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Thermal losses considerations in thermo-acoustic engine design

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ABSTRACT

Thermo-acoustic cooling as an environmentally friendly refrigeration system is one of the research areas being pursued. Although not commercially available and simple to fabricate, the designing of thermo-acoustic coolers involves significant technical challenges. Many fundamental issues related to the thermo-acoustic effects and the associated heat transfer must be addressed. The most inhibiting characteristic of current thermo-acoustic cooling devices is the lack of efficiency. The stack has been identified as the heart of the device where the heat transfer takes place. Improving its performance will make thermo-acoustic technology more attractive. Most of the existing efforts have not taken thermal losses to the surroundings into account in the derivation of the models. Five different parameters describing the stack geometry and the angular frequency of the standing wave are considered. This work explores the use of a multi-objective optimization approach to model and to optimize the performance of a simple thermo-acoustic engine. The present study highlights the importance of thermal losses in the modelling of small-scale thermo-acoustic engines using a multi-objective approach. The unique characteristic of this research is the computing of all efficient optimal solutions describing the best geometrical configuration of thermo-acoustic engines.

KEYWORDS

Thermo-acoustic engine, modelling, multi-objective optimization, heat flow, stack

INTRODUCTION

In order to broaden the applications of Micro Electro-Mechanical Systems, sensor networks and small-scale remote systems, the development of miniature power systems is critically important. A lot of efforts are directed toward the development of miniature energy sources [1]. Small-scale power systems are subjected to certain limitations. For instance, the energy densities of batteries are small and scaled-down rotating machinery are challenging to fabricate. Therefore, thermo-acoustic engine is seen as a promising candidate for small-scale electricity generation when coupled with an electroacoustic transformer [2]. Apart from being environmentally friendly, thermo-acoustic systems are potentially highly reliable because of the simplicity of their structures and the limited number of moving parts. Thermo-acoustic processes involve heat and sound interactions and the thermal-to-acoustic energy conversion [3]. A schematic of a simple standing-wave thermo-acoustic engine (or prime mover) is shown in Figure 1. A piece of porous material (called stack) is the heart of the system where acoustic power is generated in the presence of sufficiently large externally maintained temperature gradient. Two heat exchangers located on the sides of the stack supplies and rejects the heat. The acoustic modes are defined by the resonator geometry. Under appropriate conditions, heat is added to gas parcels oscillating inside the stack at the moment of their compression and extracted at the moment of their rarefaction. As a result, an acoustic power is generated and, therefore, acoustic modes can be excited [3]. In order to generate electricity, an electroacoustic transformer installed at the open end of the tube will convert some of the acoustic power into electricity.
The increased roles of thermo-viscous losses, thermal management and fabrication issues, in addition to the difficulty in integrating with heat sources, make the downsizing of thermo-acoustic systems challenging. Several previous studies report the development of miniature thermo-acoustic engines. The construction and

performance of a relatively small 14 cm tube was documented by Hofler and Adeff [4]. Much smaller systems, down to a few centimetres in length, were also built by Symko et al. [5], but their design was not reported in detail sufficient for reproduction

