THE USE OF COGNITIVELY TARGETED ASSESSMENT INSTRUMENTS IN THE PSYCHO-EDUCATIONAL ASSESSMENT OF LOW ACHIEVERS IN MATHEMATICS

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ABSTRACT

This study has been informed by theoretical developments in the expanding field of mathematical cognition, which has led to a greater understanding of how arithmetic is implemented in the brain. This in turn has led to the development of cognitively targeted assessment instruments. These instruments aim to identify deficits in numerical and other cognitive systems. The objective of this study was to investigate the use of these instruments in the psycho-educational assessment of low achievers in the subject of mathematics. Ultimately, this study sought to ascertain whether or not these assessment instruments, together with an understanding of the theory that has informed their development, would be useful to educational psychologists in their support of learners who struggle with mathematics.

This study set out to investigate this question by assessing grade eight learners who were achieving below to significantly below average marks in mathematics, with three cognition based instruments and one achievement test. A comparative analysis of the result of all participants on each assessment instrument was presented. The participants were then grouped into one of four profiles, based on similar patterns emerging in the results. An integrated discussion of each of the four profiles was given.

The discussions described support strategies based on the empirical findings of the assessments. Suggested support strategies for each of the profiles is different, which confirmed that the results of these assessment instruments should enable an educational psychologist to get a better understanding of what is actually being dealt with, which should make intervention and support more focused and less of a hit and miss approach.

This study therefore recommends that educational psychologists firstly familiarize themselves with mathematical cognition theory, particularly the theories of Dehaene (2011) and Butterworth (1997 & 2003). Secondly, that measures of mathematical cognition, such as the Dyscalculia Screener and the Number Discrimination Task, form part of psycho-educational assessment procedures for learners who are struggling with mathematics.

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CHAPTER 1

BACKGROUND AND RATIONALE

1.1 INTRODUCTION

It is widely acknowledged that South African learners are underachieving in mathematics and much research into this phenomenon has been undertaken (Maree, 2008). Most of these research endeavours have, however, focused on the environmental, pedagogical and sociological factors that negatively impact on the teaching and learning of mathematics (Maree, 2008). Recently researchers such as Reyneke (2015) and Henning (2013), amongst others, have recommended a change in focus to the psychology of learning and the learning of mathematics, particularly cognitive psychology and cognitive neuroscience (Henning, 2013). Henning (2013, p.57) suggests that "South African researchers in mathematics education would benefit from a theoretical lens that includes, but also goes beyond, theories describing learner classroom performance and teacher practice".

A similar lens to that suggested by Henning, will also be utilised in this study where the focus is on the cognition involved in the learning or 'not learning' of mathematics, more particularly focusing on the specific learning disorder in mathematics, also known as dyscalculia. This lens will also be used to investigate the phenomena of dyscalculia in terms of the practice zone of educational psychological assessment and support. This study will thus utilise a bifocal lens, which will focus on both mathematical cognition and the learning of mathematics, including a focus on new ideas about dyscalculia. In addition, the study will also look at the practice of educational psychology in terms of assessment strategies for learners who achieve poorly in mathematics.

Developments in knowledge regarding the neural bases of numerical cognition has led to increased research in this field, as has cognitive neuroscience research on dyscalculia (Wilson & Dehaene, 2007). The impetus for this study comes from the point made by cognitive neuroscientist, Stanislas Dehaene, that due to the great strides made in understanding how arithmetic is implemented in the brain, the application of cognitive neuroscience to education is no longer a “bridge too far” (2011 p. 278). Educational psychologists’ engagement with recent developments in the expanding field of mathematical cognition has been described as “overdue” (Gillum, 2012, p. 288). The possible implications of this lack of consideration is that low achievers in mathematics could be losing out on more effective support and remediation that could be informed by assessment instruments which have been informed by the theoretical developments in this field.

This study will investigate two of these cognition-based assessment instruments that could be used by educational psychologists in psycho-educational assessment procedures to identify possible causes of low achievement in mathematics. These two instruments will be used in conjunction with an assessment of working memory and an assessment of knowledge in basic mathematical operations. The study aims to argue that current psycho-educational assessment procedures, as well as most remediation strategies, are targeted at the behavioural level of mathematical operations and procedures. Thus they do not intentionally assess for specific underlying cognitive deficits, which recent studies have found to have a significant effect on the capacity to learn arithmetic (Butterworth & Laurillard, 2010).

1.2 PROBLEM STATEMENT

It is widely acknowledged that there are many interrelated factors that play a role in low achievement in mathematics (Butterworth, 2003; Maree, 2008). A wide range of both systemic or extrinsic factors as well as intrinsic or barriers within the learner have been identified and described (Dednam, 2011). Extrinsic factors could include inadequate learning environments coupled with poor-quality teachers and teaching (Maree, 2008). Intrinsic factors could include a negative study orientation towards mathematics, which has been conceptualized as a configuration of interrelated factors, including emotions, habits and attitudes (Maree, van der Walt, & Ellis, 2010c).

However, there exists a category of learners whose low achievement in mathematics cannot be ascribed to these problems (Butterworth, 2003). These children have perceptual functioning which does not display any difficulties; good command of language; and their intelligence falls within the normal range (Butterworth, 2003). Yet, these children exhibit disproportionate difficulties with number processing, arithmetic and mathematics in general, despite adequate schooling (Dehaene, 2011). This phenomenon suggests that there must exist more specific cognitive[1] mechanisms and systems that play a role in the learning of mathematics, that could be impaired (Gillum, 2012). This has indeed been the findings of research, which will be fully discussed in Chapter Two, Paragraph 2.3.

Such learners could be diagnosed as having a specific learning disorder with impairment in mathematics, as many would meet the diagnostic criteria in Diagnostic and Statistical Manual of Mental Disorders, fifth edition, (DSM-5) (American Psychiatric Association, 2013). However, since these criteria mainly describe the behavioural manifestations of the learning disorder, they will not necessarily and reliably identify learners with a specific cognitive deficit in the capacity to learn mathematics (Butterworth, 2003). I argue that in the South African context, these learners would exist in a much larger pool of learners who make up the national disaster of poor achievement in mathematics, and would most probably remain camouflaged by their cognitively non-impaired peers who achieve poorly because of other, mainly extrinsic, factors.

An assessment process that has a clear cognitive target may provide usable insights into low achievement in mathematics. These assessments may reveal the presence of cognitive deficits as the cause of low achievement on the one hand, while still acknowledging that low achievement may have an educational/instructional basis and may not necessarily be as the result of a numerical cognitive deficit on the other.

1.3 RESEARCH QUESTION

The above stated discussion of the identified problem has led to the formulation of the research question:

[1] The meaning of cognitive and cognition in terms of this study will be further deliberated in Paragraph 1.5: Theoretical Framework.

How do four assessment instruments (two mathematical cognition analysis tests, one working memory test and one achievement test) offer insight into the low mathematical performance of a group of grade eight learners?

1.4 AIMS AND OBJECTIVES

This study aims to investigate how two measures or assessments of mathematical cognition, together with an assessment of working memory and a mathematical achievement test (operational knowledge) could offer insights into a group of Grade 8 learners' low performance on mathematics.

The objectives of this study are to:

- Conduct an integrated assessment of twelve low performing grade eight learners using three cognitive measures (two of mathematical cognition and one of working memory) and one measure of mathematics achievement, measuring operational knowledge.
- Analyse the results to provide a quantitative profile for each learner.
- Use the profiles to look for commonalities in their performances.
- Suggest intervention and learning support strategies, informed by results of the assessments.

1.5 THEORETICAL FRAMEWORK

The theoretical field in which this study is located is chiefly that of cognitive psychology, which can be defined as:

> "An approach that aims to understand human cognition by the study of behaviour; a broader definition also includes the study of brain activity and structure" (Eysenck & Keane, 2015, p.2)

Cognitive neuroscience is more specifically aimed at understanding human cognition through the study of brain activity and structure (Eysenck & Keane, 2015).

Mathematical cognition is a subfield of cognitive psychology, that is concerned with the cognitive and neurological processes that underlie numerical and mathematical abilities (Campbell, 2004), and how arithmetic is implemented in the brain (Dehaene, 2011). In other words, the field of mathematical cognition is concerned with how the brain perceives, understands and processes number and quantity, and how these processes enable the performance of formal mathematics (Dehaene, 2011). Knowledge of these cognitive processes or systems has allowed researchers to link impairment in them to decreased abilities in the performance of mathematics, either as a school subject, or in day to day tasks requiring basic arithmetic, such as calculating the gratuity on a restaurant bill. Therefore, the field of mathematical cognition is also concerned with the identification of the cognitive bases of mathematical learning disorders (Dehaene, 2011), which has resulted in the development of assessment instruments designed to measure these specific cognitive systems (Gillum, 2012).

1.6 OVERVIEW OF RESEARCH METHODOLOGY

1.6.1 Research Design

It was decided to utilize a case study design in this study, based on the fact that the epistemological question that underpins the case study approach is the same question that underpins the process of psycho-educational assessment, namely, "what can be learned from the single case?" (Stake, 2005). A multiple case study design with embedded units of analysis was utilized (Yin, 2014). Each participant in the study formed a single case, and each cognitive system within each participant was an embedded unit of analysis.

1.6.2 Sample

It was decided to select learners in Grade 8 for this study. The reason for this is that it is still early enough in these learners' secondary school careers for intervention and support to be offered based on possible findings of the assessments. Additionally, the one assessment instrument utilized in this study, namely the Dyscalculia Screener (Butterworth, 2003) has an age ceiling of 14 years and 12 months. Most grade 8 learners fall within the 13 to 14 year age group.

The learners who took part in this study all attend a private boys school where English is utilized as the language of instruction. The school writes the matric examinations of the Independent Examinations Board (I.E.B), and is one of the top achieving boys' schools in this examination. This suggests a high expected standard of teaching and learning. This information, regarding the learning context of the participants, is important to note, because it will form part of the justification for the utilization of assessment instruments that have not been normed locally.

1.6.3 Data Collection Methods

Assessment Instruments

The following assessment instruments were utilized to meet the first objective mentioned above:

- Mathematical cognition:
  - Dyscalculia Screener (Butterworth, 2003)
  - Number Discrimination Task (Mazzocco, Feigenson & Halberda, 2011)
- Working memory:
  - Automated Working Memory Assessment (Alloway, 2007)
- Mathematical achievement (operational knowledge)
  - Numerical Operations subtest of the Wechsler Individual Achievement Test, UK Second Edition (Pearson Education, 2005)

The tests of mathematical cognition and working memory can collectively be referred to as cognitive tests because they aim to tap into and measure a cognitive ability or process that has not necessarily been taught. The Numerical Operations subtest is an achievement test, because it measures knowledge or skill that has been taught (Sattler & Hoge, 2006).

Documents and Artefacts

The overall term mark for the first term of the year was obtained from the class teacher, with permission of the participants.

1.6.4 Data Processing and Analysis

There were two phases in the analysis of the results. Firstly, the results of all participants on each assessment instrument were presented and discussed. Correlations between instruments will be looked for and duly discussed. Secondly, the results of each participant will be presented in an integrated fashion and the unique profile of each participant will be discussed.

I will present the data in the following way: Firstly, I will showcase the results from each assessment instrument across all participants comparatively – thus showing the results of all the participants in one particular assessment instrument such as, for instance "The Dyscalculia Screener" (Butterworth, 2003). I will furthermore present the results from the different assessment instruments for each individual participant. For instance, for Participant 1, I will show his mathematics scholastic results (term mark) and then his results on each one of the four assessment instruments. In this way I can discuss each participant's results as an integrated case, from which I can then make possible conclusions and recommendations for the participant and his parents on how his mathematical development could be supported. I will then categorise and group cases which show similarity in their results, in order to discuss a specific phenomenon which could give us insight into the possible reasons for low achievement in mathematics. From the analysis of results and reports of the findings, categories of findings will be highlighted and subsequent themes will be extracted. These themes will then be discussed in terms of possible insights into the learners' low performance in mathematics.

1.7. TRUSTWORTHINESS

In order to ensure trustworthiness, the ethical guidelines regarding the use of psychological assessments will be strictly adhered to. These include safe guarding the integrity of the assessments, strictly adhering to the administration instructions of the assessment, and ensuring that great attention is paid to the scoring of assessments (Foxcroft & Roodt, 2013).

1.8. ETHICS

The general guidelines of ethical research will be adhered to in this study. Ethical clearance has been obtained from the Research Ethics Committee of the Faculty of Education at the University of Johannesburg (Appendix A). The principle of anonymity will be maintained by using numbers to refer to the participants. The notion of voluntary participation will be communicated to all participants, and the right to withdraw at any time will be conveyed clearly. All assessment results and other data will be locked in storage and no one, besides the researcher and supervisors will have access to the data.

In terms of ethical conduct with participants, since this study is located in the practice zone of psycho-educational assessment, the guidelines for ethical practice in this field will be strongly adhered to according to the guidelines set by the Health Professions Council of South Africa (2006). These guidelines include informed consent from the parents of minors, informed assent from the participants themselves and the maintenance of confidentiality (Sattler & Hoge, 2006).

Since terms such as “low achievement” are used in this study, care will be taken to ensure that the participants do not feel demeaned by such terms. Butterworth (2003), notes that learners who struggle with mathematics often face emotional stress and do not need to be constantly reminded of the importance of mathematics. The researcher will have the responsibility of ensuring that discussion with each participant regarding the nature of their difficulties will be done in a sensitive, caring and encouraging manner, without downplaying the reality the learning difficulties they are faced with.

1.9. DEMARCATION OF THIS STUDY

The study consists of the following chapters:

Chapter One serves as an introduction to the study intended to orientate the reader to the context and theory that underpins the research. It also gives a brief overview of the study.

Chapter Two contains a detailed literature review, presenting and synthesizing the most recent theory around mathematical cognition, working memory and trends and practices related to assessment. A review of the theory that has led to the development of the assessment instruments in this study is also presented in detail.

Chapter Three presents the research design and methodology that will be utilized in this study. Since the data collection in this study will be done by using assessment instruments, a detailed description of how each instrument works will be given. A discussion of how the result of each instrument will be utilized will also be given.

Chapter Four will present the results of the assessments and will analyse the results according to the data analysis methods described in Chapter 3. The findings will be discussed and emergent themes will be identified.

Chapter Five will conclude the study with a discussion of the themes identified in Chapter 4. A discussion of limitations will follow as well as recommendations for further research.

1.10. CONCLUSION

This chapter has described the background and rationale for this study and presented an overview of how the research will take place. The next chapter will present a detailed discussion of the theory that underpins this study by reviewing and synthesising literature in the field of mathematical cognition and beyond.

CHAPTER 2

LITERATURE REVIEW:
DEVELOPMENTS IN MATHEMATICAL COGNITION THEORY AND ITS IMPLICATIONS FOR EDUCATIONAL PSYCHOLOGICAL PRACTICE

2.1 INTRODUCTION

Since educational psychologists are scientist-practitioners who stand in the privileged position between psychological research and educational practice (Gillum, 2012), this literature review will move between a focus on both theory and practice. A focal point will be a thorough exploration of the recent developments in the field of numerical and mathematical cognition, which have been informed by research in cognitive neuroscience and cognitive developmental psychology (Butterworth, Varma & Laurillard, 2011).

The literature review will begin by describing various ways of understanding low achievement in mathematics and the traditional approaches to assessment and support of these learners. Readers will then be introduced to the two main conceptual views of understanding mathematical disability on a cognitive level, namely the domain-specific and domain-general approaches. Under the domain-specific view/approach, I will review literature on recent theoretical developments in the field of numerical cognition, which have been informed by the fields of neuroscience and cognitive psychology. Under the domain-general approach, I will review literature on the role played by working memory in the learning of mathematics. From both approaches, I will introduce the reader to ways in which these theoretical developments have begun to inform practice, and how these developments can be utilized in future by educational psychologists. These suggestions for improved practice will then be tested in the study, described in chapter 3, and the results will be reported in chapter 4.

2.2 DYSCALCULIA, LOW ACHIEVEMENT IN MATHEMATICS AND CURRENT ASSESSMENT PRACTICE

The existence of a category of learners who exhibit disproportionate difficulties with number processing, arithmetic and mathematics in general, despite adequate schooling (Dehaene, 2011), suggests that there must exist more specific cognitive mechanisms and systems that play a role in the learning of mathematics, that could be impaired (Gillum, 2012).

Traditional approaches to the identification of mathematical learning disabilities, often referred to as dyscalculia, have been based on the significant underachievement of a learner on a standardized test relative to what would be expected considering education, general intelligence and age (Butterworth, 2003). The problem with this approach is that standardized mathematics tests generally test a range of skills, procedures and operations and would therefore be unlikely to reveal specific numerical cognitive deficits because they involve many combinations of cognitive processes (Wilson & Dehaene, 2007). An opinion article, authored by seventeen leading researchers from across the world, asserts that:

> "There is convincing evidence that basic numerical skills are impaired in DD [developmental dyscalculia]. Therefore, purely educational (curricular) tests are not adequate to tap the characteristic numerical deficits associated with DD". (Kaufmann et al., 2013, p.4)

The assessment of operational competence and basic mathematical knowledge, skills and vocabulary remains necessary, and there are many useful instruments available to practitioners. One such instrument is the South African standardized *Tri-Maths* (Maree, van der Walt, & Ellis, 2010a; Maree, van der Walt, & Ellis, 2010b; Maree et al., 2010c) which aims to identify basic knowledge, vocabulary and study orientation in mathematics, but does not, by the author's own admission "tap into a psychological construct" (Maree et al., 2010a, p.10). These and similar instruments, however useful, are not cognitively targeted, and are therefore unable to distinctly identify specific cognitive deficits that may be related to a learner's low performance in mathematics. Additionally, these types of tests are also unlikely to identify suitable, cognition-targeted remediation and learning support tools (Henning, 2014). At best such support is for operational/procedural knowledge, and not for mathematical reasoning and conceptual change, resulting in the possibility of treating the symptoms and not the cause. It is therefore important to differentiate between formative assessments of children's difficulties in mathematics and diagnostic assessments for dyscalculia, since the two are very different, yet both entirely necessary (Gillum, 2012).

This study included a standardized assessment of basic numerical skills, using the Numerical Operations subtest of the Weschler Individual Achievement Test, Second UK Edition (Pearson, 2005). This was done in order to obtain a basic measure of competence in mathematical operations and procedures, which will be useful to compare with a learner's performance on measures in numerical cognition.

