Numerical Modelling of an Erbium-Ytterbium Distributed Feedback Fibre Laser

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***Abstract*—A numerical model of an Erbium-Ytterbium distributed feedback fibre laser was developed. The optimized Er^{3+}-Yb^{3+} distributed feedback fibre laser is 50 mm long with a phase shift situated at 20.5 mm. The output power is 22.5 mW when the pump power is 100 mW with a slope efficiency of 35% and a threshold pump power of 10 mW. Simulations were performed to quantify the detrimental effects of cooperative up-conversion and excited state absorption on the characteristics of the laser. These phenomena reduce the output power of the laser with a rate of almost 6% at 100 mW pump power and increase the threshold from around 1mW to almost 10 mW. The results are in good agreement with experimental published work.**

***Keywords*— Erbium-Ytterbium, distributed feedback, laser modelling**

I. Introduction

Distributed feedback (DBF) fibre lasers present several advantages that includes simplicity, robustness and single longitudinal mode operation, which in turn allow long coherence lengths desirable for optical telecommunication applications [1]-[3].

The amplifying medium of a DFB fibre laser is an optical fibre that is a few centimetres long and doped with rare-earth elements. The feedback is obtained by a fibre Bragg grating (FBG) printed in the core of the rare-earth doped fibre. This type of laser emits naturally in two longitudinal modes. To obtain single longitudinal mode operation, a π phase shift is introduced in the middle of the FBG. Erbium doped fibre lasers offer the advantage of emitting single frequency emissions in the 1550 nm region where telecommunication fibres present the minimum attenuation loss [4]. However, due to the relatively short length of the gain medium, the number of available Erbium ions is small; as a result pump power absorption is strongly reduced. The straightforward solution to this problem could be to increase the concentration of doped Erbium ions. This solution however has the disadvantage of increasing the Erbium ions interactions, thus leading to detrimental effects like cooperative up-conversion and excited state absorption, which in term reduce the laser efficiency [5]. The best solution is to use Ytterbium ions as sensitizers along with Erbium ions to enhance the pump absorption thus the laser efficiency [6]-[7].

Much of the earlier work on DFB fibre lasers was experimental, rather than theoretical modelling and design. It has soon become clear that an accurate model of DFB fibre lasers was desirable since the simulation can reduce the design and development time and cost significantly. A model can also help to improve physical insight into the device or system under investigation by performing calculations and hypothetical experiments, which in many cases can be extremely difficult under laboratory conditions. However, modelling the Er^{3+}-Yb^{3+}-doped fibre comes with challenges. The first difficulty is the number of equations to be solved. The energy level scheme of the Er^{3+}-Yb^{3+}-doped fibre laser is somewhat complicated, because many different processes influence the laser behaviour, namely: radiative absorption and emission, with absorption arising both from ground and excited states (ESA), non-radiative decay and energy transfer, cooperative up-conversion, ion pair formation and clusterization[8]. A model that takes into account all the above processes is impractical. Therefore simplifications are inevitable. For example cooperative up-conversion and excited state absorption are neglected [9]-[11]. However under certain conditions such as high ion concentration and medium or high pumping, which is the case for DFB fibre lasers, these parameters can no longer be neglected without badly affecting the results of the simulation. As in any model, oversimplification has to be avoided as it leads to a large gap between simulated and experimental result, thus annihilating the model validity.

The second difficulty is the determination of the values for the parameters and coefficients that appear in the equations. Therefore, in our model, assumptions will be made based on measurements performed in existing literature [12]-[14]. The third difficulty is the computational methods which must be accurate while keeping the computation time within reasonable values. It must be remembered that a model is a tool and speed is a key requirement. Therefore a trade-off must be found between model accuracy and computational time.

In this thesis a model of Er^{3+}-Yb^{3+} co-doped DFB fibre laser is presented that takes into account most of the significant parameters and coefficients involved in a DFB fibre laser operation, and a fast computational method was applied to solve the model and present simulation results.