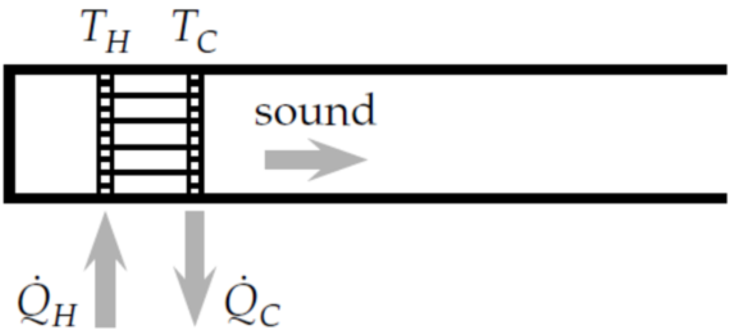


Figure 1: Thermoacoustic engine or prime mover

A study conducted at the Energy research Centre of the Netherlands [6] show that the ratio between thermal losses and acoustic power changes with increasing acoustic power for thermo-acoustic devices. It suggests that heat losses by convection, conduction and radiation need to be adequately covered in the modelling especially with regards to miniaturization of the devices where thermal losses are expected to increase [7]. McLaughlin [8] has thoroughly analysed the heat transfer for a Helmholtz-like resonator, 1.91 cm in diameter and 3.28 cm in length. The loss to conduction has been estimated as 40% of the input power. The losses from convection inside and outside of the device have been estimated as 38%. Radiation accounts for 10% of the input power. This leaves only 12% of input power that can be used to produce acoustic work (Figure 2). Although these losses are approximations not meant to be highly accurate determinations, they suggest that these losses are significant when compared to total heat input and should be considered as design criterion. Therefore, this work aims to highlight one methodology to incorporate thermal losses in the design process.

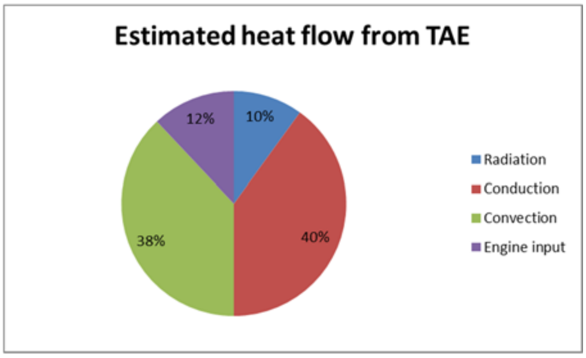


Figure 2: Heat flows from the power supply partitioned by the losses from radiation, conduction, convection and the power input

MATHEMATICAL ANALYSIS AND OPTIMIZATION

Optimization techniques as a design aid for thermo-acoustic engines have been under-utilized. Most previous studies [9, 10] have been limited to parametric studies to estimate the effect of single design parameters on device performance while ignoring thermal losses to the surroundings. These parametric studies are unable to capture the nonlinear interactions inherent in thermo-acoustic models with multiple variables. Therefore, these approaches only guarantee locally optimal solutions. In all likelihood, each optimal design is a local optimum as the solution obtained is optimal (either maximal or minimal) within a neighbouring set of candidate solution. A new approach is proposed in this study to search for global optimum. These solutions will be optimal among all possible solutions in a specific domain, not just those in particular neighbourhood of variables.
This work considers previous optimization efforts by Zink et al. [7] and Trapp et al. [11] in order to illustrate the optimization of thermo-acoustic systems. Thermal losses to the surroundings that are typically disregarded

are taken into account. These losses have been incorporated in the modelling as objectives functions. An effort to effectively implement the Epsilon-constraint method for producing the Pareto optimal solutions of the multi-objective optimization problem is carried out in this work. This has been implemented in the modelling language GAMS (General Algebraic Modelling System, www.gams.com [12]).

MODELLING APPROACH

In this section, the modelling approach for the physical standing wave engine depicted in Figure 1 is presented; the development of the mathematical model equations is included in Tartibu et al. studies [13]. The problem is reduced to a two-dimensional domain because of the symmetry present in the stack. Two constant temperature boundaries are considered; namely, one convective boundary and one adiabatic boundary, as shown in Figure 3. For the model, the stack geometry and the frequency of the sound wave are considered. The model considers variations in operating conditions and the interdependence of stack location and geometry in a quarter-wavelength (λ/4) resonator tube.

Five different parameters are considered to characterize the stack:

- L: stack length,
- H: stack height,
- Za: stack placement (with Za=0 corresponding to the closed end of the resonator tube),
- dc: channel dimension, and
- N: number of channels.