Therefore, while not overlooking the importance of the curricular aspect of assessment in supporting low achievers in mathematics, it would be remiss of educational psychologists not to take cognisance of the latest developments in the field of mathematical cognition (Gillum, 2012); both in informing curriculum development and assessment, but also in assessing and supporting individual learners who struggle specifically with mathematics.

2.2.1 Dyscalculia

The criteria for defining and diagnosing dyscalculia have been described as ambiguous (Kaufmann et al., 2013). This is of little surprise considering that research in this field is relatively new, and new theories explaining the aetiology and manifestation of dyscalculia differ in many respects (Butterworth, 2010; Piazza, 2010; Rubinsten & Henik, 2009). The biggest bone of contention lies in whether dyscalculia is caused by a single core deficit in number sense (the core deficit hypothesis), or by deficits in several cognitive systems, such as working memory and attention (Rubinsten & Henik, 2009). Those theorists who believe strongly in the core deficit hypothesis would make a diagnosis of dyscalculia only if there is clear evidence of the core deficit (Butterworth, 2003), whereas those who believe in more domain-general approaches (Kaufmann et al., 2013) would be guided by the diagnostic criteria put forward in the Diagnostic and Statistical Manual of Mental Disorders fifth Edition ,DSM-5, (American Psychiatric Association, 2013) in making a diagnosis of a Specific Learning Disorder with impairment in mathematics. However, I would argue that a

DSM-5 diagnosis alone will be of little use in supporting a mathematically low performing learner if underlying cognitive systems are not investigated. I argue that impairment or non-impairment of these cognitive systems would be crucial information in informing the support of the learner.

This study has been informed by arguments on both sides of the domain-specific and domain-general divide, as the research put forward in this literature review has produced evidence for both domain-specific and domain-general predictors of mathematical ability (Kuhn & Holling, 2014). Predictors from both domains will therefore be assessed in this study.

The study of developmental dyslexia may offer a helpful insight in terms of defining developmental dyscalculia. Most researchers of dyslexia emphasize the importance of distinguishing between genuine dyslexics and 'garden-variety' poor readers (Milne, 2005). A genuinely dyslexic learner has a certain neurological make-up that disrupts reading acquisition, whereas a 'garden-variety' poor reader has underdeveloped reading circuits due to insufficient instruction or a variety of other extrinsic factors (Milne, 2005). A similar argument could be made for dyscalculia, that there is a need to distinguish between learners who show impairment in numerical cognition systems and learners who are garden-variety poor mathematics achievers due to inadequate instruction or a variety of other extrinsic factors.

The purpose of this study is to use assessment instruments that aim to assess these specific cognitive systems, as it is hypothesized that impairment in these systems could underlie the participant's low achievement in mathematics. Educational psychologists who use these instruments in practice will need to be guided by clinical judgment and professional ethics in making, or not making a formal diagnosis.

2.2.2 Low Achievement

How do we define a 'low achiever in mathematics'? Some studies in mathematical cognition have distinguished between three groups of learners, namely maths disabled, low achieving and typical achieving (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). In the study alluded to above, participants are sorted into these groups according to how they achieved on a mathematics achievement test. In the case of the cited study, the Numerical Operations subtest of the Weschler Individual Achievement Test was used (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Those who achieved below the 15th percentile were classified as maths disabled, those who achieved between the 23rd and 29th percentile were classified as low achieving, and those who scored above the 50th percentile were classified as typical achieving.

Although this study does make use of a standardized mathematics test, the scores were not used to sort participants into different groups. The term 'low achiever' in this study is used as a general term and has been applied to learners who fall into the lowest achieving ability group in their year group, based on their performance on a mathematics test that was written at the beginning of the year of their current grade. The researcher acknowledges the fact that there are a wide range of marks within this group. Of the nine learners who participated in this study, exactly half of them obtained below 39% for the first term, with one achieving below 29%. The fact that there exists a variety of ability within the group of participants has influenced the research design and methodology, which will be fully discussed in Chapter 3.

2.3 DOMAIN-SPECIFIC APPROACHES IN MATHEMATICAL COGNITION THEORY

2.3.1 Stanislas Dehaene – *The Number Sense*

Giaquinto (2001), notes that everyone involved in the field of numerical cognition is indebted to Stanislas Dehaene for his seminal book *The Number Sense*, first published in 1997. A revised and expanded edition was published in 2011. Dehaene is a professor of experimental cognitive psychology. His interests lie in the cerebral bases of human cognitive functions such as calculation, reasoning and language (The Collège de France, n.d.) . His most significant scientific contribution has been the study of the organization of the cerebral system for number processing. Drawing on evidence gathered from positron emission tomography (PET), event related potentials (ERPs), functional magnetic resonance imaging (fMRI), and brain lesions, Dehaene demonstrated the central role played by a region of the intraparietal sulcus in understanding quantities and arithmetic, which he named 'number sense' (The

Collège de France, n.d.). Dehaene's main thesis, presented in *The Number Sense* (2011), is that we have inherited neural mechanisms for representing approximate numerical quantities, and the acquisition of numerical abilities depend on this (Giaquinto, 2001).

2.3.2 Number sense and core systems of number

According to Dehaene and other researchers in the field, there are two core systems of number that account for humans' basic number sense (Dehaene, 2011; Piazza, 2010; Feigenson, Dehaene, & Spelke, 2004). These two core systems have become known as the approximate number system (ANS) and the object tracking system (OTS). These two systems are described as distinct systems of numerical representations, that have been found to be present in human infants and other animal species, and therefore they do not emerge through learning or cultural transmission (Feigenson et al., 2004). It is widely claimed that the acquisition of symbolic number depends on the symbols for numbers acquiring meaning through being mapped onto these pre-existing core quantity representations (Piazza, 2010). Some proposals emphasise the role of the ANS in this process, while others emphasize the role of the OTS. Still others consider the combination of the two systems as crucial in the acquisition of formal arithmetic (Piazza, 2010).

2.3.2 (i) The approximate number system (ANS)

The ANS is a mental system of quantification that has shown to be present in infants, children and adults (Feigenson et al., 2004). The system yields a representation of approximate number that captures the inter-relations between different numerosities (Feigenson et al., 2004). When shown arrays of dots under conditions that prevent counting, adults are able to discriminate between numerosities, in other words they can perceive which array has more dots without counting them (Feigenson et al., 2004). This spontaneous detection of the approximate number of objects in sets, even across sensory modalities, is seen in humans of all ages, even in new born babies (Piazza, 2010). This mental system of approximate number representations is active anytime someone thinks about or uses numbers, not only when doing formal maths, but also during day to day decision making, such as choosing a queue in a grocery store that has the fewest people (Mazzocco et al., 2011).

Impairment in the basic sense of approximate quantity has been found in dyscalculic individuals (Feigenson, Libertus & Halberda, 2013) and research suggests that the acuity of the approximate number is related to mathematical ability, even in individuals with good general intelligence, visuospatial ability and working memory (Feigenson et al., 2013). There is significant evidence to suggest that the contribution of this evolved system for representing approximate quantities is a critical component of the acquisition of formal mathematical ability (Feigenson et al., 2013).

Mazzocco, Feigenson and Halberda (2011), found that impaired acuity of the ANS underlies mathematical learning disability, or dyscalculia. They studied 71 ninth graders, and found that students with mathematical learning disabilities have significantly poorer ANS precision than students in all other mathematics achievements groups. The psychophysical assessment of ANS acuity that was used in this study is described below.

The degree of uncertainty (noise) in an individual's numerical approximation increases with the quantity being represented. When an array of items appears too quickly to allow for serial counting, a specific ANS representation is activated. Figure 2.1 shows how these approximate representations can be depicted as a series of Gaussian curves, organized on a mental number line (Mazzocco et al., 2011).

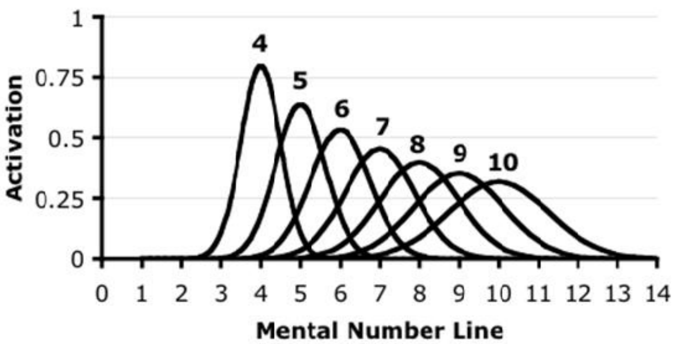

Figure 2.1. **Approximate representations depicted as a series of Gaussian curves, organized on a mental number line.** (Mazzocco et al., 2011, p. 3).

The standard deviation of each number's representation, i.e. the width of each curve, represents the amount of "noise" or error that is linked to that number's representation. As a quantity increases, so does the degree of uncertainty in an individual's numerical approximation (Mazzocco et al., 2011).

It goes without saying that there will be individual differences in the standard deviation of number representations. Larger standard deviations indicate more 'noisy' representations i.e., an increased degree of uncertainty in perceiving the quantity of that representation, and therefore poorer performance on tasks that rely on the ANS (Mazzocco et al., 2011).

A method of measuring and quantifying the precision of an individual's internal representation was used in the study by Mazzocco, Feigenson and Halberda (2011). It was based on the fact that a signature of the ANS is that discrimination of number adheres to Weber's law (Park & Brannon, 2014). Weber's law is a psychophysical law describing the relationship between the physical and the perceived magnitude of a stimulus (Piazza, 2010). The ANS adheres to Weber's law in that "the ability to discriminate two values depends on the ratio between the two values and not just their absolute difference" (Park & Brannon, 2014, p. 188). Therefore, the amount of noise in an individual's ANS can be indexed as a Weber fraction, which is derived by asking an individual to evaluate which of two quickly flashed arrays of dots is more numerous (Mazzocco et al., 2011). As the ratio between the two arrays decreases, individuals make more errors judging which of the two arrays is more numerous, in accordance to Weber's law (Halberda, Mazzocco, & Feigenson, 2008; Mazzocco et al., 2011). The *rate* of this increase in errors is a function of the amount of 'noise' in the ANS representations. A person's Weber fraction is therefore a representation of the amount of noise in the ANS. In other words, the higher the Weber fraction, the less precise is a person's ANS.

In the study by Mazzocco, Feigenson and Halberda (2011) the Weber fraction (*w*) was measured using the Number Discrimination Task. This assessment is also known as "Panamath" and has been made available online and utilised in this study. The workings of this assessment will be described in greater detail in chapter 3, more specifically paragraph 3.6.2.

Considering the amount of literature that indicates that an impairment in ANS as underlying dyscalculia, or poorer performance in mathematics generally, this study will include an assessment of ANS precision in a group of low achieving grade 8 learners. The assessment used for this purpose will be the already developed Panamath task, as described above. The result of this assessment, together with results of other assessments will form the unit of analysis of this study, and will be used to test the hypothesis that cognitive factors, in this case, impairment in the ANS, may underlie these learners' low achievement in mathematics.

2.3.2 (ii) The object tracking system (OTS)

The second core system of number has to do with precisely keeping track of small numbers of individual objects (Feigenson et al., 2004). An important defining property of this system is that it is limited in capacity to three or four individual objects at a time (Piazza, 2010). The existence of this core system of number is evident in enumeration tasks, where subjects can determine the number of objects in small collections of three or four items (such as dots) with high speed and accuracy (Piazza, 2010). This phenomenon is known as *subitizing*, which can be defined as "the rapid, accurate and confident judgement of the number of items in small collections 'at a glance', without counting" (Piazza, 2010, p. 542).

Some theorists (Carey, 2001) propose that the OTS is foundational in the acquisition of symbolic numbers because it provides the notion of exact number (Piazza, 2010). Additionally, it is also claimed that the ANS cannot provide semantic foundation to the representations of symbolic number because it lacks these two properties (Carey, 2001 cited in; Piazza, 2010).

However, there appears to be little evidence to date to suggest that the object tracking system is impaired in dyscalculic learners (Butterworth, 2010; Piazza, 2010), and where there is some evidence to suggest impairment in this system, enumeration of the entire range from one to nine is also impaired (Butterworth, 2010). This points to the existence of a different domain specific system specialized in dealing with numerical representations, and it is the impairment of this system that Brian Butterworth has hypothesized that underlies dyscalculia.

2.3.3 Brian Butterworth - a different domain-specific system

Brian Butterworth, professor of cognitive neuropsychology, followed Stanislas Dehaene's seminal book *The Number Sense*, published in 1997, with *The Mathematical Brain*, published in 1999. The work covers much of the same ground as *The Number Sense* (Joyce, 2001) although the core system of number put forward by Butterworth as underlying the acquisition of formal arithmetic is slightly different. Butterworth argues that a deficit in *numerosity coding,* not in the ANS or the OTS, is responsible for dyscalculia and that the ANS and the OTS are not sufficient to support the typical development of arithmetic skills (Butterworth, 2010).

Numerosity coding has to do with treating a collection of objects as a set, in which a set can be a type of object that can itself take a property. This property does not imply something common between the objects of the set, but could be a property of the set itself, which in this case is the numerosity of the set (Butterworth, 2010). Several studies have found that human infants can use the numerosity of visual arrays as a discriminative stimulus, and that infants can select collections of objects and treat them as a single unit (Butterworth, 2010).

As far back as 1978, Gelman and Gallistel hypothesized that pre-counting children possess 'numerons', that is an ordered sequence of numerosity concepts, e.g. the numeron for one, the numeron for two and so on (Butterworth, 2010). When a child learns to count, they are essentially learning to associate an ordered sequence of counting words to an ordered sequence of numerons, which they should already possess (Butterworth, 2010). So therefore the concept of 'fiveness' pre-exists the knowledge that the word five refers to the numerosity of fiveness (Butterworth, 2010).

Halberda and Feigenson (2008, p. 655) also argue for "a third core capacity – the ability to bind representations of individuals into sets" as responsible for the acquisition of formal arithmetic. They argue that conceiving of objects as a set cannot come from object tracking, the approximate number system, or language (Halberda & Feigenson, 2008), and that "conceiving of a set requires representing the hierarchical relationship between individual items and the larger structure into which they are bound" (Halberda & Feigenson, 2008, p. 655).

The main prediction made from the numerosity coding hypothesis is that dyscalculic individuals will suffer from a deficit in enumerating sets (Butterworth, 2010), and therefore tasks that measure number comparison and dot enumeration will indicate an individual's capacity for numerosity (Butterworth, 2003).

Based on this assumption, Butterworth developed the *Dyscalculia Screener* in 2003 (Butterworth, 2003). Using his findings that dyscalculic children performed significantly worse on two tests of numerosity processing, namely dot enumeration and number comparison, a computerized test was developed that aims to measure these two aspects of numerosity processing, as well as single digit arithmetic (Butterworth & Laurillard, 2010). The test has been standardized with UK norms for learners aged between 6 and 14 years (Butterworth & Laurillard, 2010). The measurement will be discussed in more detail in Chapter 3, Paragraph 3.4.2.

This study included the Dyscalculia Screener in the assessment of the group of grade 8 learners in order to further test the hypothesis that the impairment of cognitive factors, in this case numerosity coding, may underlie these learners' low achievement in mathematics.

2.4 DOMAIN-GENERAL APPROACHES

Whereas basic numerical capacities, such as the ANS, the OTS and numerosity coding can be conceptualized as domain-specific factors relating to mathematical ability, research has also highlighted domain-general factors that play a role in the learning of mathematics, particularly that of working memory (Kuhn & Holling, 2014).

The domain-general approach conceives of dyscalculia as resulting from impairment or dysfunction in supporting cognitive systems, such as phonological skills, processing speed and working memory (Mazzocco et al., 2011). The role of working memory as a domain-general cognitive factor that plays a role in the learning of mathematics was emphasized in this study. This is due to a large amount of research on this role during recent years, as well as the development of reliable assessments of working memory (Alloway, Gathercole, Kirkwood, & Elliott, 2008).

2.4.1 The Baddeley Model of Working Memory

In 1974, Baddelely and Hitch proposed the concept of working memory and provided a framework for understanding the role of temporary information storage in the performance of a variety of cognitively complex tasks (Baddeley, 2000). It differed from earlier understandings of short term memory in that it proposed a multi-component system as opposed to a unitary store, and it emphasised that the system plays a role in carrying out cognitively complex tasks, and does not exist only for memory *per se* (Baddeley, 2000). This concept has been widely used in many areas of cognitive science and cognitive psychology (Baddeley, 2000).

The framework consisted of a three component model, namely the phonological loop, the visual spatial sketchpad and the central executive. However, Baddeley found that several phenomena did not fit comfortably with the original model, and as a result, the model was reformulated in 2000, to include a fourth component, namely the episodic buffer (Baddeley, 2000).

The revised model is represented in Figure 2.2 below:

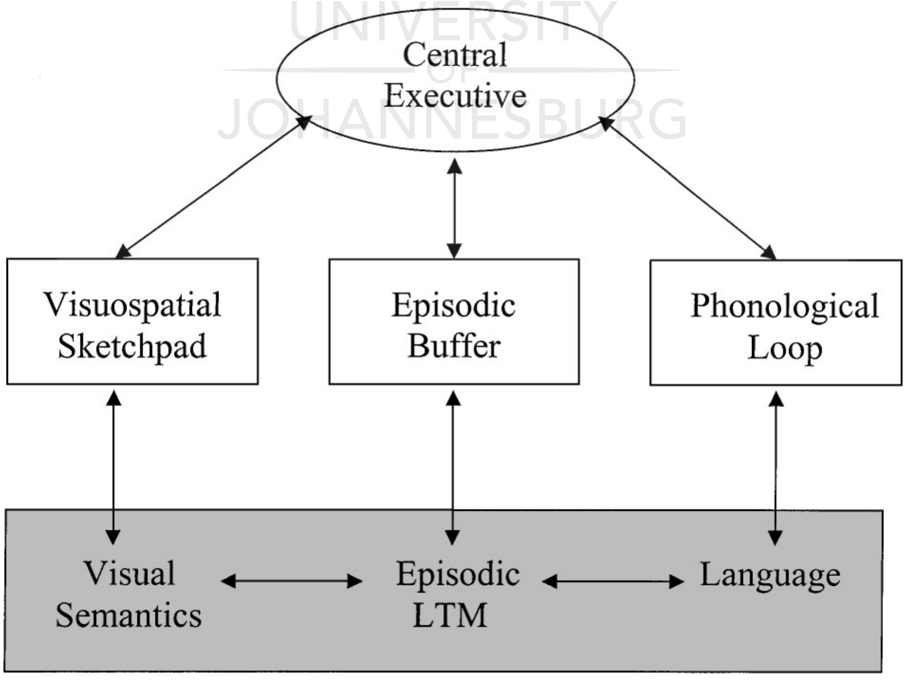

***Figure 2.2*: The multicomponent model of working memory.**

(Baddeley, 2000).

