<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>NATIONAL DIPLOMA MECHANICAL ENGINEERING</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUBJECT</td>
<td>APPLIED STRENGTH OF MATERIALS 3</td>
</tr>
<tr>
<td>CODE</td>
<td>ASM 301</td>
</tr>
<tr>
<td>DATE</td>
<td>SUMMER SSA EXAMINATION 2015 8 DECEMBER 2015</td>
</tr>
<tr>
<td>DURATION</td>
<td>(SESSION 1) 08:00 - 11:00</td>
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<tr>
<td>WEIGHT</td>
<td>40:60</td>
</tr>
<tr>
<td>TOTAL MARKS</td>
<td>95</td>
</tr>
<tr>
<td>FINAL MARKS</td>
<td>100</td>
</tr>
<tr>
<td>ASSESSOR</td>
<td>A. MASHAMBA</td>
</tr>
<tr>
<td>MODERATOR</td>
<td>F. KIENHÖFER</td>
</tr>
<tr>
<td>NUMBER OF PAGES</td>
<td>5 PAGES + 5 ANNEXURE</td>
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**INSTRUCTIONS**

1. ANSWER ALL QUESTIONS.
2. SHOW ALL CALCULATIONS.
3. ANSWERS WITHOUT UNITS WILL BE IGNORED.
4. ALL DIMENSIONS ARE IN mm UNLESS STATED OTHERWISE.
5. FOR MISSING DATA, ASSUME TYPICAL ENGINEERING VALUES.
QUESTION 1

A simply supported compound steel beam carries a point load of 60 kN as shown in Figure Qn 1. Section AC of the beam has a flexural rigidity \((EI)\) of 10 \(MNm^2\) and Section CE has a flexural rigidity of 5 \(MNm^2\). By using Möhr’s moment area method, determine;

a) the deflection at Point B and  
b) the deflection at Point D
QUESTION 2

A steel connecting rod \((E = 200 \text{ GPa}, S_y = 220 \text{ MPa})\) has a length of 500 mm and a diameter of 16 mm as shown in Figure Qn 2.

The connecting rod is pin-jointed on both ends such that one end is connected to the piston and the other end is connected to a crankshaft rod that is rotating anti-clockwise at a slow angular velocity, \(\omega\). When the connecting rod makes an angle \(\beta\) to the horizontal, during the compression stroke, it just buckles. At this angle, \(\beta\), the fluid pressure in the cylinder is 0.4 MPa. The cylinder has an internal diameter of 200 mm. The Rankine constant for pinned joints, \(a\) is \(\frac{1}{6800}\).

![Figure Qn 2](image)

Figure Qn 2

a) Check and show if Euler buckling valid for the connecting rod. \((8)\)
b) Calculate the angle, \(\beta\) at which buckling occurs. \((6)\)
c) If the angular velocity \(\omega\) were high, would the buckling pressure for the system in Figure Qn 2 be higher or lower than 0.4 MPa? Explain your answer. \((2)\)
QUESTION 3

A connecting bracket is subjected to a tensile force of 15 kN as shown in Figure Qn 3.

![Diagram of a connecting bracket](image)

Section a - a

Figure Qn 3

Determine;

a) the principal stresses at Point A and
b) the principal stresses at Point B.

(15) (5)

QUESTION 4

A solid steel rod (\(E = 205\) GPa, \(v = 0.28\)) of diameter 50 mm just fits inside a copper tube (\(E = 100\) GPa, \(v = 0.34\)) of thickness 3 mm. An axially compressive force of 50 kN is then applied to the solid steel rod only, as shown in Figure Qn 4.

![Diagram of a solid steel rod and copper tube](image)

Figure Qn 4

Calculate;

a) the pressure that will be induced at the interface of the solid steel rod and the copper tube and
b) the percentage change in volume of the solid steel rod.

(10) (8)

[20] [18]
**QUESTION 5**

A bronze hub \( (E = 100 \text{ GPa}, \nu = 0.34) \) of 25 mm wall thickness is be shrunk onto a solid steel shaft \( (207 \text{ GPa}, \nu = 0.28) \) 100 mm in diameter to make a compound shaft.

a) If an interface pressure of 69 MPa is required, determine the interference between the hub and the shaft. \hspace{1cm} (7)

b) If the contact length and the coefficient of friction between the hub and the shaft is 150 mm and 0.18 respectively, determine the maximum torque that can transmitted by the compound shaft. \hspace{1cm} (3)

**QUESTION 6**

A shaft with a diameter of 50 mm when subjected to pure torsion can transmit a maximum torque of 1.2 kNm. A similar shaft is subjected to a torque of 720 Nm and a bending moment \( M \). Determine the maximum allowable value of \( M \) according to the maximum shear stress theory. \hspace{1cm} [17]

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**TOTAL MARKS: 95**

**FINAL MARKS: 100**
### ANNEXURE 1: FORMULA SHEET

1. **Quarter elliptical leaf-spring**
   - Maximum bending stress: \( \sigma = \frac{6WL}{nb^2} \)
   - Maximum deflection: \( \delta = \frac{6WL^3}{nEb^3} \)

2. **Deflection of beams**
   - Deflection of beam at position B relative to A:
     \[
     \gamma_{B/A} = \frac{\sum_{i=1}^{n} \int_{A}^{B} M(x)_i dx \cdot \bar{x}_i}{\sum_{i=1}^{n} EI_i}
     \]
   - Slope of beam at position B relative to A:
     \[
     \theta_{B/A} = \frac{\sum_{i=1}^{n} \int_{A}^{B} M(x)_i dx}{\sum_{i=1}^{n} EI_i}
     \]
   - For \( i = 1, 2, \ldots, n \) areas under the bending moment diagram. Where \( \bar{x}_i \) is the centroidal distance of area \( i \) from position B.