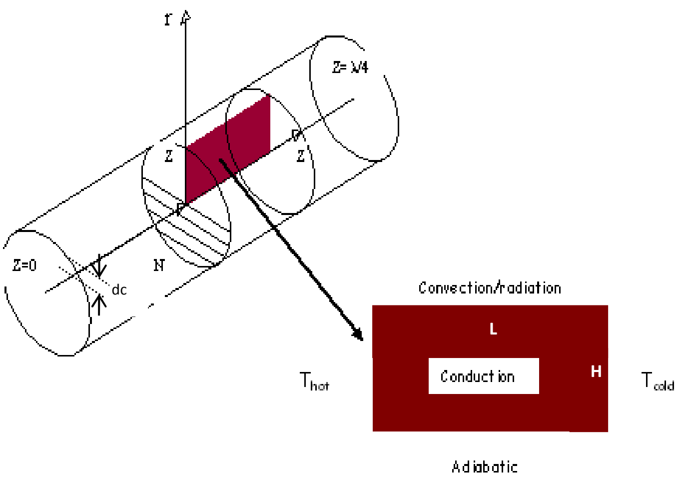


Figure 3: Computational domain

Those parameters have been allowed to vary simultaneously. Five different objectives as described by Trapp et al. [11], namely two acoustic objectives — acoustic work (W) and viscous resistance (R_V) — and three thermal objectives — convective heat flow ($\dot{Q}_{conv}$), radiative heat flow ($\dot{Q}_{rad}$) and conductive heat flow ($\dot{Q}_{cond}$) — are considered to measure the quality of a given set of variable values that satisfy all the constraints. Ultimately, optimizing the resulting problem generates optimal objective function value $G^* = \left[W^*, R_V^*, \dot{Q}_{conv}^*, \dot{Q}_{rad}^*, \dot{Q}_{cond}^* \right]$ and optimal solutions $x^* = \left[L^*, H^*, dc^*, Za^*, N^* \right]$. Since the five objectives are conflicting in nature [11, 14], a multi-objective optimization approach has been used.

ILLUSTRATION OF THE OPTIMISATION PROCEDURE OF THE STACK

Boundary conditions

The five variables— L, H, dc, Za, N —may only take values within the certain lower and upper bounds. The feasible domains for a thermo-acoustic stack are defined as follows:

$$
\begin{aligned}
&L_{min} \le L \le L_{max}\\
&H_{min} \le H \le H_{max}\\
&dc_{min} \le dc \le dc_{max}\\
&Za_{min} \le Za \le Za_{max} - L\\
&N_{min} \le N \le N_{max}
\end{aligned}
\tag{1}
$$

$L,\ H,\ dc,\ Za \in \Re+$ and $N \in Z+$

with $dc_{min} = 2\delta_k$ and $dc_{max} = 4\delta_k$ [10]

δ_k is the thermal penetration depth given by:

$$\delta_k = \sqrt{\frac{2K}{\rho c_p \omega}} \tag{2}$$

where $\omega=2\pi f$ is the angular frequency of the sound wave, f is the design frequency and K the thermal conductivity. Here, ρ and c_p are the density and isobaric specific heat of the gas, respectively.

$$N(dc + t_w) \le 2H \tag{3}$$

where t_w represents the wall thickness around a single channel

The following boundary conditions are defined:

1. constant hot side temperature (T_h) or (T_{hot});
2. constant cold side temperature (T_C) or (T_{Cold});
3. adiabatic boundary, modelling the central axis of the cylindrical stack:

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0; \tag{4}$$

4. free convection and radiation to surroundings (at T_∞) with temperature dependent heat transfer coefficient (h), emissivity (ε), Stefan Boltzmann constant (k_B) and thermal conductivity (K):

$$K\left.\frac{\partial T}{\partial r}\right|_{r=H} = h(T_S - T_\infty) + \varepsilon k_b (T_S^4 - T_\infty^4) \tag{5}$$

Acoustic power

$$W = \omega L N\left(\frac{\pi H^2}{2(dc + t_w)}\right)\left[\delta_k \frac{(\gamma - 1)p^2}{\rho c^2(1+\varepsilon)}(\Gamma - 1) - \delta_v \rho u^2\right] \tag{6}$$

where γ, c and Γ are the isentropic coefficient, the speed of sound and the critical temperature gradient respectively.