This model conceptualized working memory as consisting of four components. The central executive is a domain-general component that controls attention and processing (Alloway et al., 2008). The temporary storage of information is mediated by two domain-specific stores – the phonological loop providing for temporary storage of verbal material, and the visuospatial sketchpad allowing for the maintenance and manipulation of visual and spatial representations (Alloway et al., 2008). These two components are also known as the “slave systems” (Baddeley, 2000). The episodic buffer represents what binds different kinds of information from the slave systems into integrated chunks (Alloway et al., 2008) or coherent episodes (Baddeley, 2000).

2.4.2 The Role of Working Memory in Learning Mathematics

According to the Baddelely model as described above, working memory refers to a mental workplace in which information can be temporarily stored and manipulated in order to support other complex cognitive activities (Gathercole & Pickering, 2000). Gathercole and Pickering (2000), have found that working memory skills are closely related to academic progress in early primary school and therefore the assessment of working memory could offer a useful method of screening children for risk of poor academic progress.

Several studies (Geary et al., 2009; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; McLean & Hitch, 1999) have found that children with significantly lower achievement in mathematics compared to their peers, scored below average on measures that assess one or several working memory systems. These studies have suggested that learning difficulties in arithmetic might reflect links between working memory and long term memory. A study by McLean and Hitch (1999), found that children with specific difficulties in arithmetic were impaired relative to age-matched controls on a number of tasks tapping different aspects of working memory. These results from this study speak to one particular suggestion as to how this could happen; namely that children could fail to develop long-term memory for basic number facts or operational procedures because information in their working memory decays too quickly for relevant associations to be formed (McLean & Hitch, 1999).

A helpful way of understanding the role of visuospatial memory in the learning of mathematics is by conceptualizing visuospatial memory as a mental blackboard that supports number representation such as place value. Learners with poor visuospatial memory therefore have less room in their mental blackboard to retain and process relevant numerical information (Alloway, 2006).

Further links between visuospatial memory and the learning of mathematics have been found by Klingberg (2013). He found that the same area of the brain that Dehaene (2011) found responsible for numerical representation, namely the intraparietal sulcus, is also activated when someone retains visuospatial information in working memory. One of the functions of visuospatial working memory is the creation of a mnemonic map, which remembers where a person has seen something – in other words, the position of an object. Klingberg (2013) argues that this same system that keeps information on different positions in working memory, also retains the image of a mental number line. The mental number line represents numbers as points or positions, with low numbers to the left and high numbers to the right (Klingberg, 2013). Klingberg argues that it is likely that the mneomic map of the mental number line would reside in the parietal cortex.

2.4.3 Assessing Working Memory

Susan Gathercole and Tracy Packiam Alloway developed the Automated Working Memory Assessment (AWMA) in 2007. The development of this assessment was based on the revised Baddeley conceptualization of working memory (Alloway et al., 2008). The assessment provides for three measures each of both the verbal and visuospatial aspects of short-term and working memory, which equals a total of 12 subtests (Alloway et al., 2008). Working memory is assessed with tasks that involve the simultaneous storage and processing of information, whereas short term memory is assessed with tasks that involve only storage of information.

Research highlighting the close association between working memory and a wide range of academic abilities, created the need for a reliable measure of working memory, since screening and assessment is the first and very important step in being able to provide appropriate intervention. The AWMA was developed to serve this need (Alloway, 2007)

In the assessment, the verbal short-term memory tests correspond to the phonological loop on the Baddeley model, and the visuospatial short-term memory tests correspond to the visuospatial sketchpad. Both the working memory tests correspond with the central executive (Alloway, 2007). The assessment is carried out using a computer, and test scores are calculated automatically by the programme.

2.5 CONCLUSION

This literature review has provided an exploration of the recent developments in the field of numerical and mathematical cognition, which have been informed by research in cognitive neuroscience and cognitive developmental psychology (Butterworth, Varma & Laurillard, 2011). Various ways of understanding low achievement in mathematics and the traditional approaches to assessment and support of these learners have been unpacked. Readers were introduced to the two main conceptual views of understanding mathematical disability on a cognitive level, namely the domain-specific and domain-general approaches. Under the domain-specific view/approach, literature on recent theoretical developments in the field of numerical cognition was reviewed. Under the domain-general approach, literature on the role played by working memory in the learning of mathematics was reviewed. The reader was then introduced to ways in which these theoretical developments have begun to inform practice.

With the above in mind, I will now in the next chapter, show how I plan to embark on this study in order to investigate whether the current ideas around phenomenon of dyscalculia could be utilized to give insight into the low achievement in mathematics of the participants.

CHAPTER 3

RESEARCH DESIGN AND METHODS: MEASURING COGNITIVE FACTORS THAT COULD UNDERLIE LOW ACHIEVEMENT IN MATHEMATICS

3.1 INTRODUCTION

Butterworth, Varma, & Laurillard (2011) emphasize that when it comes to children with low numeracy, neuroscience research offers a clear cognitive target for assessment and intervention that is largely independent of a learner's educational circumstances. The assessment of individual cognitive capacities should allow for the differentiation of dyscalculia from other causes of low numeracy (Butterworth et al., 2011). The development of such assessment instruments based on emerging numerical cognition theory, as well as theory on human cognition in general, is clear evidence of Stanislas Dehaene's claim that the application of neuroscience to education is no longer "a bridge too far" (2011, p. 278).

The literature review in the previous chapter has outlined theoretical developments in the field of mathematical cognition, and how these developments have led to the development of these assessment instruments, especially those measurements which could be utilised to diagnose dyscalculic learners. This chapter will outline the research design and methods that have been employed to direct the empirical work of this study and aid in the analysis of the results. The study aimed to assess nine grade 8 learners who have demonstrated low achievement in mathematics during the first term of their grade 8 year.

These learners will be assessed with the instruments that have been briefly described in Chapter 2. This study hypothesized, based on the theory explored in the previous chapter, that impairment in one or more cognitive systems underlies the low achievement in these learners and may be indicative of dyscalculia. The literature suggests that the identification of such an impairment, or lack thereof, will be useful information for an educational psychologist in supporting the affected learner (Gillum, 2012)

Low achievement in mathematics that is disproportionate to a learner's achievement in other learning areas is a real-life phenomenon that has a concrete manifestation (Yin, 2014). Mathematical cognition research has identified particular cognitive variables related to this phenomenon – that of cognitive systems within an individual who manifests this real life phenomenon in the performance in the subject of mathematics. This study aimed to assess these identified cognitive systems using assessment instruments developed for these particular purposes. This study, therefore, focused on the information gained on each assessment instrument used, and each learner constituted an individual case (Yin, 2014), which in turn will hopefully give us information on cognitive variables related to mathematics performance across the different cases.

3.2 RESEARCH DESIGN – A MULTIPLE-CASE DESIGN WITH EMBEDDED UNITS OF ANALYSIS

3.2.1 Case Study Methodology

Stake (2005), suggests that case studies draw attention specifically to what can be learned about the single case. Therefore, the epistemological question that underpins the case study research method is "what can be learned about the single case?" (Stake, 2005). Stake emphasizes that case studies should be designed to "optimize understanding of the case rather to generalize beyond it" (2005, p. 45).

This epistemological question is appropriate to this study, because the same question would underpin the practice of psychological assessment, which can be defined as an activity aimed to further the process of accumulating information and forming a judgment about various characteristics and domains of functioning of an individual (Sattler & Hoge, 2006). Since educational psychologists are tasked with assessing learners who present with difficulties in their learning in order to better understand what may be contributing to their difficulties (Foxcroft & Roodt, 2013), I argue that a purely qualitative study aiming to just describe the make-up of a sample of learners would not yield appropriate information on the phenomenon under scrutiny in this study (that of low achievement); nor would it allow for enough individual attention or usable information to each case. The individual cases that would be analysed could, in the end, also contribute to a bigger depiction of the phenomenon of low performance in mathematics, which could in turn contribute to an integrated multi-case study.

3.2.2 Multiple-case study design

Following a case study by Kos (as cited in Creswell, 2005) in which four learners with reading disabilities were studied, each learner in this study will form a bounded system or case (Creswell, 2005), thus an assessment of each case will be undertaken. The embedded unit of analysis in this study is the cognitive system that is being assessed with each instrument.

Yin (2014) has proposed four possible types of designs for case studies (see figure 3.1 below). The design in the bottom right quadrant is a multiple-case design with embedded units of analysis (Yin, 2014). This design has been selected as the most suitable for this study.

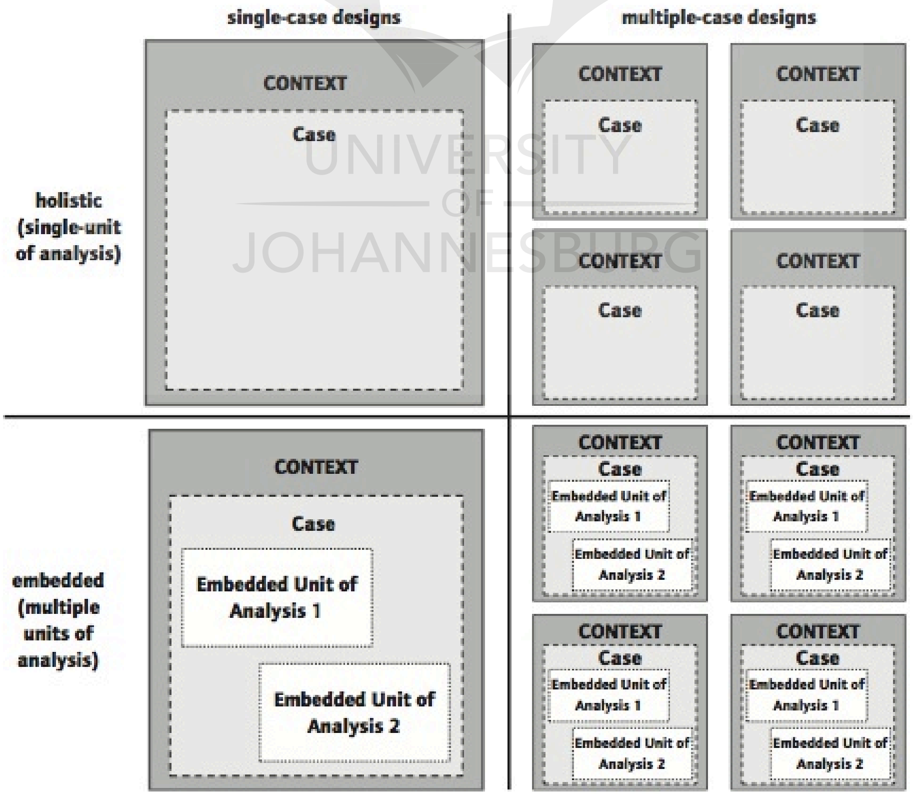

***Figure 3.1*. Basic Types of Design for Case Studies.** (Yin, 2014).

Figure 3.2 is a diagrammatic representation of the application of the multiple case study design of this study. It represents how the multiple-case design with embedded units of analysis will apply to this study. The context is a low performing mathematics class in Grade 8. The case is each learner, and the embedded units of analysis are the cognitive systems being assessed.

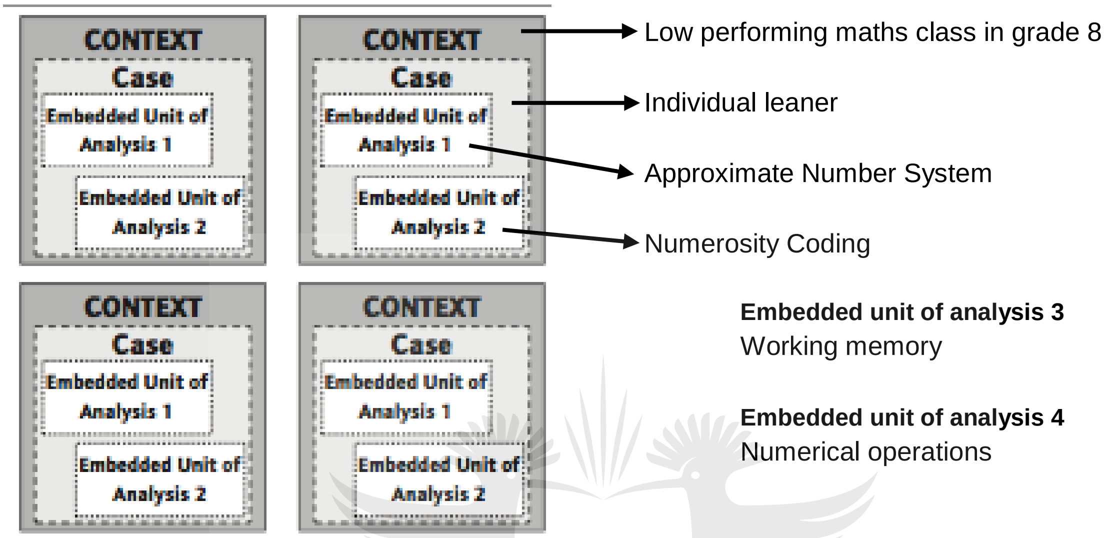

***Figure 3.2*. Application of the multiple-case design with embedded units of analysis to this study.** (Yin, 2014).

Multiple-case designs are also known as collective case studies (Stake, 2005) and allow for a number of cases to be studied jointly in order to investigate a phenomenon. It may or may not be known in advance that the individual cases manifest some common characteristic (Stake, 2005). In this study it is known that a common characteristic among the cases is low achievement in mathematics, and the learners have been chosen because it is believed that understanding their cognitive systems that play a role in the performance of mathematics could lead to a better understanding or a better theory regarding a larger collection of cases (Stake, 2005).

3.3 CASE SELECTION

The school where the research will take place practices ability grouping in the subject of mathematics for all year groups. Ability grouping, also known as a tracking, is the practice of grouping learners with a certain ability range into different classes

(Wadesango & Bayaga, 2013). The practice is a controversial one, and a considerable amount of research is critical of it (Linchevski & Kutscher, 1998). Discussion of the advantages and disadvantages of this practice is beyond of the scope of this study. However, this practice has been mentioned because it has informed the selection of the cases for the study.

Grade 8 learners at this school have been grouped into eight mathematics classes according to their achievement (marks) in the subject. The overall subject marks for the first term in the highest of these classes range from 94% to 61%. The marks in the lowest achieving class range from 61% to 19%. It is possible for learners to move classes based on improvement or decline in marks. It must be emphasized that all learners in the lowest achieving class were invited to participate, not just those with the lowest marks. Therefore, the group of participants presented with a mixed bag of marks, ranging from 19% to 57%. Regardless of the term mark, all participants were assessed in the same manner.

3.4 DATA COLLECTION METHODS

3.4.1. The use of norm referenced assessment instruments

All four assessment instrument used in this study are norm referenced. A norm referenced assessment aims to aims to yield information on the standing or ranking of the individual being assessed relative to a comparison group (Cohen & Swerdlik, 2002). Generally speaking, the closer the match between the demographics and context of the individuals who have made up the norm group to the demographics and context of the individual being assessed, the more appropriate and reliable the test may be for a given purpose (Cohen & Swerdlik, 2002).

The obvious challenge here for South African psychologists is twofold; Firstly, tests developed in South Africa with local norms are almost non-existent. Secondly, the tests which are freely available and that have been developed overseas do not have local norms. In other words, the norm group (also known as a normative sample) is made of up individuals local to the country where the test was developed. This is the case for all four assessment instruments being utilized in this study. In an investigation into psychological assessment in post-apartheid South Africa, Cockcroft & Sumaya (2014) emphasize that tests need to be developed, adapted and standardized appropriately for the local context. Until such development and standardization occurs, which is very costly and thus not often attempted, we have no choice but to use the assessments available.

The Professional Board for Psychology has not expressly forbidden the use of psychological tests that have been developed in other countries (The Professional Board for Psychology, 2009). The reason being, that without these tests, South African educational psychologists will have almost no tests to use in assessment batteries. Special care needs be taken when interpreting the results of such tests (The Professional Board for Psychology, 2009). The fact that the assessment instruments utilized in this study are normed against a foreign normative sample is acknowledged, and therefore caution needs to be exercised when interpreting the results. However, since the participants in this study come from a private, English speaking school with a high academic standard (as mentioned in Chapter 1, paragraph 1.6.2), it would not be unreasonable to assume that the educational background of the participants has some similarity to the educational background of the normative sample group of the instruments used.

3.4.2 The Dyscalculia Screener

3.4.2 (i) Description of the measurement

The Dyscalculia Screener provides a quick and reliable way of identifying dyscalculia by using item-timed tasks that test an individual's capacity for numerosity (Butterworth, 2003). It is made up of four computerized, item-timed tests. Since it is the speed of response to numerical questions that is measured, the test has to take into account whether a person responds slowly, or is simply, by nature, a slow responder. The test does this by including a test of simple reaction time, and the computer programme will then adjust the learner's reaction time as a function of the reaction time test (Butterworth, 2003).

The normative sample consisted of 549 school children of various ages from schools across England. This researcher acknowledges that the normative sample is therefore not local. However, the school from which the participants in this study come from would be similar in some respects to schools in England, as the school is a private, English first language school.