II. MATHEMATICAL MODEL

The Er^{3+}-Yb^{3+} co-doped medium is described by a set of rate equations derived from the transition between energy levels due to ion-ion and ion-light interaction. The ion–ion interactions that were considered were cooperative up-conversion (CUP) among Er^{3+}-ions and energy transfer between Er^{3+}- and Yb^{3+}-ions. Ion-light interactions included absorption at the ground state, stimulated emission, and absorption at an ESA. In addition to these transitions, spontaneous emission and non-radiative decays were also considered. The following rate equations were used to model the respective population densities:

$$\frac{dN_3}{dt} = R_{13}(N_1 - N_3) - N_3A_3 + K_{tr}N_1N_6 + C_{up}N_2^2 + R_{ESA,}N_2 - R_{ESA,3}N_3 \quad (1)$$

$$\frac{dN_2}{dt} = W_{12}N_1 - W_{21}N_2 - N_2A_2 + N_3A_3 - 2C_{up}^2N_2^2 - R_{ESA,2}N_2 \quad (2)$$

$$\frac{dN_4}{dt} = R_{ESA,3}N_3 - A_4N_4 - N_4R_{43} \quad (3)$$

$$\frac{dN_6}{dt} = R_{56}N_5 - R_{65}N_6 - N_6A_6 - K_{tr}N_1N_6 \quad (4)$$

$$N_1 + N_2 + N_3 + N_4 = N_{Er} \quad (5)$$

$$N_5 + N_6 = N_{Yb} \quad (6)$$

N_1, N_2, N_3, N_4 and N_{Er} are the E_r^{3+} concentrations of the $4I_{15/2}$, $4I_{13/2}$, $4I_{11/2}$, and $2H_{11/2}4S_{3/2}$ energy levels and total Erbium-ion concentration respectively. Similarly, N_5, N_6 and N_{Yb} are the Y_b^{3+} concentrations at the $2F_{5/2}$, $2F_{7/2}$ levels and the total Yb^{3+}-ion concentration. C_{up} , is the homogeneous up-conversion (HUC) coefficients of Er^{3+}-ions, and K_{tr} is the energy transfer coefficients from Ytterbium- to-Erbium-ions.

A_2, A_3, A_4, and A_6 are the rate of spontaneous emissions from energy levels N_2, N_3, N_4 and N_6 respectively. It should be noted that the rate of the up-conversion is the inverse of the lifetime of the concerned energy level.

$$A_i = \frac{1}{\tau_i} \quad (7)$$

where τ_i is the lifetime of the i^{th}-energy level.

In the following equations, R_{13} and, R_{56} , correspond to Er^{3+}- and Yb^{3+}-pump absorption rates respectively and R_{65} correspond to the Yb^{3+}-emission rate. W_{12} and W_{21} represent the stimulated absorption and emission rates respectively.

$$R_{13} = \Gamma_p \frac{\sigma_{13}}{h\nu_p A_c} P_p \quad (8)$$

$$R_{56} = \Gamma_p \frac{\sigma_{56}}{h\nu_p A_c} P_p \quad (9)$$

$$R_{65} = \Gamma_p \frac{\sigma_{65}}{h\nu_p A_c} P_p \quad (10)$$

$$W_{12} = \Gamma_s \frac{\sigma_{12}}{h\nu_s A_c} P_s \quad (11)$$

Table 1: Values of parameters used in the equations obtained from existing literature or determined from known assumptions.

symbol	Parameters	Value
λ_s	Pump wavelength	980 nm
λ_p	Signal wavelength	1550 nm
σ_{12}	Absorption cross section of Er^{3+} at λ_s	$8.9x10^{-25}$ m^2
σ_{13}	Absorption cross section of Er^{3+} at λ_p	$2x10^{-25}$ m^2
σ_{21}	Emission cross section of Er^{3+} at λ_s	$8.7x10^{-25}$ m^2
σ_{31}	Emission cross section of Er^{3+} at λ_p	$2x10^{-25}$ m^2
σ_{56}	Absorption cross section of Yb^{3+} at λ_p	$8.7x10^{-25}$ m^2
σ_{65}	Emission cross section of Yb^{3+} at λ_p	$11.6x10^{-25}$ m^2
σ_{23}	ESA cross section of Er^{3+} at λ_s	$1x10^{-25}$ m^2
R_{21}	Spontaneous emission rate of Er^{3+}	100 s^{-1}
R_{32}	Nonradiative decay rate of Er^{3+}	100000 s^{-1}
R_{31}	Spontaneous emission rate of Er^{3+}	30000 s^{-1}
R_{41}	Spontaneous emission rate of Er^{3+}	100000 s^{-1}
R_{43}	Nonradiative decay rate of Er^{3+}	100 s^{-1}
R_{65}	Spontaneous emission rate of Yb^{3+}	1000 s^{-1}
C_{22}	Cooperative upconversion coefficient of Er^{3+}	$1.2x10^{-24}$ m^3s^{-1}
K_{tr}	Energy transfer coefficient Yb^{3+} to Er^{3+}	$5x10^{-22}$ m^3s^{-1}
α_s	Background loss at λ_s	0.15 m^{-1}
α_p	Background loss at λ_p	0.20 m^{-1}
N_{Er}	Total Erbium ion population	$2.4x10^{25}$ m-3
N_{Yb}	Total Ytterbium ion population	$1.2x10^{25}$ m^{-3}
r	Core Radius	2.3μm
Γ_p	Overlap factor at λ_p	0.64
Γ_s	Overlap factor at λ_s	0.43

The feedback of the DFB fibre laser was obtained by a grating printed inside the core of the doped fibre. Therefore, the simulation of the model required the solution of two types of equations. The propagation equations inside the grating structure and the rate equation of the active medium.