$$\Pi = \frac{2A}{dc + t_w} \tag{7}$$

$$p = p_{max}\cos\left(\frac{2\pi Za}{\lambda}\right) \tag{8}$$

$$u = u_{max}\sin\left(\frac{2\pi Za}{\lambda}\right) \tag{9}$$

$$\text{with } u_{max} = \frac{p_{max}}{\rho c} \tag{10}$$

$$\varepsilon = \frac{(\rho c_p \delta_k)_g}{(\rho c_p \delta_s)_s}\frac{\tanh((i+1)y_0/\delta_k)}{\tanh((i+1)l/\delta_s)} \tag{11}$$

Viscous resistance

$$R_V = \frac{\mu \Pi L}{A_C^2 \delta_V N} = \frac{2\mu}{\delta_V}\frac{L}{(dc + t_w)\pi H^2 N} \tag{12}$$

where μ, A_C and δ_V (given by $\sqrt{\frac{2\mu}{\rho\omega}}$) are respectively the dynamic viscosity of the gas, the area of the channel and the viscous penetration depth.

Convective heat flow

$$\overset{o}{Q}_{conv} = hA\left(T_S - T_\infty\right) \tag{13}$$

$$T_S = T_h e^{\ln\left(\frac{T_C}{T_h}\right)\frac{Za}{L}} \tag{14}$$

$$\overset{o}{Q}_{conv} = H\int_0^{2\pi}\int_0^{L} h(T(z))(T(z) - T_\infty)dzd\phi \tag{15}$$

$$h(Ts) = \frac{k_g}{2H}Nu \tag{16}$$

$$Nu = 0.36 + \frac{0.518Ra_D^{\frac{1}{4}}}{\left[1+\left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} \tag{17}$$

$$Pr = \frac{\nu}{\alpha} \tag{18}$$

$$Ra = \frac{g\beta(T_S - T_\infty)8H^3}{\nu\alpha} \tag{19}$$

where Pr is the Prandtl number; T_S is the surface temperature; T_∞ is the (constant) temperature of the surroundings; ν is the viscosity of the surrounding gas; and α is the thermal diffusivity of the surrounding gas (air).

$$\overset{o}{Q}_{conv} = 2\pi HLh\left[\frac{T_C - T_H}{\ln\left(\frac{T_C}{T_H}\right)} - T_\infty\right] \tag{20}$$

$$Za \geq L\ \log\left(\frac{T_{inf}}{T_C}\right) \tag{21}$$

Radiative heat flow

$$\overset{o}{Q}_{rad} = k_B\varepsilon As\left(T^4 - T_\infty^4\right) \tag{22}$$

$$\overset{o}{Q}_{rad} = Hk_B\int_0^{2\pi}\int_0^{L}\varepsilon\left(T_{(z)}{}^4 - T_\infty^4\right)dzd\phi \tag{23}$$

$$\overset{o}{Q}_{rad} = 2\pi HLk_B\varepsilon\left[\frac{T_C^4 - T_H^4}{4\ln\left(\frac{T_C}{T_H}\right)} - T_\infty^4\right] \tag{24}$$

Conductive heat flow

$$\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x} \tag{25}$$

$$\overset{o}{Q}_{cond} = \int_0^{2\pi}\int_0^{H}\left(k_{zz}\frac{\partial T}{\partial r}\right)drd\phi \tag{26}$$

$$k_{zz} = \frac{k_S t_w + k_g dc}{t_w + dc} \tag{27}$$

Therefore: $$\left.\frac{\partial T}{\partial z}\right|_{z=L} = \frac{\ln\left(\frac{T_C}{T_H}\right)\left(\frac{T_C}{T_H}\right)}{L} \tag{28}$$

And after integration $$\overset{o}{Q}_{cond} = \frac{k_{zz}}{L}\pi H^2 T_C \ln\left(\frac{T_H}{T_C}\right) \tag{29}$$

Details description of the derivations of Equations 3 to 29 are available in reference [7], [11] and [13].