The reason that speed of response is cardinal in this test is because the tasks are very easy, and all learners should get most of the answers correct. An item from the Dot Enumeration Task is shown in Figure 3.3:

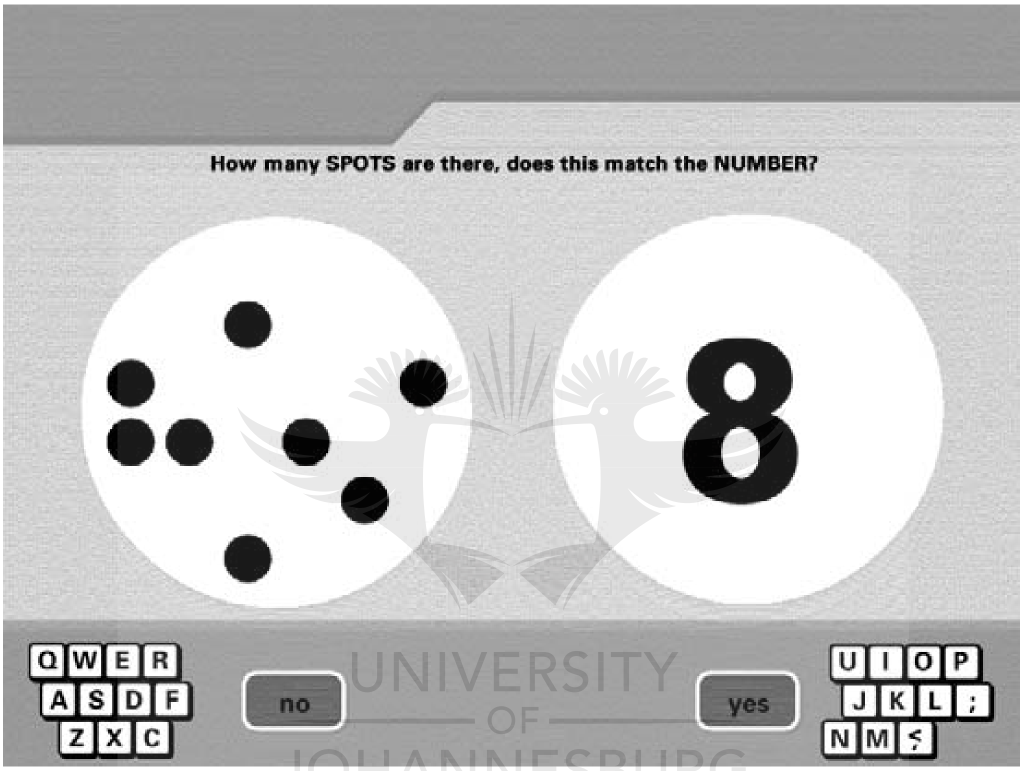

***Figure 3.3*. An item from the Dot Enumeration Task of the Dyscalculia Screener.** (Butterworth, 2003).

The learner will need to say whether the number of dots is equal to the symbolic number opposite it, by pressing a “no” or a “yes” key. Most people will get the correct answer, therefore accuracy is not being measured, but rather reaction time. It is hypothesized that the reaction time will identify learners who count slowly as a result of a deficient capacity for identifying numerosities (Butterworth, 2003).

The test also hypothesizes that dyscalculic learners would have failed to form efficient connections between numerals and their meanings, due to deficits in their capacity to recognize numerosities (Butterworth, 2003). Therefore, a test of number discrimination is included, which requires the leaner to select the larger of two numbers (Butterworth, 2003). This task also exploits the fact that the physical size of numerals, i.e how tall and wide they are, can be varied. Some number comparisons make the numerically smaller number physically bigger, which results in slowed time to select the larger of the numbers (Butterworth, 2003). In other words, incongruent physical size interferes with numerical comparison (Girelli, Lucangeli, & Butterworth, 2000). These effects are known as 'Stroop' effects, named after the scientist who described how task irrelevant features can influence task performance. In the case of numerical Stroop, the task irrelevant feature is the physical size of the numeral. It is hypothesized that the acquisition and refinement of numerical knowledge will determine the extent of task irrelevant interference (Girelli et al., 2000). An example from the Numerical Stroop task is show in Figure 3.4, where the bigger of the two number, in this case five, is smaller in size.

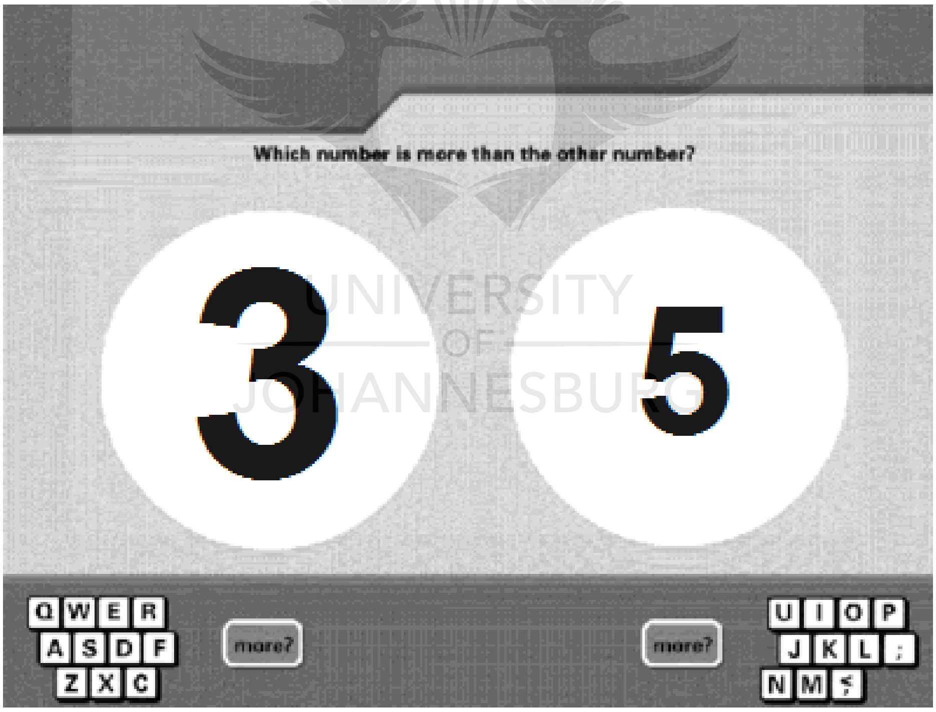

***Figure 3.4.* An item from the Numerical Stroop Task of the Dyscalculia Screener.** (Butterworth, 2003).

3.4.2 (ii) Use of the results from the Dyscalculia Screener in this study

The results of the two capacity tests of this instrument will be utilized and analysed as empirical data in Chapter 4, because these tests are hypothesized to directly tap a numerical cognitive system (Butterworth et al., 2011; Butterworth, 2003). Below average performance in these two tests is said to be "typical of someone with dyscalculic tendencies" (Dyscalculia Screener Teacher/Practitioner Report). Average to above average performance is seen as age appropriate and indicative of the fact that the individual is "unlikely to have dyscalculia" (Dyscalculia Screener Teacher/Practitioner Report). Each participant's results on these two capacity tests will be reported on and analysed both in comparison to other participants' results and in comparison to the individual participant's results on the other measures.

3.4.3 The Number Discrimination Task

3.4.3 (i) Description of the measurement

This task which had been developed by Mazzocco et al., assesses the individual's Approximate Number Sense (ANS) which has been discussed at length in Chapter 2 Paragraph 2.3.2.1. The measurement is freely available for download from the Panamath.org website (www.panamath.org). The assessment is computer based, and participants are shown spatially intermixed arrays of blue and yellow dots on the computer screen for approximately 200 milliseconds. Participants are required to indicate which array was more numerous by pressing a colour-coded key (Mazzocco et al., 2011). The main aim of the test is to measure the acuity of the ANS.

Figure 3.5 is an example of two items from the Number Discrimination Task. The arrays of dots within the grey squares are momentarily flashed on the computer screen. (A line has been included on this diagram to indicate which dots are yellow and which are blue). The first example is harder to get right because the ratio between the values is smaller, whereas the second example is easier because the ratio is bigger. The rate of the increase in errors an individual will make as the ratio between the two arrays of dots increase, is a function of the amount of noise in the ANS representations.

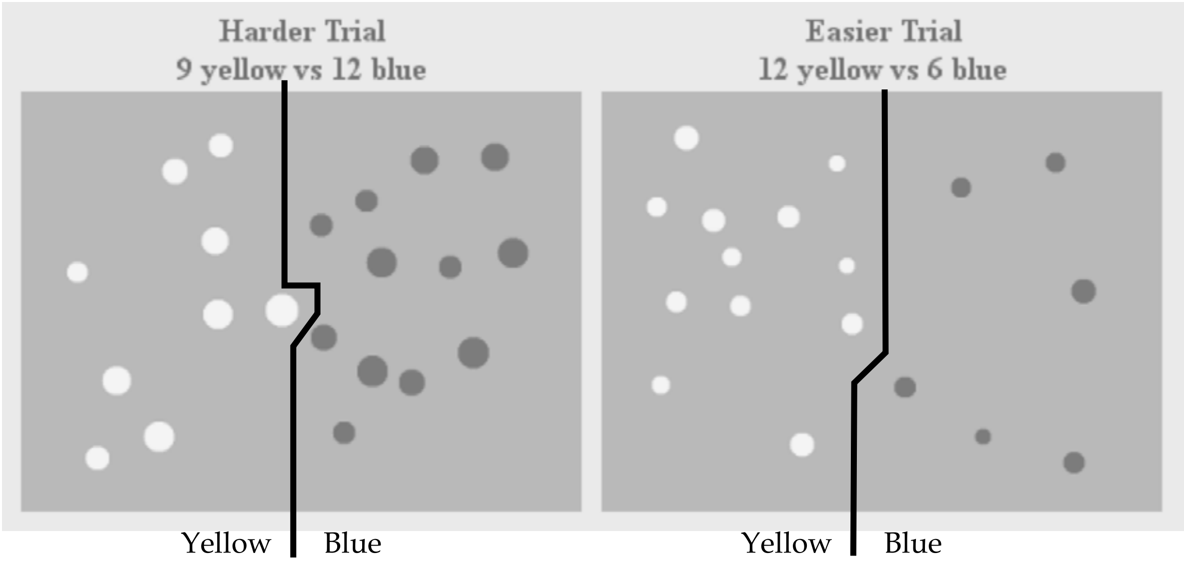

***Figure 3.5*. Two items from the Number Discrimination Task**

(The lines separating the different colour dots, together with the labels 'Yellow' and 'Blue' are not present in the actual task)

3.4.3 (ii) Use of the results from the Number Discrimination task in this study

The Panamath task automatically generates an electronic report at the end of the measurement (see Appendix E for an example of such a report), which indicates the individual's Weber fraction in relation to the 90th and 10th percentile of the age referenced norm group[2]. The Weber fraction will be utilised and analysed as empirical data in Chapter 4. Each participant's Weber fraction will be reported on and analysed both in comparison to other participants' Weber fractions and in comparison to the individual participant's results on the other measures.

3.4.4 The Automated Working Memory Assessment (AWMA)

3.4.4 (i) Description of the measurement

The Automated Working Memory Assessment is a computer based measure of working memory. It is intended to serve as a practical and convenient means for educational professionals to screen individuals for problems in working memory (Alloway, 2007). The instrument has been standardized for use within the age range of four to 22 years. The normative sample used to standardize the age range of 12 –

[2] Standardization data was not available due to technical problems with the Panamath website

18 years included 351 children from eight secondary schools in England (Alloway, 2007). Again, cognisance has been made of the fact that these norms are not local.

The tests used in this measure have been established to provide reliable and valid assessments of verbal and visuospatial short-term and working memory (Alloway, 2007). Verbal and visuospatial short-term memory are measured using tasks that involve storage with minimal processing, whereas verbal and visuospatial working memory are measured using tasks involving both the storage and processing of information (Alloway, 2007). Figure 3.6 is a diagrammatic representation of the relationship between the different subtests of the AWMA and the components of the working memory model.

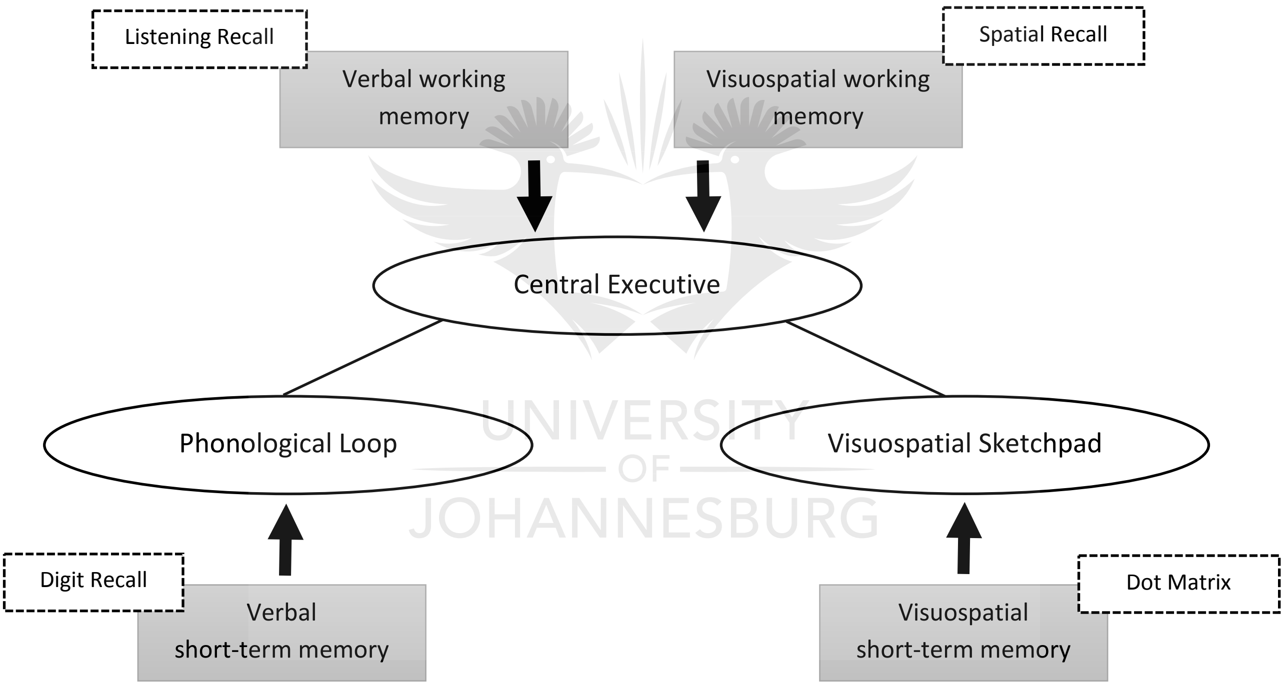

***Figure 3.6*. A diagrammatic account of working memory and its relationship to the AWMA.** (Alloway, 2007).

(The tests used to tap each component are named in the dashed line boxes.)

The assessment consists of multiple tasks that make use of a variety of stimuli to measure the four different memory components, namely verbal short-term memory, verbal working memory, visuospatial short-term memory and visuospatial working memory, as shown in figure 3.6 (Alloway, 2007). The Long Form of the AWMA consists of 12 tests, three for each of the four components. Due to the amount of time taken to administer the Long Form, the Short Form of the AWMA was utilized in this study. The Short Form consists of four tests, one for each of the memory components. It is recommended as a screening procedure to screen individuals who are suspected to have memory difficulties (Alloway, 2007). The one limitation of using the Short Form is that only one variety of stimuli is used per memory component, which means that a low score could be the result of a difficultly in processing regarding that particular kind of stimuli, and not necessarily a memory problem (Alloway, 2007). In cases of an atypical working memory profile on the Short Form, a recommendation will be made that the participant be assessed with the Long Form.

3.4.4 (ii) Use of the results from the AWMA in this study

The AWMA automatically generates a report at the end of the assessment. The raw scores are automatically converted to standardized scores for each of the four components of memory. The four standardized scores are not combined into a composite score, as the differences between the four components provide more insight into an individual's memory functioning than a combined score (Alloway, 2007). A working memory profile is therefore created, and these profiles will be utilized and analysed as empirical data in Chapter 4. Each participant's profile will be reported on and analysed both in comparison to other participants' profiles and in comparison to the individual participant's results on the other measures.

3.4.5 The Numerical Operations Subtest of the Wechsler Individual Achievement Test, Second Edition (WIAT-II)

3.4.5 (i) Description of the measurement

The Wechsler Individual Achievement Test - Second UK Edition (WIAT-II) is a standardized achievement test which aims to measure accomplishment or achievement in various academic areas (Johnson & Christensen, 2014). It is an individually administered test for assessing the achievement of learners between the ages of four and 16 years 11 months (Pearson Education, 2005).

The WIAT-II contains nine subtests, each aimed to measure various academic domains, one of which is mathematics (Pearson Education, 2005). The mathematics domain is measured by two subtests, namely Numerical Operations and Mathematical

Reasoning. The Numerical Operations subtest is a measure of mathematical calculation skills including the solving of written calculation problems and simple equations involving all basic operations (addition, subtraction, multiplication, and division) (Pearson Education, 2005). It is therefore an assessment of basic procedural knowledge or knowledge of mathematical operations (Henning, 2013).

3.4.5 (ii) Use of the results from the Numerical Operations subtest in this study

The Numerical Operations subtest was used in this study for the purposes of comparing results on a test of mathematical operations and procedures with results on tests of mathematical cognition.

3.4.6 School Mark in Mathematics

The participants' overall mark in the subject of mathematics in term 1 was utilized as an indication of their general performance in mathematics. These marks will be presented as the starting point of the data analysis in Chapter 4.

3.5 RESEARCH PROCEDURE

After obtaining ethical clearance (Appendix A) and consent to conduct this research from the headmaster of the school, I will invite learners in the lowest performing mathematics class in Grade 8 to take part in this study. I will address the learners as a group. Those who are interested will be given information letters to take home to their parents or guardians. As the learners are all minors, consent from their parents or guardians needed to be obtained. The absence of any detriment to the learner if he decided not to participate will be clearly communicated to learners and their parents or guardians.

Each of the nine participants were assessed individually by the researcher after school in the afternoon. The assessments will took place in a therapy room that is regularly used for psychological assessment. The administration procedures for each instrument was discussed in section 3.4 of this chapter. The instruments were all administered in the same order, and the same instructions were given to all participants. Although each participant was assessed on a different day, they were all assessed at roughly the same time of day, over a period of roughly three weeks.

The assessments were then scored according to instructions, and the process of data analysis began. Participants, together with their parents, were invited for individual feedback.

3.6 DATA ANALYSIS

Regarding the use of quantitative data in case study research, Gillham (2010) distinguishes between descriptive and inferential presentation of data. Descriptive refers to ways of summarizing and presenting numerical data, while inferential refers to techniques which allow one to draw inferences such as the extent of correlations and significance of differences between groups or cases (Gillham, 2010). The three phases of data analysis in this study were descriptive, in that results will be presented, but at the same time inferential, as correlations between participants' results were looked for.