The optical field propagation inside the grating structure is given by the following pair of coupled differential equations:

$$\frac{dR}{dz} = (g - i\hat{\sigma})R(z) - ikS(z)$$
$$\frac{dS}{dz} = -(g - i\hat{\sigma})S(z) - ik^{*}R(z) \qquad (12)$$

where R and S are the amplitudes of co- and counter-propagating electric field given by:

$$R(z) = A(z)exp(-i\Delta\beta z + \phi/2) \qquad (13)$$

and

$$S(z) = B(z)exp(i\Delta\beta z - \phi/2) \qquad (14)$$

In equations (12), k is the "AC" (associated coupling) coefficient and $\hat{\sigma}$ is a general "DC" (demi coupling) self-coupling coefficient. g Accounts for possible gain, e.g. in DFB fibre lasers. In a passive grating, $g = 0$.

These equations were solved using the transfer matrix method along with a shooting algorithm to calculate the laser output power for a given input power.

III. RESULTS

A. *Spatial gain distribution*

The gain distributions at 20 mW and 100 mW pump power for a 50 mm long Er^{3+}-Yb^{3+} co-doped DFB fibre laser with a phase shift in the centre of the fibre Bragg grating are shown in figure 1.

At a low pump power of 20 mW, the high rate of homogeneous upconversion depopulate the metastable level at the rate that a small pump power cannot compensate for. At a higher pump power however, the pump rate overcomes the rate of upconversion and the population inversion is maintained. In this case the gain distribution becomes symmetrical with respect to the phase shift. The maximum gain achieved was around 2.8 m^{-1}.

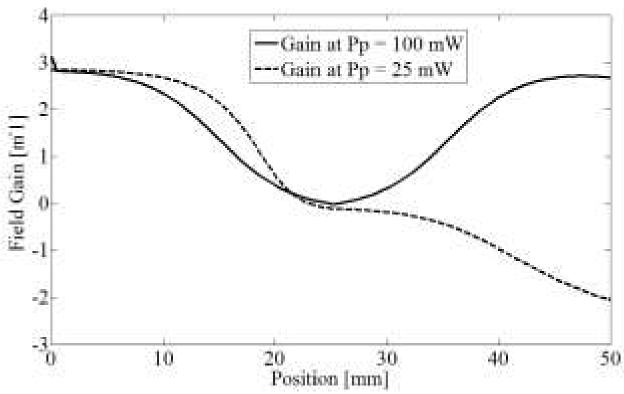

Figure 1: Gain distributions inside the DFB fibre laser for 20 mW and 100 mW pump power.

The gain profile at 100 mW pump power is solely the result of intensity distribution inside the cavity due to the phase shift in the FBG. In fact, the high intensity at the phase shift location quickly depopulated the metastable level $4I_{13/2}$. As a result the gain reached its lowest value at the phase shift position. Under certain conditions of pumping power and coupling coefficients the gain can reach negative values at the FBG phase shift which corresponded to loss. Finally the influence of the homogeneous upconversion on the gain caused a reduction of the average gain of the laser, even at high pump powers.

In figure 2 a laser with similar characteristics than the one described in the above section was modelled and simulated neglecting HUC and ESA and the gain of the DFB fibre laser modelled without HUC and ESA is shown. When pumped at 100 mw pump power, the maximum gain was 3.5 m^{-1} instead of 2.8 m^{-1}, recorded when the 2 processes were taken into account. This provided evidence that the HUC and ESA processes reduced the gain significantly and any accurate model should include them