SOLUTION METHODOLOGY OF THE MULTI-OBJECTIVE MATHEMATICAL PROGRAMMING PROBLEMS

All the expressions involved in the mathematical programming formulation (MPF) have been presented in the previous section. Together with the following expressions, they represent a mixed-integer nonlinear programming (MINLP) problem:

$$(\text{MPF}) \min_{L,H,Za,dc,N} \xi = w_1(-W) + w_2 R_V + w_3 \overset{o}{Q}_{conv} + w_4 \overset{o}{Q}_{rad} + w_5 \overset{o}{Q}_{cond} \tag{30}$$

This mathematical model characterizes the essential elements of a standing wave thermo-acoustic engine. Restricted cases of objectives functions in order to identify general tendencies of the structural variables to influence individual objective components is described in Tartibu et al. [13] studies . To illustrate the proposed approach, a thermo-acoustic couple (TAC) as described in Atchley et al. [15], which consists of a parallel-plate stack placed in helium-filled resonator is considered. All relevant parameters are given in Table 1 and Table 2.

Table 1: Specifications for thermo-acoustic couple

Parameter	Symbol	Value	Unit
Isentropic coefficient	γ	1.67	
Gas density	ρ	0.16674	kg/m^3
Specific heat capacity of the gas	c_p	5193.1	J/kg.K
Dynamic viscosity of the gas	μ	$1.9561.10^{-5}$	kg/m.s
Maximum velocity	u_{max}	670	m/s
Maximum pressure	p_{max}	114003	Pa
Speed of sound	c	1020	m/s
Thickness plate	t_w	1.91.10-4	m
Frequency	f	600-700-800	Hz
Thermal conductivity Helium	k_g	0.16	W/(m.K)
Thermal conductivity stainless steel	k_S	11.8	W/(m.K)

Table 2: Additional parameters used for programming

Parameter	Symbol	Value	Unit
Temperature of the surrounding	T_∞	298	K
Constant cold side temperature	T_C	300	K
Constant hot side temperature	T_H	700	K
Wavelength	λ	1.466	m
Thermal expansion	β	$1/T_\infty$	1/K
Thermal diffusivity	α	2.1117E-5	m^2s-1

Critical temperature gradient	Γ	3	

All proposed MINLP models are solved by GAMS 23.8.1, using LINDOGLOBAL solver on a personal computer Pentium IV 2.1 GHz with 4 GB RAM.

For the case of multiples objective optimization, all five objective components are considered by regarding acoustic work (W), viscous resistance (R_V), convective heat flow ($\overset{o}{Q}_{conv}$), radiative heat flow ($\overset{o}{Q}_{rad}$) and conductive heat flow ($\overset{o}{Q}_{cond}$) as five distinct objective components. The optimization task is formulated as a five-criterion mixed-integer nonlinear programming problem (MPF) that simultaneously minimizes the negative magnitude of the acoustic work (W) (since it is the only objective to be maximized), the viscous resistance (R_V), the convective heat flow ($\overset{o}{Q}_{conv}$), the radiative heat flow ($\overset{o}{Q}_{rad}$) and the conductive heat flow ($\overset{o}{Q}_{cond}$).

$$(\mathrm{MPF}) \min_{L,H,Z_a,dc,N} \xi = \left\{ -W_{(L,H,Z_a,dc,N)}, R_{V(L,H,Z_a,dc,N)}, Q_{conv(L,H,Z_a,dc,N)}, Q_{rad(L,H,Z_a,dc,N)}, Q_{cond(L,H,Z_a,dc,N)} \right\} \quad (31)$$

subject to constraints in Equations 3 and 21, and variable restrictions in Equation 33. In this formulation, (L, H, Za, dc, N) denotes the geometric parameters.

There is no single optimal solution that simultaneously optimizes all the five objectives functions. In these cases, the decision makers are looking for the "most preferred" solution. To find the most preferred solution of this multi-objective model, the augmented ε-constraint method (AUGMENCON) as proposed by Mavrotas [16] is applied. The AUGMENCON method has been coded in GAMS. The code is available in the GAMS library (http://www.gams.com/modlib/libhtml/epscm.htm) with an example. While the part of the code that has to do with the example (the specific objective functions and constraints), as well as the parameters of AUGMENCON have been modified in this case, the part of the code that performs the calculation of payoff table with lexicographic optimization and the production of the Pareto optimal solutions is fully parameterized in order to be ready to use.

