DEPARTMENT OF MATHEMATICS

MODULE MAT2B10
MULTIVARIABLE AND VECTOR CALCULUS
Science Stream

CAMPUS APK

EXAM SUPPLEMENTARY EXAM 2014

EXAMINER(S) MRS C DUNCAN
INTERNAL MODERATOR MR F SCHULZ

DURATION 2 HOURS

MARKS 45

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: 1 + 10

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE CLEARLY ANY ADDITIONAL WORKING OUT
Question 1

(1.1) Define clearly what is meant by saying “$f(x, y)$ is continuous at the point $(a, b)$”.

(1.2) Is the function

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + 2y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

continuous at $(0, 0)$?
Question 2

The directional derivative of $f(x, y)$ at the point $P = (0, 4)$ in the direction of the origin is $-2$. If $\nabla f(0, 4) = \langle k, k \rangle$ for some $k \in \mathbb{R}$, what is the directional derivative at $P$ in the direction of $\theta = \pi/3$?
Question 3

Consider the volume represented by the following triple integral:

\[ V = \left[ \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz \, dy \, dx \right] = \left[ \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} dz \, dy \, dx \right] \]

(3.1) Explain, in words, the represented volume.

(3.2) Rewrite the first term only in the order \( dx \, dz \, dy \).
(3.4) Rewrite the total volume $V$ in spherical coordinates using only one triple integral. (2)

(3.4) Rewrite the total volume $V$ in cylindrical coordinates using only one triple integral. (3)
Question 4

(4.1) Find \( \iiint_{E} \frac{1}{(x^2 + y^2 + z^2)^{n/2}} \, dV \), where \( E \) is the region bounded between the spheres with center the origin and radii \( r \) and \( R \), where \( 0 < r < R \).
(4.2) For what values of $n$ does the integral in (4.1) have a limit as $r \to 0^-$. (3)
Question 5

Use an appropriate change of variable to evaluate the double integral

\[ \int \int_R \cos \left( \frac{x - y}{x + y} \right) \, dA \]

where \( R \) is in the first quadrant and bounded by the lines \( x + y = 1 \) and \( x + y = 3 \).
Consider the following vector field:

\[ \mathbf{F} = (3yx^2 + 7, x^3 + 5). \]

(6.1) Does there exist a scalar function \( f \) such that \( \mathbf{F} = \nabla f \)? Justify your answer clearly. (2)

(6.2) Hence, determine the work done by \( \mathbf{F} \) along a curve \( C \) with starting point \( (1,0) \) and terminal point \( (3,0) \). (4)
Question 7

The force exerted by an electric charge at the origin on a charged particle at a point \((x, y, z)\) with position vector \(\mathbf{r} = (x, y, z)\) is \(\mathbf{F} = \frac{K\mathbf{r}}{||\mathbf{r}||^3}\), where \(K\) is a constant.

Find the work done by this latter force as the particle moves along a straight line from \((2, 0, 0)\) to \((2, 1, 5)\).
Question 8

Given a vector field $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$. Show that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P \, dx + Q \, dy + R \, dz$$

along a smooth curve $C$. 