The first phase of the data analysis will involve presenting the results of all participants on each assessment instrument. I will showcase the results from each assessment instrument across all participants comparatively – thus showing the results of all the participants in one particular assessment instrument. This will allow for correlations between instruments to be looked at and duly discussed.

The second phase of data analysis will present the results of each participant in an integrated fashion and the unique profile of each participant will be discussed. For instance, for Participant 1, I will show his mathematics scholastic result (term mark) and then his results on each one of the four assessment instruments. In this way I can discuss each participant's results as an integrated case, from which I can then make possible conclusions and recommendations for the participant and his parents on how his mathematical development could be supported.

Lastly, I will categorise and group cases which show similarity in their results, in order to discuss a specific phenomenon which could give us insight into the possible reasons for low achievement in mathematics. From the analysis of results and reports of the findings, categories of findings will be highlighted and subsequent themes will be extracted. These themes will then be discussed in terms of possible insights into the learners' low performance in mathematics.

3.7 ETHICAL CONSIDERATIONS

Seven ethical considerations regarding case study research have been described by (deRoche & deRoche, 2010), which overlap with the ethical guidelines laid down by the Health Professions Council of South Africa (2006). Five of the seven have been identified as necessary to ensure an ethical research process in this study. They are:

1. Do not harm participants
2. Maintain their privacy
3. Bring them available benefit
4. Inform them about the research
5. Involve them only voluntarily

A brief discussion of how each of these ethical considerations will be applied to this study will follow.

1. Do not harm participants. As mentioned in Chapter 1, paragraph 1.8, care will be taken to ensure that participants are not harmed in this study. Particular attention will be paid to ensuring that participants do not feel demeaned by terms such as "low achievers", and that results of the assessments are carefully and sensitively explained to the participants' and their parents.

2. Maintain their privacy. This ethical consideration has to do with the maintenance of confidentiality of all information pertaining to the participants, not only results obtained on the assessments.

3. Bring them available benefit. The participants will be entitled to the results of the assessments together with a consultation regarding their meaning and implication. Where necessary, referrals will be made for further investigation and support.

4. Inform them about the research. This ethical consideration relates to consideration number 5 and has to do with informed consent. The participants were fully informed about the study as a group, and their parents or guardians will be given detailed written information about this study which was attached to a consent form. See Appendix D for an example of this form. They will then be free to accept, decline or ignore the invitation. The researcher will keep the signed informed consent form, while the participants and their parents will keep the information sheet.

5. Involve them only voluntarily. As mentioned in paragraph 3.4, the absence of any detriment to the learner if he decided not to participate was clearly communicated to learners and their parents or guardians. The participants will also be free to withdraw from the study without any consequence. As the researcher, I will go to great lengths to accommodate the participants so that their other commitments, such as sport or homework time, will not be compromised.

In addition to this, it must be noted that ethical clearance was obtained from the Research Ethics Committee of the Faculty of Education at the University of Johannesburg (Appendix A).

3.8 CONCLUSION

This chapter has outlined the research design of this study, together with a detailed discussion of case selection, data collection, research procedure, data analysis and ethical considerations. Chapter 4 will present and analyse the data that has been collected as per the discussion in this chapter.

CHAPTER 4

RESEARCH RESULTS AND DISCUSSION OF FINDINGS: SHEDDING LIGHT ON LOW ACHIEVEMENT

4.1 CLASSROOMS AS AUTHENTIC LABORATORIES

The strides that have been made in understanding how arithmetic is implemented in the brain led to Stanislas Dehaene concluding his book, The Number Sense, with the statement: “the classroom should be our next laboratory” (2011, p. 278). The ability to identify numerical processing and working memory deficits has paved the way for researchers to begin to design and implement innovative educational programmes, and neuroscience researchers have the tools in hand to study their impact and effectiveness (Dehaene, 2011). Understanding the links between numerical cognition and formal mathematics should enable better support for individuals with mathematical learning disabilities (Feigenson, Libertus, & Halberda, 2013). This study focuses on the first step on a road that will hopefully lead to effective, cognition based learning support – that of assessment and identification.

This chapter deals with the presentation and analysis of the data of this study. I will logically present and analyse the results of the assessments that were conducted on the participants. The previous chapter outlined mechanics of the assessment instruments and what specifically each assessment aims to measure. This was carefully outlined in Chapter 3 and repetition will make it superfluous. This chapter will analyse and report the data in terms of assessment results and interpretation. I will begin this chapter by presenting the results of each assessment instrument – indicating equivalences and differences – with the purpose of gaining a better understanding of how these instruments could shed light on the participants’ low achievement. Following onto that, I will categorise each of the participant’s assessment results (referred to as individual cases) into four categories showing comparative profiles. Discussion of how the results offer insight into low achievement in mathematics will take place throughout these sections.

I will then discuss the identification of themes that have emerged from the insights gained from the analysis of each instrument and each category, and how they may provide possible usable findings (Butterworth, 2003). These themes will pave the way for further discussion in Chapter 5 of their significance in answer to the research question.

4.2 MULTIPLE CASE RESULTS: AN ANALYSIS OF THE FINDINGS OF EACH ASSESSMENT

4.2.1 Starting point: Term mark in mathematics

Although not assessed by means of a separate assessment instrument, the mathematics term marks of the participants were considered as an indication of their mathematics achievement at the time of the study. Figure 4.1 shows the mathematics term marks for each participant.

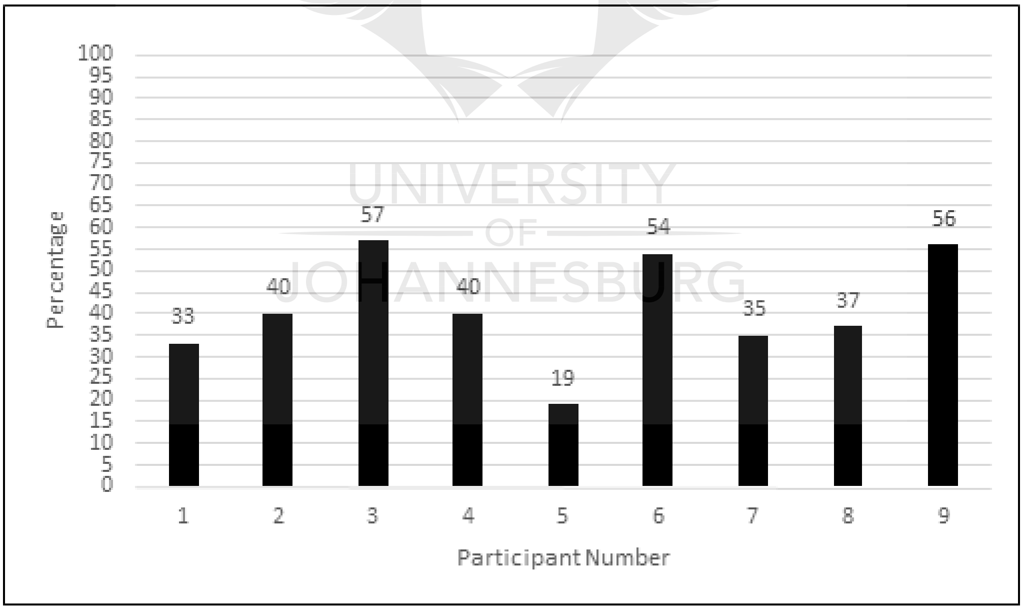

Figure 4.1. **Mathematics term marks for each participant**

Four out of the nine participants failed mathematics in the first term of their Grade 8 year, as they had obtained an overall term mark of less than 39%. The lowest mark is 19%, and the highest mark is 57%. It is clear that there is varied achievement within this group. This has proven useful in this study as correlations between the various term marks and results on the assessments can be made.

4.2.2 The Dyscalculia Screener

Butterworth claims that the dot enumeration subtest of the Dyscalculia Screener directly taps the cognitive system of numerosity coding - a deficit in which, according to Butterworth, is the cause of dyscalculia (Butterworth & Laurillard, 2010). The numerical stroop subtest measures whether or not efficient connections between numerals and their meanings have been formed, which dyscalculic learners often fail to form, due to deficits in their capacity to recognize numerosities (Butterworth, 2003).

The results of these two subtests of the Dyscalculia Screener (dot enumeration and numerical stroop) are presented as stanines. Stanine is short for “standard nines”, and divides the standard age score into nine broader bands. The stanine reflects how the learner performs in comparison with the standardization sample of the instrument, with nine being the highest score and one being the lowest (Butterworth, 2003).

The stanines of the dot enumeration and numerical stroop subtests of the nine participants are represented in Figure 4.2.

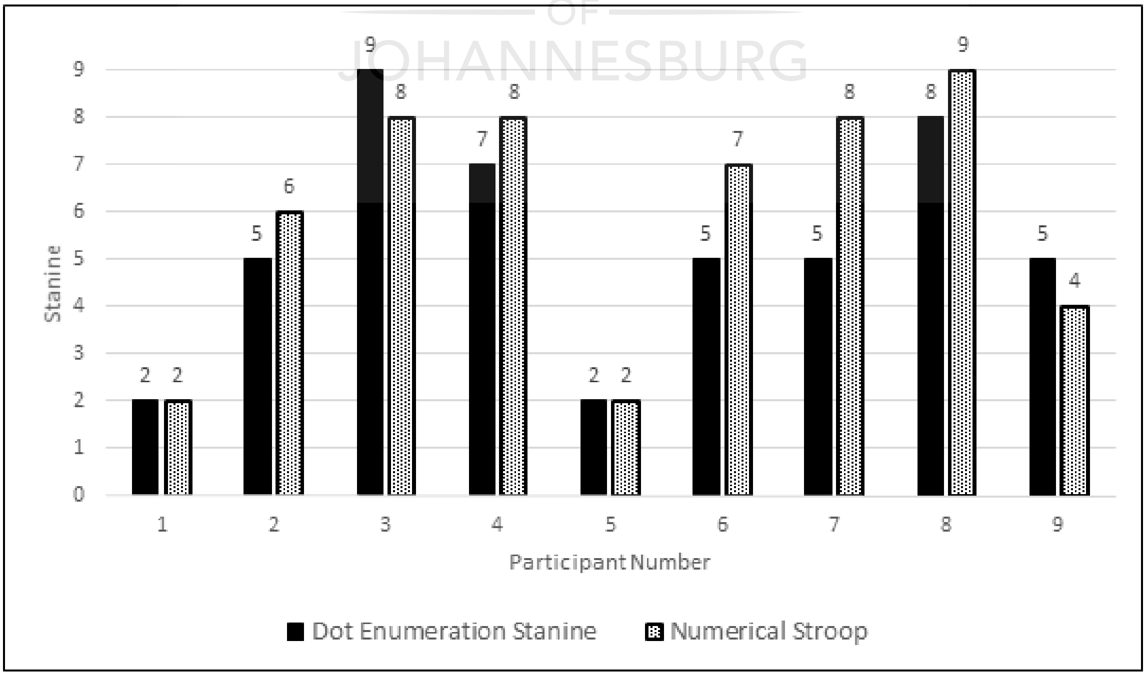

Figure 4.2. **Dyscalculia Screener results for each participant**

Most participants obtained similar scores for the dot enumeration and numerical stroop subtests. Five participants had a difference of one stanine between the two tests, with the biggest difference being three stanines. Two participants obtained equal scores.

A correlation seems to exist between the stanine score and term marks of participants 1 and 5. According to Butterworth, the low score of ‘2’ is indicative of dyscalculia (2003). These two participants are the lowest achieving in terms of their term marks, having obtained 33% and 19% respectively.

Although the participant with the highest dot enumeration stanine also has the highest term mark (57%), attention must also be drawn to participants 8 and 4. Although their term marks are low (40 and 37 % respectively), their dot enumeration stanines are well above average, indicating no evidence of dyscalculia. Therefore, according to Butterworth, these two learners are not achieving poorly in mathematics due to dyscalculia, whereas participants 1 and 5 likely are (2003).

4.2.3 Panamath: The Approximate Number system

As discussed in chapter 3 (Paragraph 3.6.3) the Panamath assessment measures an individual’s approximate number system by identifying the amount of ‘noise’ in this system, which is indexed as the Weber fraction. The greater the amount of ‘noise’, the higher the individual’s Weber fraction and the less accurate the system. The Weber fractions of the participants are indicated in the graph below (Figure 4.3). The graph also indicates the 90^{th} and 10^{th} percentile, or percentile rank, for the 14-year-old age group. The percentile rank is interpreted as a percentage of scores in a reference group that fall below a particular raw score.

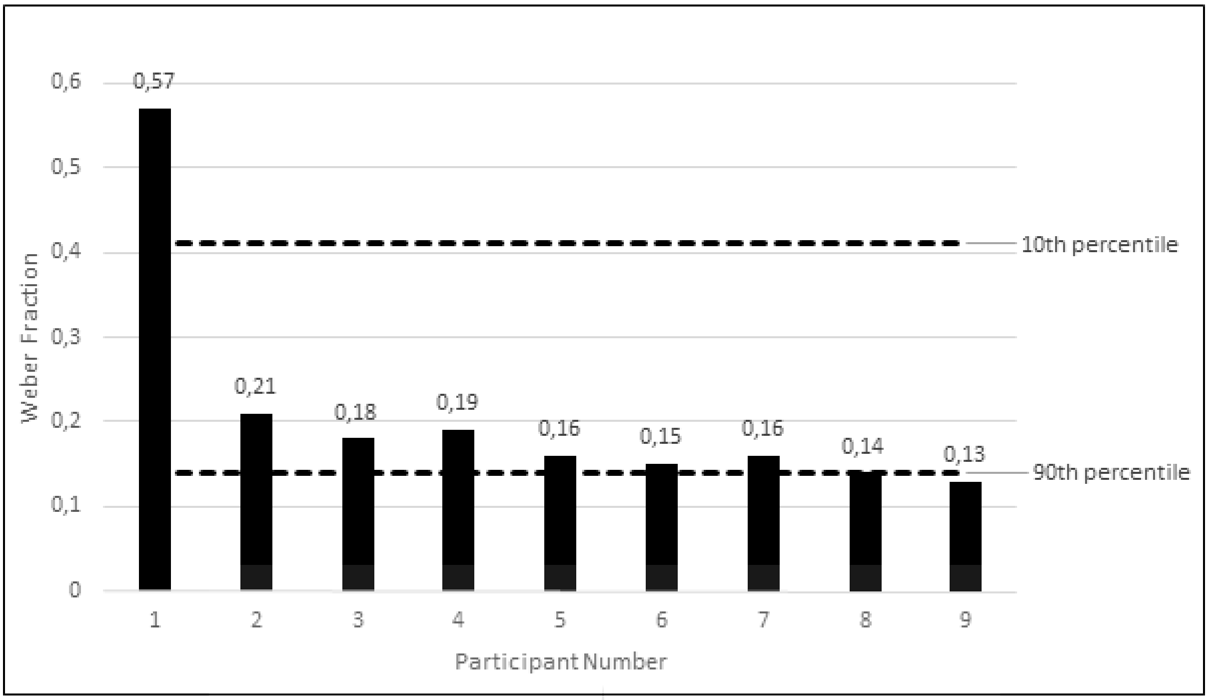

Figure 4.3. **Weber Fraction of each participant obtained on the Panamath assessment**

Only participant 1's Weber fraction is of significance. This participant's Weber fraction is 0.16 point higher than the 10^{th} percentile for his age group, which is 0.41, indicating significant impairment in the approximate number system (Mazzocco et al., 2011). It would seem that this system is not impaired in the other eight participants.

4.2.4 The Automated Working Memory Assessment (AWMA)

The AWMA reports on the results obtained for each of the four components of working memory that are measured, namely verbal short-term memory, verbal working memory, visuospatial short-term memory and visuospatial working memory. The instrument does not combine the scores into an overall composite score, because the differences between scores obtained for each component are significant in understanding a learner's working memory profile (Alloway, 2007). Therefore, an overall combined score would not be of value. Rather the raw scores obtained on the individual tests are converted into standardized scores. Standardized scores describe an individual's performance with respect to the performance of others in the same age band (Alloway, 2007).

The average range of this assessment is one standard deviation below (85 – 100) and one standard deviation above (100 – 115) the mean (Alloway, 2007). A below average score would fall between 70 and 85. A score below 70 would be considered extremely low. Above average scores would fall between 115 and 130. A score above 130 would thus be considered extremely high (Alloway, 2007). Figure 4.4 reflects the standard scores obtained by each participant on each of the four components of their working memory.

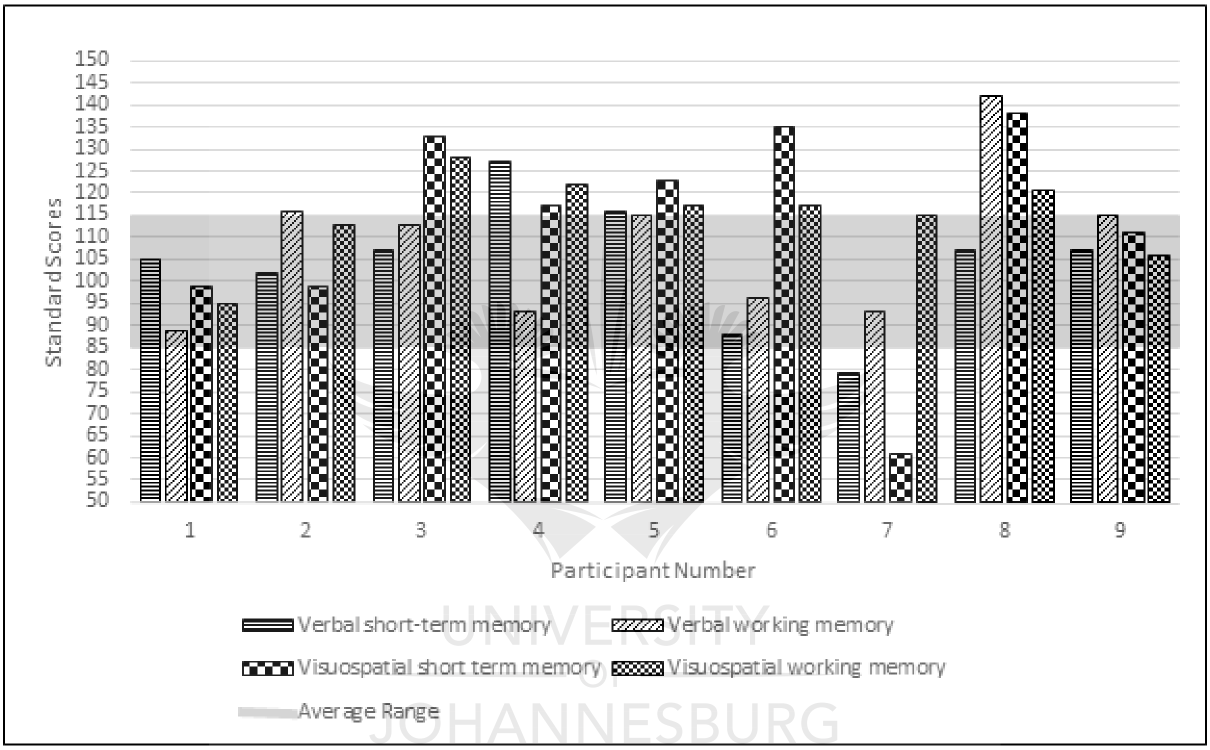

***Figure 4.4*. Working memory standard scores for each participant obtained on the Automated Working Memory Assessment**

All but one of the participants' scores for all four components of working memory fell within the average to above average range, indicating adequate to excellent working memory and therefore ruling out the possibility that working memory deficits could underlie low achievement in mathematics for these participants. Only one participant's working memory profile was of significance, that of participant 7. This participant scored below average and well below average for the verbal short-term and visuospatial short-term memory components respectively.