Practically, the ε-constraint method is applied as follows: from the payoff table the range of each one of the p-1 objective functions that are going to be used as constraints is obtained. Then the range of the ith objective function is divided into qi equal intervals using (q_i-1) intermediate equidistant grid points. Thus in total (q_i + 1) grid points that are used to vary parametrically the right hand side (ε_i) of the ith objective function are obtained (detailed descriptions available in reference [16]). The total number of runs becomecon (q_2 + 1) × (q_3 + 1) × … × (q_p + 1). The augmented ε-constraint method for solving model (Equation 31) can be formulated as:

$$\max\left(W_{(L,H,Za,dc,N)} + dir_1\ r_1 \times \left(\frac{s_2}{r_2} + \frac{s_3}{r_3} + \frac{s_4}{r_4} + \frac{s_5}{r_5} \right) \right)$$

Subject to

$$\begin{aligned} & R_{V(L,H,Za,dc,N)} - dir_2\ s_2 = \varepsilon_2 \\ & \overset{o}{Q}_{conv(L,H,Za,dc,N)} - dir_3\ s_3 = \varepsilon_3 \\ & \overset{o}{Q}_{rad(L,H,Za,dc,N)} - dir_4\ s_4 = \varepsilon_4 \\ & \overset{o}{Q}_{conv(L,H,Za,dc,N)} - dir_5\ s_5 = \varepsilon_5 \\ & s_i \in \Re^+ \end{aligned} \quad (32)$$

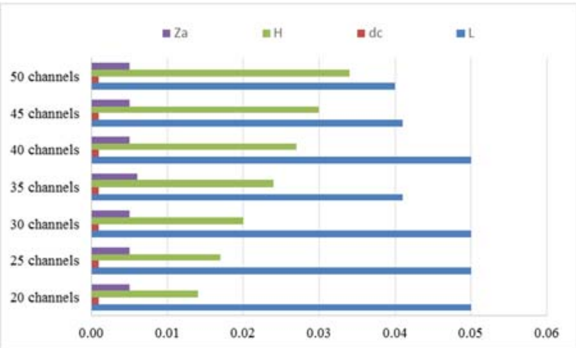
where dir_i is the direction of the ith objective function, which is equal to -1 when the ith function should be minimized, and equal to +1, when it should be maximized. Efficient solutions to the problem are obtained by parametrical iterative variations in the ε_i. s_i are the introduced surplus variables for the constraints of the MMP problem. $r_1\ s_i / r_i$ are used in the second term of the objective function, in order to avoid any scaling problem. The formulation of Equation 32 is the augmented ε-constraint method due to the augmentation of the objective function W by the second term. The following constraints (upper and lower bounds) have been enforced on variables in order for the solver to carry out the search of the optimal solutions within each identified ranges:

$$
\begin{aligned}
&L.lo = 0.005; \;\; L.up = 0.05; \\
&Za.lo = 0.005; \\
&H.lo = 0.005; \\
&dc.lo > 2.\delta_k; \;\; dc.up < 4.\delta_k
\end{aligned}
\quad (33)
$$

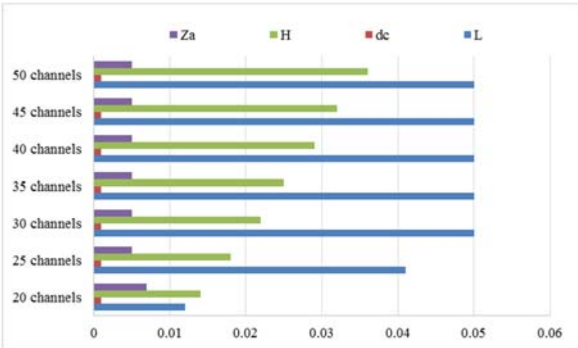
We use lexicographic optimization for the payoff table (detailed descriptions available in reference [13]); the application of the model (Equation 32) will provide *only* the Pareto optimal solutions, avoiding the weakly Pareto optimal solutions. Efficient solutions to the proposed model have been found using AUGMENCON method and the LINDOGLOBAL solver. To save computational time, the early exit from the loops as proposed by Mavrotas [16] has been applied. The range of each five objective functions is divided in four intervals (five grid points). The integer variable N has been given values of 20-25-30-35-40-45-50. In addition, the frequency was set (arbitrary) to 600 Hz-700 Hz- 800 Hz. The maximum CPU time taken to complete the results is 2050.212 sec. The following section report only sets of Pareto solutions obtained.

