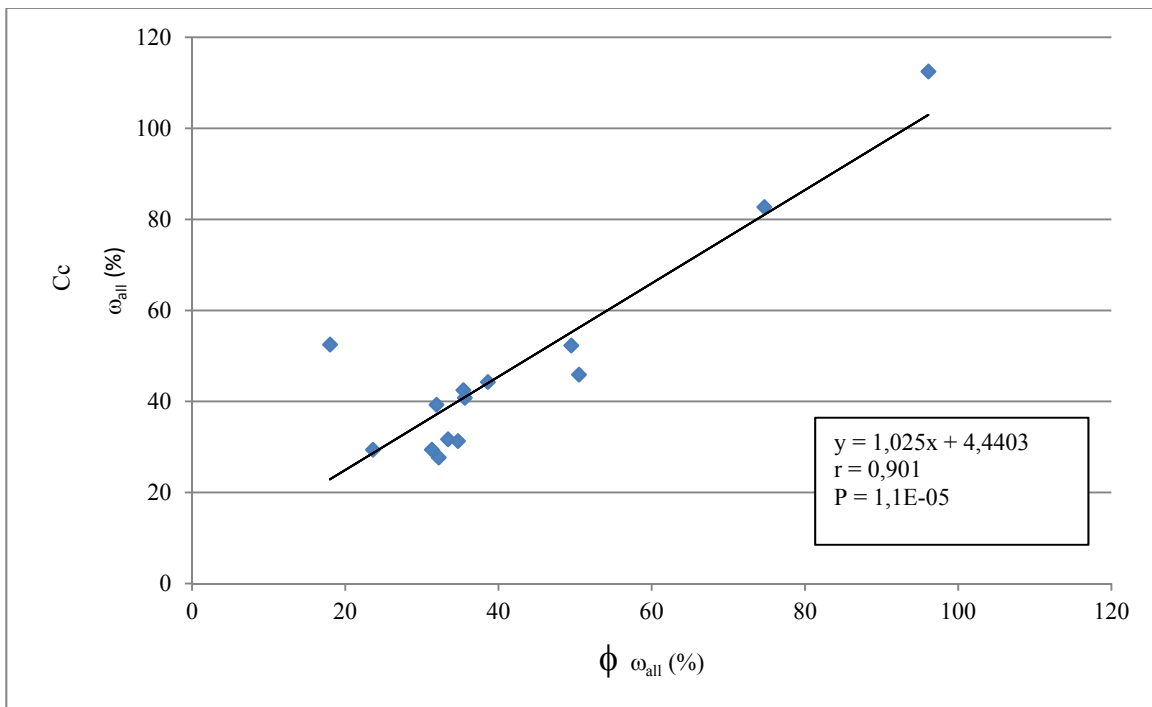
The coefficients of variation shown in Table 4 were plotted, in ascending order (decreased accuracy) of specific creep ( $C_c$ ) overall coefficients of variation ( $\omega_{all}$ ), as shown in Figure 1, with a view to identify any trend relating to these coefficients.



**Fig. 1.** Coefficients of variation for  $C_c$ ,  $E$  and  $\phi$

Referring to Figure 1, it is evident that the accuracy of the predicted specific creep ( $C_c$ ) generally increased with the accuracy of the predicted creep coefficient ( $\phi$ ).

The relationship of  $C_c$  with  $\phi$ , in terms of overall coefficients of variation ( $\omega_{all}$ ), is shown in Figure 2. This linear relationship yielded a correlation coefficient ( $r$ ) of 0,901. Furthermore, this relationship was highly significant, being at the 0,001 % level of probability.



**Fig. 2.** Relationship between predicted  $C_c$  and predicted  $\phi \omega_{all}$  values



From the above, it was concluded that the creep coefficient component of code-type prediction models has a significant influence on the predicted creep magnitude.

## 6. Conclusions

Based on the results of this investigation, the following was concluded:

- The CEB-FIP (1990) yielded the most accurate predictions of creep coefficient, giving an overall coefficient of variation ( $\omega_{all}$ ) of 27,7 %.
- When considering specific mixes, the CEB-FIP (1978) model yielded by the least accurate predictions of creep coefficient for all the high strength mixes (Q1, G1 and A1). This model also yielded the least accurate results, with the highest overall coefficient of variation ( $\omega_{all}$  of 112,5 %).
- The current European model (EC 2, 2004), Australian model (AS 3600, 2009) and GL (2004) model yielded less accurate predictions than their immediate superseded versions.
- The accuracy of the creep coefficient predictions did not increase with the complexity of the method applied or with increasing number of variables accounted for by the method. On that basis, it is recommended that the relatively simple SANS 10100 (2000)/BS 8110 (1985) model, which exhibited the second greatest degree of accuracy, of all the models investigated by the author, be used for predicting creep coefficients for South African conditions.
- The accuracy of predicted specific creep ( $C_c$ ) generally increased with the accuracy of the predicted creep coefficient ( $\phi$ ). This highly significant relationship ( $P = 0,001$  %) yielded a correlation coefficient ( $r$ ) of 0,901.
- From the above, it was concluded that the creep coefficient component of code-type prediction models has a significant influence on the predicted creep magnitude. Hence, it may be inferred that inaccurately predicted creep coefficients will in turn lead to inaccurate creep predictions.

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