4.2.5 Numerical Operations Subtest of the Wechsler Individual Achievement Test, Second Edition (WIAT-II)

This test, as described in paragraph 3.6.5, is a test of basic operational knowledge in mathematics. In other words, it tests a learner's knowledge of the four operations in mathematics, namely addition, subtraction, multiplication and division (Pearson Education, 2005).

The test converts a participant's raw score into a standardized score according to norms. The mean is 100, which is the 50th percentile. The 90th percentile is 119 and the 10th percentile is 81. The participant's standardized scores, as well as an indication of the 90th, 50th and 10th percentiles are represented in Figure 4.5 below:

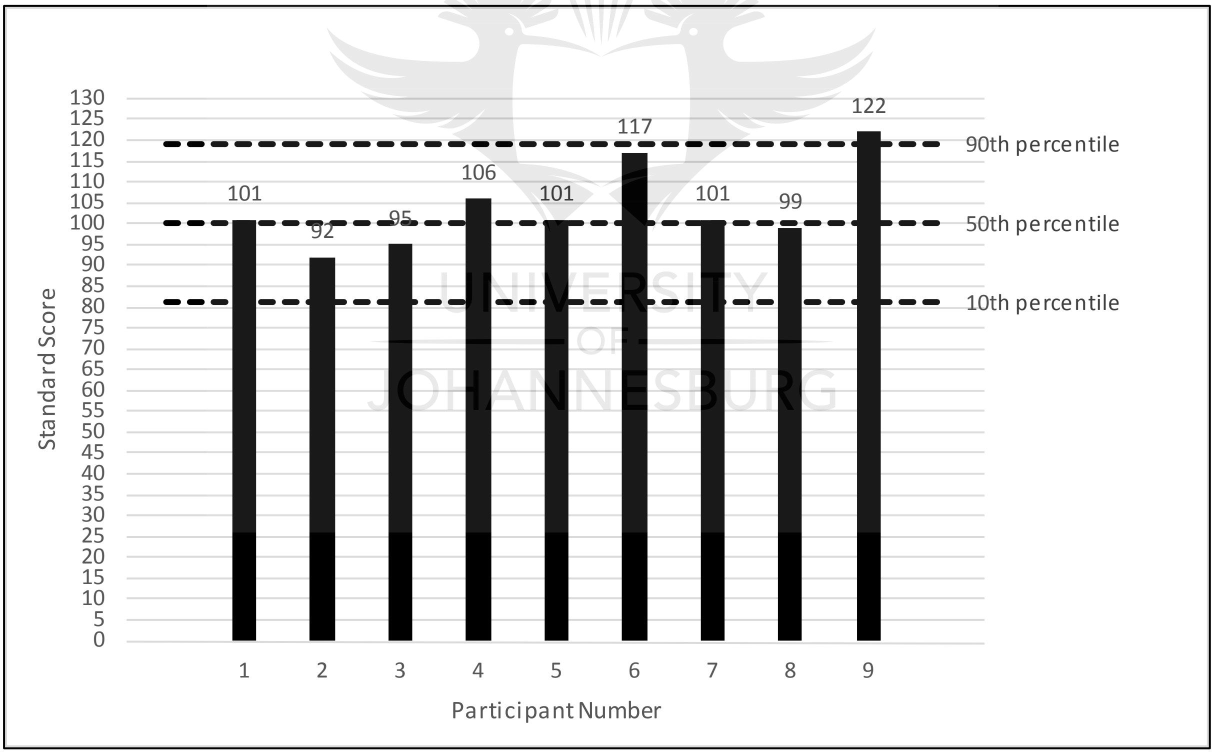

***Figure 4.5*. Standardized score obtained on the Numerical Operations subtest of the WIAT-II by participant**

Seven of the participants scored within the average range (between 90 and 110). One participant scored in the lower limits of the below average range, one in the high average range and one in the superior range. There exists little to no correlation between the scores obtained on this assessment with the participants' term marks or with their scores on measures of numerical cognition. The possible significance of these results will be discussed further in Paragraph 4.4 of this chapter.

4.3 GROUPING SIMILAR CASES: AN ANALYSIS AND DISCUSSION OF SIMILAR PROFILES

When reporting on the individual case in a multiple-case study, it should be indicated how and why a particular proposition was demonstrated or not demonstrated (Yin, 2014). The proposition of this study was that cognitive factors, as measured by the described instruments, could underlie a learner's low achievement in mathematics. The reporting on individual cases will describe when and how this proposition was either demonstrated or not demonstrated. This was my initial aim as well (paragraph 3.8), but this changed when I started with the report and analysis of the data.

Since there was similarity between certain cases, it made more sense to group individual cases into categories according to a similarity in results. Giving the same analysis and discussion of very similar cases proved unnecessarily repetitive and monotonous in the report. I therefore decided to group the cases into categories and present an analysis of each category. Although each case is unique (no case has exactly the same scores), the cases do fit into profiles of a similar nature, where scores fall within the same range.

The following four categories or profiles of cases presented themselves in analysis:

1. Dyscalculia present
2. Dyscalculia not present, but achievement low
3. Dyscalculia not present and achievement average
4. Dyscalculia not present but deficit in working memory found

An integrated discussion of the cases forming each of the four profiles will follow the presentation of the results.

4.3.1 Dyscalculia Present – participants 1 and 5

4.3.1 (i) Presentation of findings

Participants 1 and 5 obtained 33% and 19% respectively for mathematics in term 1 of grade 8. The participants' results on the four assessments used are represented comparatively on bar graphs in Figures 4.6 to 4.9.

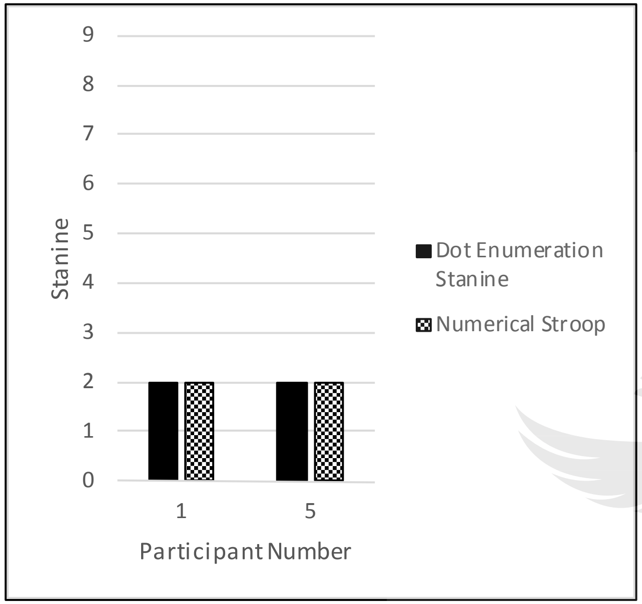

***Figure 4.6*. Dyscalculia Screener results for participants 1 and 5**

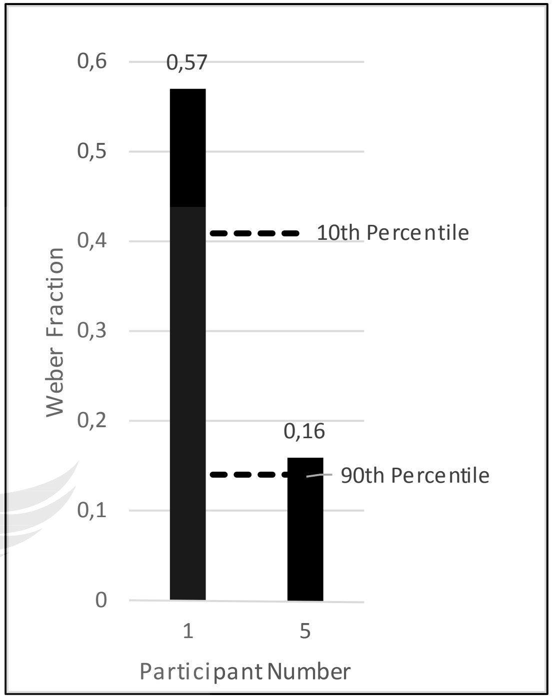

***Figure 4.7.* Weber Fraction of participants 1 and 5 as obtained on the Panamath assessment**

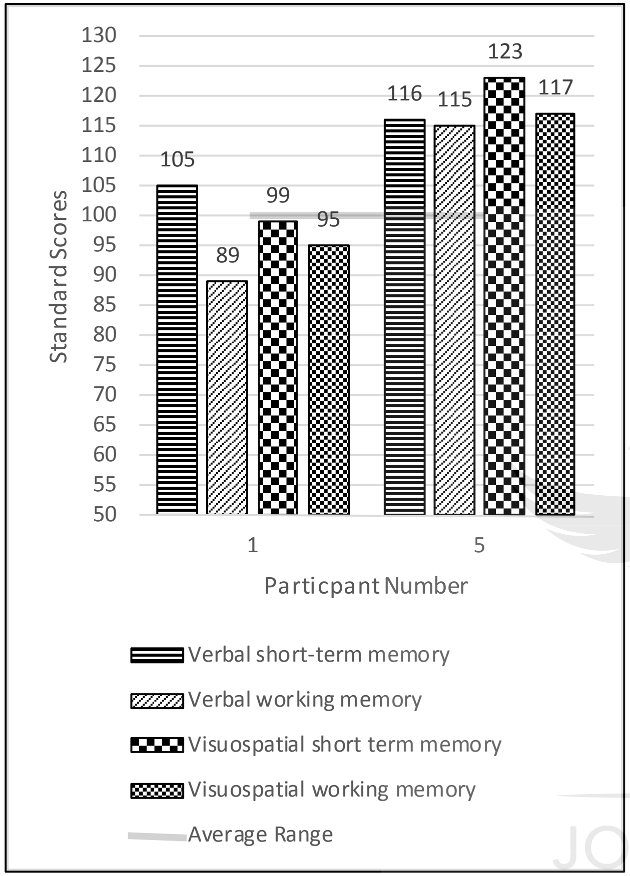

Figure 4.8. **Working memory standard scores for participants 1 and 5 obtained on the Automated Working Memory Assessment**

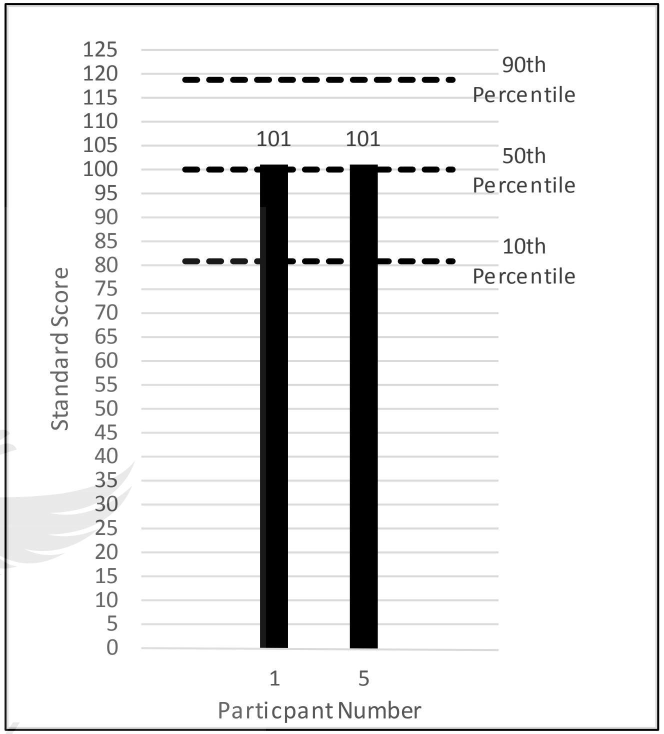

Figure 4.9. **Standardized score obtained on the Numerical Operations subtest of the WIAT-II by participants 1 and 5**

4.3.1 (ii) Integrated discussion of the profile

Both participants' mathematics term marks are significantly low. The participants' results on the Dyscalculia Screener are significant in that they both obtained a stanine of two for both the dot enumeration and numerical stroop subtests. According to Butterworth, (the developer of the test) these results are indicative of impairment in the cognitive capacity for numerosity coding, therefore these participants would be classified as having "dyscalculic tendencies" (Butterworth, 2003).

Participant 1 has also demonstrated significant impairment in the approximate number system, as indicated by the Weber fraction being higher than the 10th percentile. In other words, the amount of 'noise' in the participant's approximate number system is high. It is higher than less than 10% of the participant's age group, indicating impaired acuity of the system (Mazzocco et al., 2011). Participant 5 demonstrated little to no impairment in the approximate number system. Although these two participants differed significantly on this score, it was still decided to group them into one profile, as they both demonstrated impairment in numerosity coding, which is enough to classify them as dyscalculic. The results for participant 1 indicates that impairment is evident in more than one core system of number.

Both participants' results on the AWMA do not show any impairment in any component of working memory. Participant 1 scored within the average range for all four components and participant 5 scored within the above average range for three of the four components of working memory. Impairment in working memory for both participants is not evident and therefore can be ruled out as a contributory factor in their low achievement in mathematics.

Both participants' scored one point higher than the 50th percentile on the WIAT-II numerical operations subtest. There exists no correlation between this result and the participant's term mark, and the results on the measures of mathematical cognition. This finding has significant implications that will be discussed in Paragraph 4.4 of this chapter.

It would seem likely that the hypothesis of this study, namely that impairment in one or more cognitive system may underlie the low achievement in mathematics and be indicative of dyscalculia, has been demonstrated in these two cases. Considering the findings on both measures of numerical cognition for participant 1, and on the Dyscalculia Screener specifically for participant 5, a diagnosis of dyscalculia would seem clinically sound.

The hypothesis of this study has suggested that the information obtained on these assessments would be useful to an educational psychologist in supporting learners who demonstrate low achievement in mathematics in a number of ways. Firstly, that these learners are unlikely to significantly benefit from additional tuition purely focused on curriculum content and procedural knowledge. Any remediation or learning support would need to also include cognitively targeted interventions and recent developments of this kind of remediation will need to be taken into account (Butterworth et al., 2011; Dehaene, 2011). Additionally, this information can be useful in informing the long term planning of such a learner's academic career, including subject choice and possible career options.

4.3.2 Dyscalculia not present, but achievement low – participants 2, 4 and 8

4.3.2 (i) Presentation of findings

The mathematics term marks for participants' 2, 4 and 8 are represented on the bar graph in Figure 4.10. The participants' results on the four assessments are represented comparatively on bar graphs, in Figures 4.11 to 4.14. A detailed discussion of this profile will follow.