RESULTS AND DISCUSSIONS

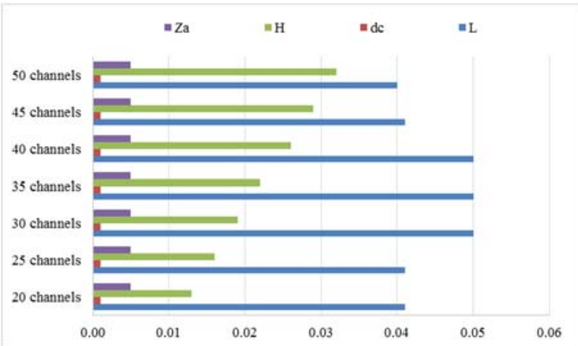
Figure 4 represents the Pareto optimal solutions graphically; these results shows that there are numerous optimal solutions that optimizes the geometry of the stack and highlights the fact that the geometrical parameters are interdependent (no clear relation between variables could be derived), supporting the use of a multi-objective approach for optimization of thermo-acoustic engines. To maximize acoustic work W and minimize viscous resistance and thermal losses simultaneously, there is a specific stack length (L) to which correspond a specific stack height (H), a specific stack spacing (dc) and a specific number of channels (N). It should be noted that in all cases, locating the stack closer to the closed end produced the desired effect. All Pareto optimal solutions can be computed in order for the decision maker to select his preferred choice.



(a) Frequency = 600 Hz



(b) Frequency = 700 Hz



(c) Frequency = 800 Hz

Figure 4: Optimal structural variables

These optimal solutions are then used to construct Figure 5 representing respectively acoustic work, viscous resistance and thermal losses plotted as a function of N, L, dc and H. The unique contribution of this work is the ability to quantify all the thermal losses and select the best geometrical configuration of the stack accordingly. Therefore, the designer can simultaneously maximise acoustic work and minimise losses (viscous resistance as well as heat flows) by considering the thermal efficiency (η) which can be defined as the ratio of the work output over the sum of the work output and losses as follows:

$$\eta = \frac{W}{W + R_v + Q_{conv} + Q_{rad} + Q_{cond}} \tag{34}$$

This ratio can be used to compare the results obtained by the proposed augmented ε-constraint method and identify the preferred solution.