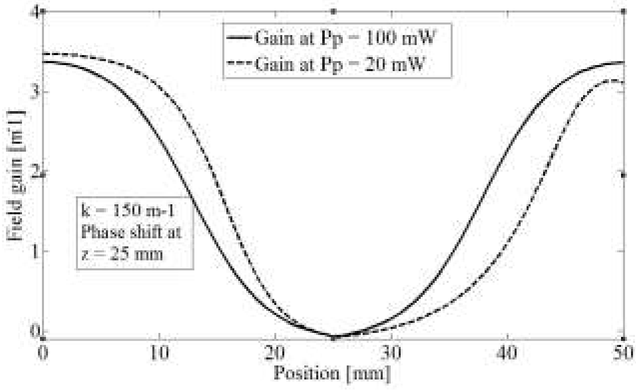

Figure 2: Gain distributions inside the DFB fibre laser for a 100 mW pump power, modelled without HUC and ESA

B. *DFB fibre laser output power.*

In figure 3, the output power as a function of pump power for a 50 mm long DFB fibre laser is shown. The grating was uniform, its coupling coefficient was 150 m^{-1} and the phase shift was located in the middle of the FBG.

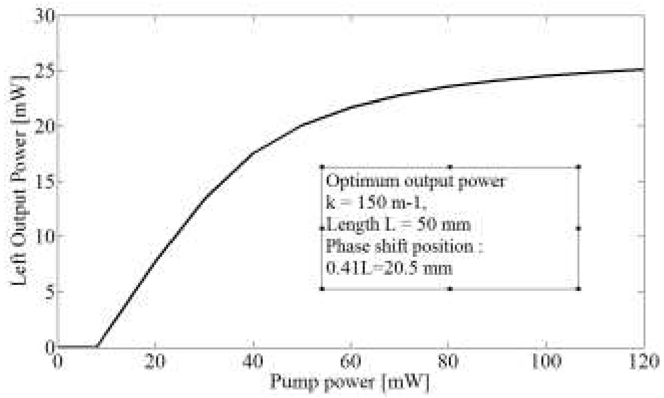

Figure 3: Output power as a function of pump power for a 50 mm long DFB with a phase shift in the middle of the FBG.

The pump power was varied from 0 to 120 mW in the simulation. The threshold power was ≈ 10 mW. After reaching the lasing threshold, the laser output power increased steadily with a slope efficiency of 60 %. When the pump power reached 40 mw the slope efficiency decreased rapidly. Beyond 60 mW the slope efficiency was only 11%, therefore the output power remained almost constant with respect to increased pump power. This decrease in slope efficiency is caused by the finite energy transfer rate between Yb^{3+-} and Er^{3+}ions, which leads to the reduction of absorbed pump photons relative to the launched pump power. The relatively high threshold of the laser can be explained by homogeneous upconversion. In fact, when the pump power was low, HUC depopulated the metastable level of the active medium at a rate higher than the pump rate and pump photons are lost instead of being converted into lasing photons, thus the cavity loss is higher than the gain and lasing was impossible.

C. *Influence of cooperative upconversion and excited state absorption on DFB fibre laser output.*

The influence of HUC and ESA can be observed and quantified by neglecting the two phenomena in the model and calculate output power as a function of pump power. The results are shown in figure 4. The graph shows that HUC and ESA processes caused detrimental effects on output power and threshold power.

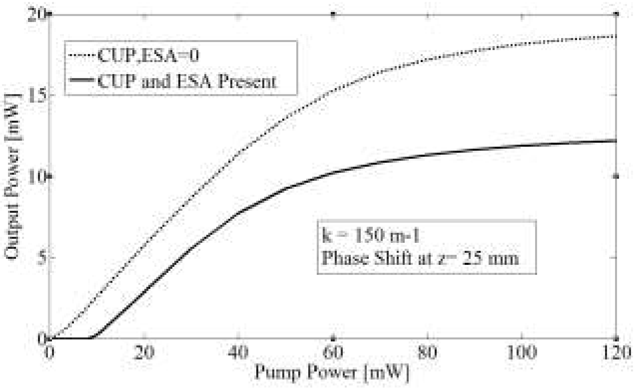

Figure 4: Output power as a function of pump power for a 50 mm long DFB fibre laser with phase shift in it centre, without HUC and ESA in the model (dotted line) and with HUC and ESA (solid line).

Without HUC and ESA the threshold was found to be less than 1mW whereas if the two processes were taken into account, the threshold increases to 10 mW. This shows evidence that HUC and ESA deteriorated the laser efficiency.

IV. CONCLUSION

An Erbium-Ytterbium co-doped distributed feedback fibre laser was modelled. Simulations were performed and detrimental effects like cooperative up-conversion and excited state absorption were highlighted. It showed that the efficiency of the laser was reduced when the HUC and ESA phenomena were considered. The model can be used to design DFB fibre lasers for optical communication.

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