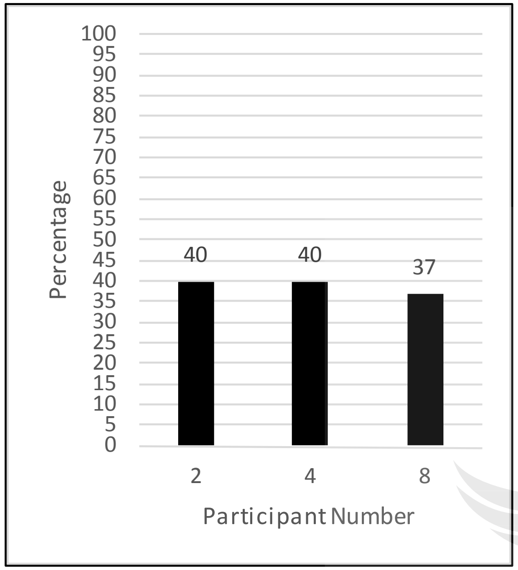

Figure 4.10. **Mathematics term marks for participants 2, 4 and 8**

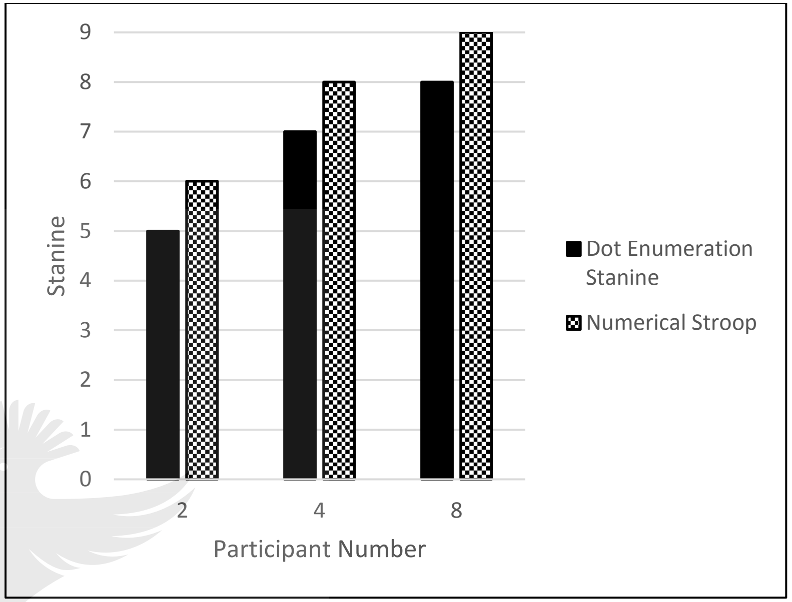

Figure 4.11. **Dyscalculia Screener results for participants 2, 4 and 8**

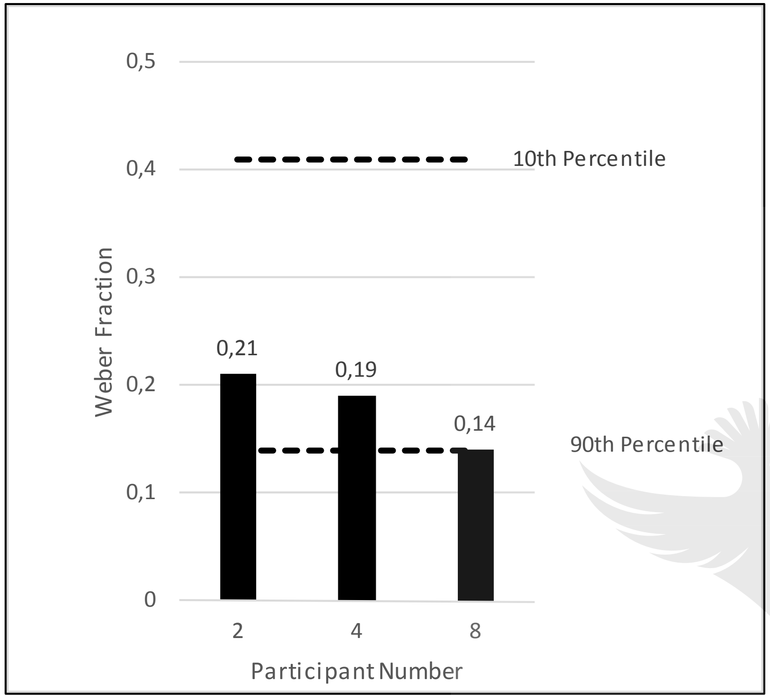

***Figure 4.12*. Weber Fraction of participants 2, 4 and 8 as obtained on the Panamath assessment**

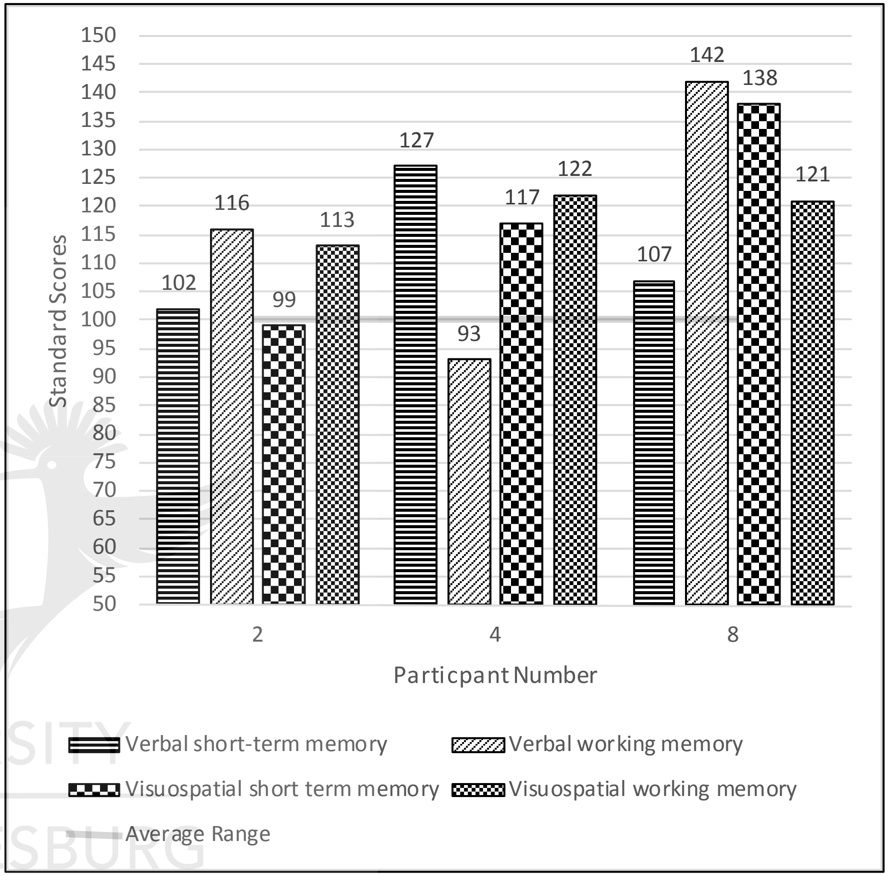

***Figure 4.13.* Working memory standard scores for participants 2, 4 and 8 obtained on the Automated Working Memory Assessment**

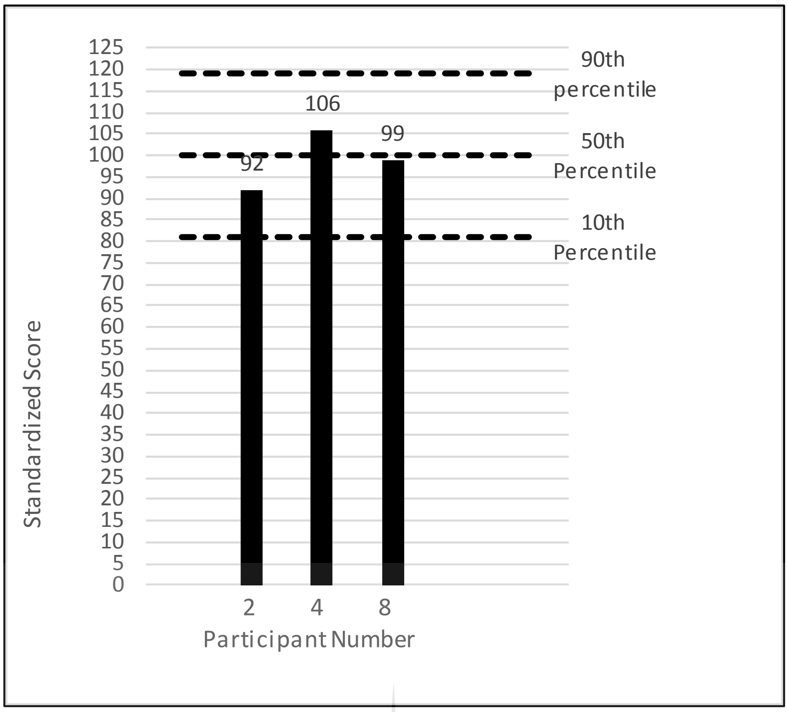

***Figure 4.14*. Standardized score obtained on the Numerical Operations subtest of the WIAT-II by participants 2, 4 and 8**

4.3.2. (ii) Integrated discussion of profile

Two of the three participants in this profile group obtained 40% for mathematics in term 1, with one obtaining 37%. Although the two participants who obtained 40% would have been considered to have a pass mark, the marks still indicate lower than average performance.

These participants all obtained stanines of 5 and above for both subtests on the Dyscalculia Screener. Only stanines of 3 or less suggest that an individual will have dyscalculic tendencies (Butterworth, 2003). Therefore, impairment in numerosity coding can be ruled out in all three of these participants.

None of the participants' results show impairment in the approximate number system. Although participant 2 shows more noise in his approximate number system than the other two, the Weber fraction is still not high enough to indicate impairment in the ANS (Mazzocco et al., 2011).

All three participants demonstrated average to superior working memories. Participant 8 obtained superior scores for the verbal and visuospatial working memory. As with profile 1, impairments in working memory can be ruled out.

The participants' scores on the WIAT-II numerical operations subtest are slightly varied. Participants' 4 and 8 demonstrated a good level of competence in numerical operations. As with profile 1, these scores suggest that operational competence does not necessarily mean one will perform well across the subject of mathematics, since operations can be 'automatized' – that is, performed mechanically through the rote memorization of steps, without necessarily grasping the underlying mathematical concept (Henning, 2013; Jensen, 2006). Participant 4 on the other hand has a below average score of 92, suggesting possible gaps in basic procedural knowledge.

This category presents a picture of adequate numerical cognition, ruling out the possibility of dyscalculia, adequate working memory, but inadequate mathematical achievement. This would be useful information for an educational psychologist. Having now ascertained that there are no deficits in numerical cognition, the educational psychologist supporting this learner would need to investigate further, looking for gaps in procedural and operational knowledge, as well as looking at general intellectual and emotional functioning. Additional standardised achievement tests, as well as a qualitative error analysis of the WIAT-II numerical operations subtest, could assist with this process.

4.3.3 Dyscalculia not present and achievement average – participants 3, 6 and 9

4.3.3 (i) Presentation of findings

As discussed in Chapter 3 (paragraph 3.4), all learners who formed part of the lowest achieving mathematics class in their grade were invited to participate in this study. This resulted in learners with a varied range of school performance marks to take part in the study. A logical conclusion would thus be that if the participants with higher marks demonstrated impairment in numerical cognition the hypothesis of this study would be questionable. None of the three higher achieving participants did, which suggests that adequate numerical cognition, or the absence of impairment in core systems of number, is required in order to obtain and maintain average to above average achievement in mathematics.

The mathematics term marks for participants 3, 6 and 9, are represented on the bar graph in figure 4.15. The participants' results on the four assessments used are represented comparatively in bar graphs, in figures 4.16 to 4.19. An integrated discussion of this profile will follow.

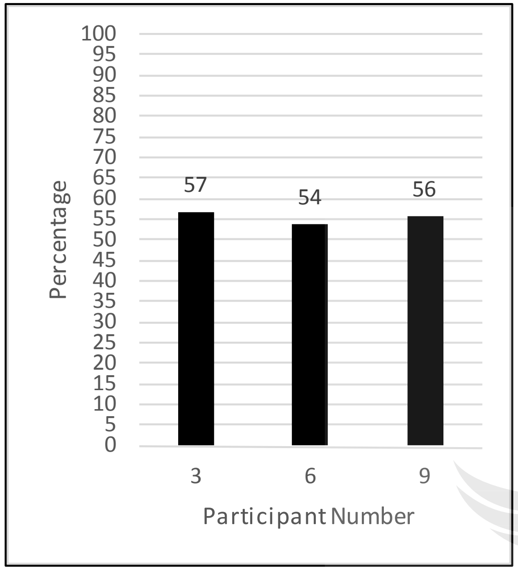

***Figure 4.15*. Mathematics term marks for participants 3, 6 and 9**

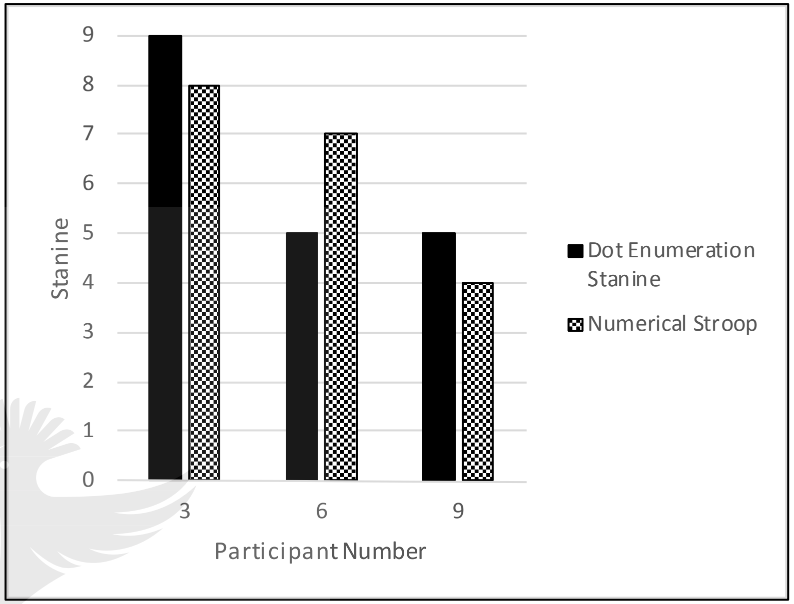

***Figure 4.16*. Dyscalculia Screener results for participants 3, 6 and 9**

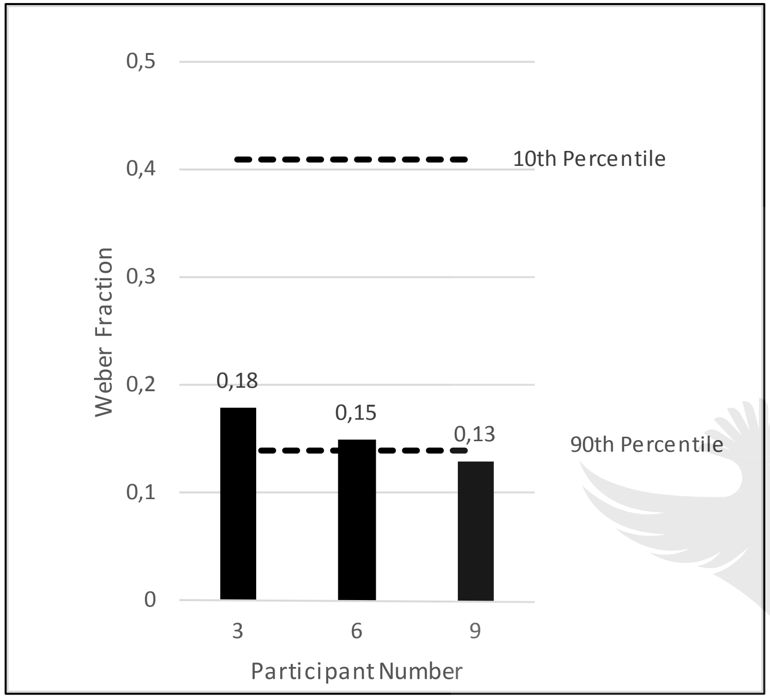

***Figure 4.17.* Weber Fraction of participants 3, 6 and 9 as obtained on the Panamath assessment**

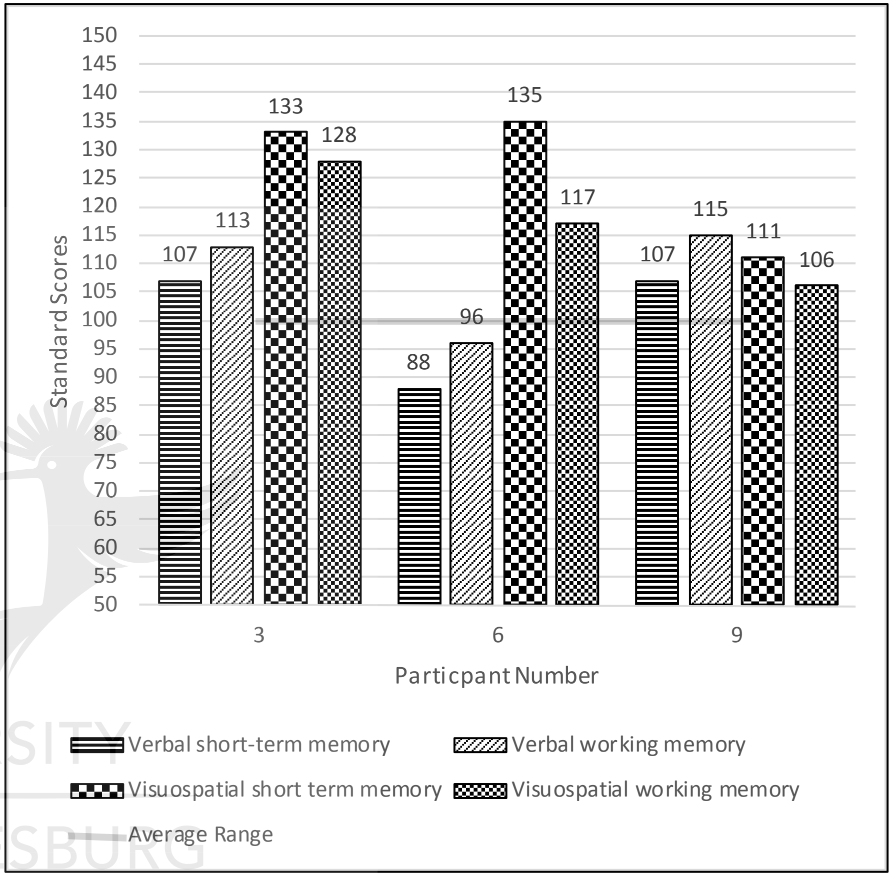

***Figure 4.18*. Working memory standard scores for participants 3, 6 and 9**

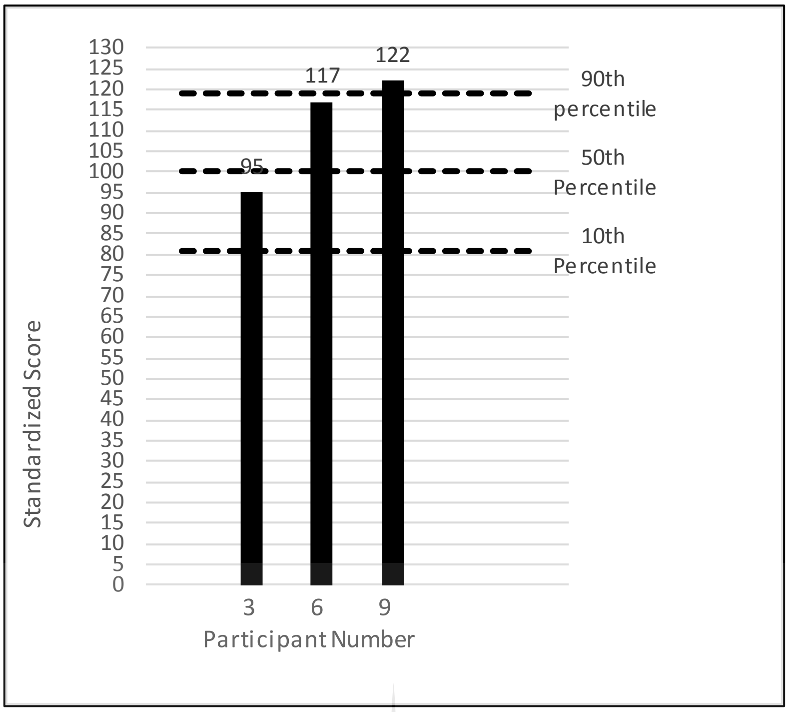

***Figure 4.19*. Standardized score obtained on the Numerical Operations subtest of the WIAT-II by participants 3, 6 and 9**

4.3.3. (ii) Integrated discussion of the profile

These three participants obtained the highest term marks in the group, ranging from 54% to 57%, even though these results would be considered average achievement in mathematics in general.