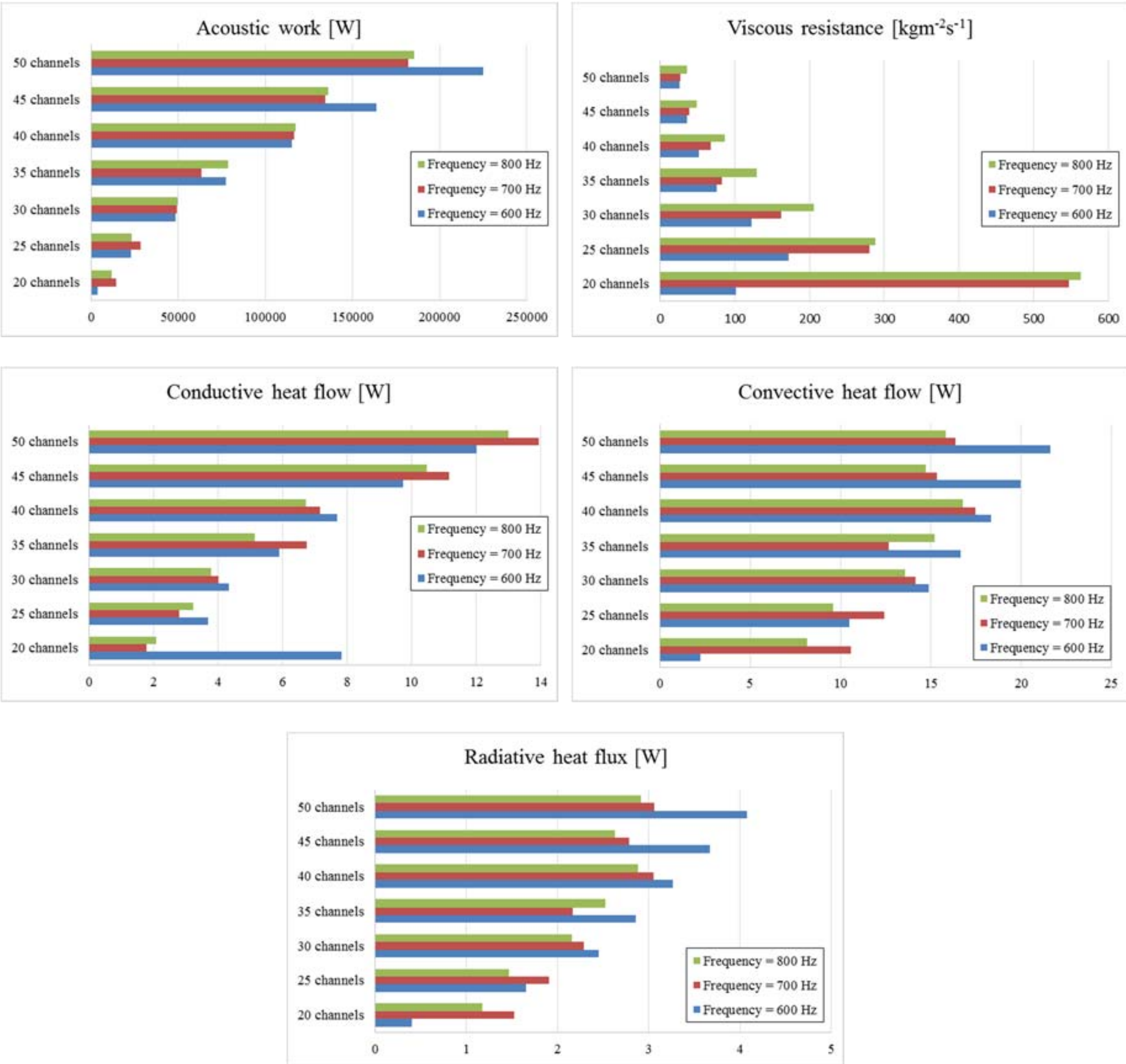


Figure 5: Acoustic work, viscous resistance and heat flows

CONCLUSION

Previous studies reveal that scaling down the device leads to an increase of the ratio between the surface area and the active volume, resulting in higher thermal losses. These thermal losses - the convective, radiative and conductive heat flow - need careful attention since they are not adequately included in current modelling approaches. An estimate of heat losses shows that they are significant relative to the total energy supplied to the thermo-acoustic engine (TAE). This provides a clear motivation to include the aforementioned losses in the modelling approach in an effort to improve the performance of the devices. A new mathematical modelling approach is proposed to model and optimize thermo-acoustic engines while taking into account thermal losses. The idea of incorporating thermal losses in the modelling and optimization of TAE gives the decision maker a clear picture of expected magnitude and the ability to search for the configuration that will simultaneously minimize them. This approach is used to compute the optimal set of parameters describing the geometry of the device: the stack length, stack height, stack position from the closed end of the TAE, stack spacing and the number of channels. These are the variables in the mathematical modelling formulation. The performance of the device is measured through the acoustic (work output and viscous resistance) and the thermal losses (convective, radiative and conductive heat flow) that have been used as objective functions to measure the quality of each set of variable values that satisfies all of the constraints. This problem has been formulated as a five-criterion mixed-integer nonlinear programming problem. This formulation allows for identifying the implication of each objective emphasis on the geometry of the stack. A case study is used for illustration. A set of objective functions and Pareto optimal solutions are computed in this work and guidance for the decision maker's selection of the preferred solution is suggested.

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