Participant 3 obtained a stanine of 9, the highest possible score on the dot enumeration subtest, and a stanine of 8 on the numerical stroop subtest of the Dyscalculia Screener. Although participants 6 and 9 obtained slightly lower scores, these are not indicative of an impairment in numerosity coding (Butterworth, 2003).

The Weber fractions of all three participants indicate little to no impairment in their respective approximate number systems (Mazzocco et al., 2011). Interestingly, participant 9, has the lowest Weber fraction of all the participants, indicating the approximate number system with least 'noise'.

As with profile 2, all three participants demonstrated average to superior working memories. As with profiles 1 and 2, no deficits in working memory are evident.

Participants 6 and 9 achieved the highest scores on the WIAT-II numerical operations subtest. It may be the case that a better grasp of mathematical concepts enables an even better execution of basic mathematical operations. This assumption lies in the fact that the scholastic term marks of participants 6 and 9 suggest a better grasp of mathematical concepts when compared to the other participants, enabling an enhanced performance of basic operations.

This profile presents a picture of adequate numerical cognition and working memory and competence in numerical operations. I therefore propose that these cases serve to demonstrate that non-impairment in numerical cognition and working memory enables adequate performance in mathematics.

4.3.4 Dyscalculia not present but deficit in working memory – participant 7

4.3.4 (i) Presentation of findings

This participant's scholastic term mark is 35%, indicating low achievement. No evidence of dyscalculia is present on the Dyscalculia Screener, and no impairment in the ANS is evident. The participant obtained a standard score of 101 on the WIAT-II numerical operations subtest, indicating competence in basic numerical operations.

This participant would fit the second profile (paragraph 4.3.2) of low achievement but no evidence of dyscalculia, except for the working memory profile, which is atypical. The profile is represented in figure 4.15.

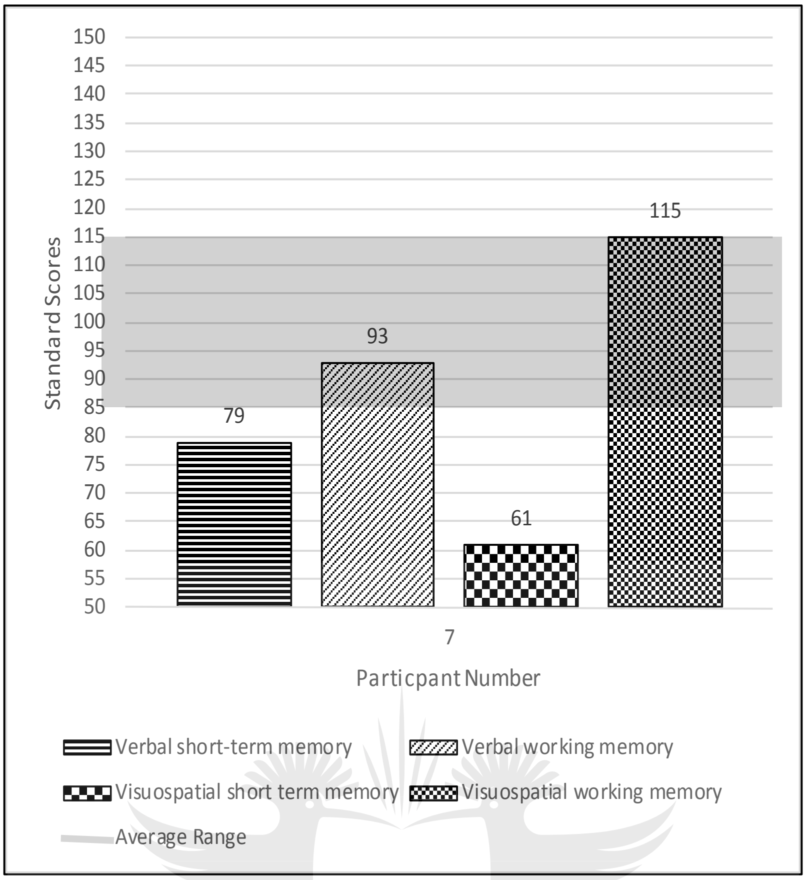

Figure 4.20: Working memory profile of participant 7

4.3.4 (ii) Integrated discussion

Not only are significant deficits evident, but the profile is also unusual, in that the participant's short term memory components are weaker than the working memory components. The short term memory components are usually stronger than working memory, where overall working memory is at a deficit (Alloway, 2007). This participant's working memory profile is typically only found in children with Autism Spectrum Disorder, where communication deficits are linked to deficits in verbal short term memory (Alloway, 2007).

Alloway (2015, personal correspondence), confirmed that this profile is still possible in a child who does not have autism spectrum disorder, because the tests use different stimuli to tap into the various components of working memory, and the child may have a difficulty processing those particular stimuli.

Levine (1994) also notes that it is possible for short term memory to be weaker than working memory in some children. In such a case, Levine encourages children to manipulate information in the mind, for example making use of paraphrasing, in order to engage the stronger working memory and enable long term storage. It may be the case that because the working memory subtests of the AWMA require mental manipulation of a stimulus, rather than just direct recall, this participant did better on these subtests.

However, since the profile is atypical, it should be interpreted with caution, and further assessments should be recommended. The long version of the AWMA would be useful in this case, as would a full psycho-educational assessment. This participant was therefore referred for further assessment to clarify.

4.4. IDENTIFICATION OF BROADER FINDINGS AND THEMES

Through the process of data analysis and discussions, broader findings, or themes, have begun to emerge. This chapter will end with a discussion of how these themes were identified. The themes will be discussed more fully in Chapter 5.

I found that the most significant finding of the assessments was the identification of dyscalculia in two participants, and not in the other seven. This identification was only possible using the assessments of numerical cognition. While the participants in category 2 would have more than likely met the criteria for a specific learning disorder in mathematics according to the DSM-5 (American Psychiatric Association, 2013), they would not, according to mathematical cognition theory, (especially that of Butterworth) be considered dyscalculic. In other words, in the case of category 1, the assessments of numerical cognition have allowed for identification of the problem at a cognitive level, and not just at the level of behaviour. In the case of the other 3 categories, the assessments of numerical cognition have demonstrated the absence of markers of dyscalculia. With this knowledge gained from the assessments, the approach to intervention and support for dyscalculic and non-dyscalculic learners will differ significantly.

This finding leads us back to the research question of this study, which is:

How do four assessment instruments (two mathematical cognition analysis tests, one working memory test and one achievement test) offer insight into the low mathematical performance of a group of grade eight learners?

Regarding the two assessments of numerical cognition, assessments offer insight because they would provide educational psychologists with crucial information regarding the cognitive abilities of the learner they are working with in terms of arithmetic and mathematics. Considering that this information should lead to a different approach to support and intervention, it could also be argued that not only are the assessments useful and beneficial, they are also very necessary. This has led to the formulation of the first theme of this study, namely, that ***cognitively targeted assessment instruments are useful and necessary in the psycho-educational assessment of low achievers in mathematics.***

The cases making up category 2 draw our attention to another important finding – that not all learners who struggle with mathematics are dyscalculic, at least from a cognitive point of view. However, the cases of category 1, as discussed above, show that some learners who struggle with mathematics are in fact dyscalculic from a cognitive point of view. I argue that this is a very important distinction to make, because, as mentioned previously, it will affect the approach to support and intervention. This leads to the second theme, that ***dyscalculia may or may not underlie low achievement in mathematics***.

The third theme of the findings has been informed by the somewhat surprising results obtained on the numerical operations subtest of the WIAT-II. My prediction was that most participants would do poorly on this test, because they were doing poorly in mathematics in general. That fact that none of them did, including the two dyscalculic participants, is significant and suggests two things. Firstly, as mentioned in paragraph 4.2.3 (ii), numerical operations can be ‘automatized’ (Henning, 2013; Jensen, 2006). This leads to the second suggestion, that the performance or non-performance of numerical operations and mathematical procedures cannot be used to identify dyscalculia, at least from a cognitive point of view. I would argue that the subject of mathematics, at least at grade 8 level and above, requires more than just performance of mathematical operations and procedures, but rather the solving of complex mathematical problems, which would require adequate numerical cognition. The third theme therefore states that ***dyscalculia cannot be identified using tests of procedural knowledge or curriculum content.***

4.5. CONCLUSION

This chapter has dealt with the presentation and analysis of the data of this study. The findings from the assessments that were conducted on the participants have been logically presented and analysed. The results of each assessment instrument – indicating equivalences and differences – were presented and discussed. Following that, each of the participant's assessment results (referred to as individual cases) were grouped into four categories showing comparative profiles. The themes emerging from the discussion of the four categories were then presented. These themes will pave the way for further discussion of their significance in answer to the research question in Chapter 5.

CHAPTER 5

DISCUSSION OF FINDINGS, SUMMARY, LIMITATIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter will conclude this study by providing a discussion of each of the three themes identified in the previous chapter. A summary of the study will then be given. Following that, a discussion of the limitations of the study and recommendations for further research will be given.

5.2 DISCUSSION OF THEMES

5.2.1 Theme 1: Cognitively targeted assessment instruments are useful and necessary in the psycho-educational assessment of low achievers in mathematics.

The assessment process used in this study was able to either identify or rule out dyscalculia, which led to the second theme, that dyscalculia may or may not underlie low achievement in mathematics. Markers of dyscalculic tendencies were found in two of the participants, and not in the other seven. According to Butterworth (2011; 2003) this information is necessary and valuable in order to effectively support all nine participants, because the factors underlying their low achievement are now better (though not yet entirely) understood. Two different approaches to learning and general support have been outlined in the discussion of profile one and profile two respectively, and it is the results of the assessments that have informed these two different approaches.

As with dyslexia (as mentioned in Chapter 2, Paragraph 2.2.3), a child whose reading is poor is not necessarily dyslexic, and research in that field identified the need to differentiate genuinely dyslexic children from those who are garden-variety poor readers (Milne, 2005). This study suggests the same is true with regards to mathematics, that genuinely dyscalculic learners need to be differentiated from learners who are not.

5.2.2 Theme 2: Dyscalculia may or may not underlie low achievement in mathematics.

This is an important differentiation to make in order to support effectively the learner in question, as previously mentioned. More importantly, genuine dyscalculia needs to be either identified or ruled out in order to save a dyscalculic learner from undue stress (Butterworth, 2003). While cognition targeted learning support may be helpful to younger dyscalculic learners, in the long run they are more than likely going to struggle with mathematics despite ongoing intervention, as is the nature of learning disorders (Gillum, 2012). For such a learner to continue with mathematics for the rest of their school career could be akin to requiring a learner who cannot draw to take art. I argue that in the case of a genuinely dyscalculic learner, it would probably be in the learner's best interests to apply for an exemption from mathematics. The National Senior Certificate allows for learners who have dyscalculia to apply for exemption from mathematics and mathematical literacy (Umalusi Council for Quality Assurance in General and Further Education and Training, 2013). Another alternative may be for the affected learner or to take mathematical literacy instead of pure mathematics. Further investigation into the curriculum content of mathematical literacy would be needed in order to gauge whether or not a dyscalculic leaner could cope with this subject.

5.2.3 Theme 3: Dyscalculia cannot be identified using tests of procedural knowledge or curriculum content

The fact that the two participants in this study who showed markers of dyscalculia in the cognition tests scored one point above the 50^{th} percentile on the WIAT-II numerical operations subtest, proves the point made by Kaufmann et. al. that "…purely educational (curricular) tests are not adequate to tap the characteristic numerical deficits associated with developmental dyscalculia" (2013, p.4).

These two learners managed to perform basic numerical operations on par with 50%

of their age group, despite showing significant impairment in at least two systems of numerical cognition. This is not entirely uncommon in dyscalculic individuals, and may be as result of “dogged determination using strategies inappropriate for their age” (Butterworth, 2003).

As mentioned in the discussion of Category 2 (paragraph 4.3.2.), mathematical operations can be “automatized” without understanding or grasping the mathematical concepts behind the operation. Jensen (2006, p.221) explains that:

> “the automatizing of a basic cognitive skill involves short-circuiting its defining operations, thereby greatly quickening its retrieval. The sheer recall of simple arithmetic number facts, such as adding, subtracting, or multiplying single-digit numbers, which are memorized and practiced repeatedly requires a conceptually much less complex level of cognition than understanding abstractly the mathematical operations that define addition, subtraction, and multiplication”.

This suggests that a combination of good teaching, extra tuition and hard work can result in even dyscalculic leaners being able to automatize mathematical operations, almost akin to learning a recipe or set of steps off by heart (This is one reason that leads me to believe that a dyscalculic leaner may be able to cope with mathematical literacy). But when it comes to problem solving and the application of mathematical operations that requires more than just factual and procedural recall, both the dyscalculic and non-dyscalculic learners seem unable to perform adequately – as indicated by their term marks.

5.2 SUMMARY

This study has been informed by theoretical developments in the expanding field of mathematical cognition, which has led to a greater understanding of how arithmetic is implemented in the brain, which has in turn led to the development of cognitively targeted assessment instruments. The objective of this study was to investigate the use of these instruments in the psycho-educational assessment of low achievers in the subject of mathematics. Ultimately, this study sought to ascertain whether or not these assessment instruments, together with an understanding of the theory that has informed their development, would be useful to educational psychologists in their support of learners who struggle with mathematics.

This study set out to investigate this question by assessing grade eight learners who were achieving below to significantly below average marks in mathematics, with three cognition based instruments and one achievement test. A multiple-case study design with embedded units of analysis was utilized (Yin, 2014). This was deemed the most appropriate research method, since the epistemological question that underpins the case study research method would be the same question that underpins the practice of psycho-educational assessment. The question being “what can be learned about the single case?” (Stake, 2005). Each learner assessed made up a single case, with the embedded units of analysis being the cognitive system that the assessment instruments aimed to measure.

The results of the assessments were presented and analysed in Chapter 4. A comparative analysis of the result of all participants on each assessment instrument was presented. The participants were then grouped into one of four profiles, based on similar patterns emerging from the results. An integrated discussion of each of the four profiles was given.

The first profile is made up learners who presented with dyscalculia, which in this study, is understood as resulting from an impairment in a core system of number, and can only be identified with assessment instruments aimed to directly measure that core system. The next two profiles are made of learners where dyscalculia has been ruled out, and achievement is either low or average. The fourth profile also excludes dyscalculia, but presents with an atypical working memory profile.

The integrated discussion of each profile described support strategies based on the empirical findings of the assessments. Attention must be drawn to the fact that suggested support strategies for each of the profiles is different. Therefore, the results of these assessment instruments should enable an educational psychologist to get a better understanding of what is actually being dealt with, which should make intervention and support more focused and less of a hit and miss approach.

Three main themes, related to the research question, emerged during the data analysis and were discussed in Chapter 4. The first theme related directly to the research question regarding the use of cognitively targeted assessment instruments, namely that they are not only useful, but also very necessary in the psycho-educational assessment of low achievers in mathematics.

This leads into the third theme, that the differentiation of genuinely dyscalculic learners from their cognitively non-impaired peers is not possible using tests of procedural knowledge or curriculum content, whether standardized or not.

This study therefore recommends that educational psychologists firstly familiarize themselves with mathematical cognition theory, particularly the theories of Dehaene (2011) and Butterworth (1997 & 2003). Secondly, that measures of mathematical cognition, such as the Dyscalculia Screener and the Number Discrimination Task, form part of psycho-educational assessment procedures for learners who are struggling with mathematics.

5.3 LIMITATIONS

As this study was only an initial focus on the possibility of identifying indicators of dyscalculia with a group of learners, the study has obviously presented with some limitations. I will now discuss these limitations briefly.

The first limitation of this study is the size of the sample. Only nine learners took part. Although each case presented useful information, the findings would carry more weight if similar patterns emerged among a bigger sample. Secondly, the learners who took part are all male, and come from a similar educational background, therefore the sample cannot be considered representative of South African learners in general.

Thirdly, because of the very specific focus of the study, the other possible factors that could play a contributory role in a learner's low achievement in mathematics were not measured. These include general intelligence, attentional problems such as Attention Deficit/Hyperactivity Disorder and emotional factors, particularly that of anxiety. Although measures to ensure trustworthiness and integrity of the assessments were strictly adhered to, there are certain factors that cannot be entirely ruled out as having an effect on the assessment results, such as fatigue of the learner or the learner experiencing emotional turmoil on the day of the assessment.

Lastly, the factor of working memory is a limitation in this study because it was shown to be largely non-contributory, despite plenty of research that has found working memory to be impaired in low achievers in mathematics (Geary et al., 2009; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; McLean & Hitch, 1999). This could however be accounted for by the small sample.

It should also be noted at this point that, despite being well trained and experienced in psycho-educational assessment, I found the Automated Working Memory Assessment (Alloway, 2007) difficult to administer. The slightest distraction or lapse in concentration can result in the assessor not correctly capturing a response. For this reason, the administration of this assessment was practiced a number of times before being administered to the participants. The developers of this assessment should consider using touch-screen technology to make the instrument easier to administer and more accurate.

5.4 RECOMMENDATIONS FOR FURTHER RESEARCH

After the analysis of each case in this study and the deliberation of results, the following recommendations could be made regarding further research of the psycho-educational assessment of low achievers in mathematics:

- Replicate this study using a bigger research sample and one that is more representative of learners in South Africa.
- Replicate the study using a research sample made up of learners in lower grades.
- Investigate cognitively targeted intervention programmes, based on mathematical cognition theory.
- Analyse the curriculum content of the subject of mathematical literacy, and investigate whether a dyscalculic learner would cope with the subject.

5.5 CONCLUSION

The research question put forward in this study has essentially been answered - that cognitively targeted assessment instruments can “shed light” or provide insight into low achievement in mathematics which ultimately can be of benefit to learners who struggle with mathematics. One does need to keep in mind that no psychological assessment is foolproof, and sound clinical judgement in terms of what is in the learner’s best interest remains paramount in any psycho-educational assessment or intervention.

The purpose of this study was to describe developments in the field of mathematical cognition and to describe how these developments have led to the development of cognitively targeted assessment instruments. The study then set out to investigate the usefulness of these assessments in the practice zone of educational psychological assessment. The instruments were found to be useful, beneficial as well as necessary in the psycho-educational assessment of low achievers in mathematics.

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