---

**Diagram 1**

- \( \bar{x} = \frac{3}{8} B \)
- \( \text{Area} = \frac{2}{3} BH \)

**Diagram 2**

- \( \bar{x} = \frac{1}{4} B \)
- \( \text{Area} = \frac{1}{3} BH \)
3. Buckling of Struts

- Euler Buckling: \( P_E = \frac{\pi^2EI}{L_a^2} \)
- Rankine Buckling: \( P_R = \frac{S_yA}{[1 + \alpha (\frac{L_b}{L})^2]} \)
- Validity Limit: \( \left( \frac{L_b}{K} \right)_{lim} = \sqrt{\frac{2\pi^2E}{S_y}} \)

4. Transformation of Stress

- Direct and shear plane stresses on an oblique plane \( \theta \) degrees (anticlockwise) from the vertical axis:
  \[
  \sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos2\theta + \tau_{xy}\sin2\theta \\
  \tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin2\theta - \tau_{xy}\cos2\theta
  \]

- Maximum principal direct stresses:
  \[
  \sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}
  \]

- Maximum principal shear stress:
  \[
  \tau_{max} = \pm\frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \pm\frac{1}{2}(\sigma_1 - \sigma_2)
  \]

- Direction of maximum principals stresses:
  \[
  \tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}
  \]

5. Analysis of Strain

- Bi-axial strain:
  \[
  \varepsilon_x = \frac{\Delta x}{x} = \frac{\sigma_x}{E}; \quad \sigma_x = \frac{E}{(1 - \nu^2)}(\varepsilon_x + \nu \varepsilon_y) \\
  \varepsilon_y = \frac{\Delta y}{y} = \frac{\sigma_y}{E} - \frac{\nu \sigma_x}{E}; \quad \sigma_y = \frac{E}{(1 - \nu^2)}(\varepsilon_y + \nu \varepsilon_x) \\
  \varepsilon_A = \frac{\Delta A}{A} = \varepsilon_y + \varepsilon_x
  \]

- Tri-axial volumetric strain:
  \[
  \varepsilon_x = \frac{\Delta x}{x} = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E} \\
  \varepsilon_y = \frac{\Delta y}{y} = \frac{\sigma_y}{E} - \frac{\nu \sigma_x}{E} - \frac{\nu \sigma_z}{E} \\
  \varepsilon_z = \frac{\Delta z}{z} = \frac{\sigma_z}{E} - \frac{\nu \sigma_x}{E} - \frac{\nu \sigma_y}{E} \\
  \varepsilon_V = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\nu)
  \]
Strain in circular shafts:
\[ \varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E} (\sigma_L - 2\nu \sigma_D) \]
\[ \varepsilon_D = \frac{\Delta D}{D} = \frac{1}{E} (\sigma_D - \nu \sigma_D - \nu \sigma_L) \]
\[ \varepsilon_V = \frac{\Delta V}{V} = \varepsilon_L + 2\varepsilon_D \]

Strain in thin cylinders:
\[ \varepsilon_L = \frac{\Delta L}{L} = \frac{\sigma_L}{E} - \frac{\nu \sigma_L}{E} = \frac{pd}{4tE} (1 - 2\nu) \]
\[ \varepsilon_H = \frac{\Delta H}{H} = \frac{\sigma_H}{E} - \frac{\nu \sigma_L}{E} = \frac{pd}{4tE} (2 - \nu) \]
\[ \varepsilon_V = \varepsilon_L + 2\varepsilon_H = \frac{pd}{4tE} (5 - 4\nu) \]

Strain in thin spheres:
\[ \varepsilon_H = \frac{\Delta H}{H} = \frac{1}{E} (\sigma_H - \nu \sigma_H) = \frac{pd}{4tE} (1 - \nu) \]
\[ \varepsilon_V = 3\varepsilon_H = \frac{3pd}{4tE} (1 - \nu) \]

Elastic constants:
\[ E = 2G(1 + \nu); \quad E = 3K(1 - 2\nu) \]

Direct and shear plane strains on an oblique plane \( \theta \) degrees (anticlockwise) from the vertical axis:
\[ \varepsilon_\theta = \frac{1}{2} (\varepsilon_x + \varepsilon_y) + \frac{1}{2} (\varepsilon_x - \varepsilon_y) \cos 2\theta + \frac{1}{2} \gamma_{xy} \sin 2\theta \]
\[ \gamma_\theta = -(\varepsilon_x - \varepsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \]

Maximum principal direct strains:
\[ \varepsilon_{L,2} = \frac{1}{2} (\varepsilon_x + \varepsilon_y) \pm \frac{1}{2} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2} \]

Direction of maximum principals strains:
\[ \tan 2\theta = \frac{\gamma_{xy}}{(\varepsilon_x - \varepsilon_y)} \]

Shear strain: \( \gamma_{xy} = \frac{\tau_{xy}}{G} \)

6. Thick Cylinders

Radial stress: \( \sigma_r = A - \frac{B}{r^2} \)
Hoop stress: \( \sigma_c = A + \frac{B}{r^2} \)
Stresses in thick cylinders due to an internal pressure \( P_i \) and external pressure, \( P_o \):

\[
\sigma_r = \frac{Pr_i^2 - Pr_o^2}{(r_o^2 - r_i^2)} - \frac{(P_i - P_o)r_i^2 r_o^2}{(r_o^2 - r_i^2) r^2}
\]

\[
\sigma_c = \frac{Pr_i^2 - Pr_o^2}{(r_o^2 - r_i^2)} + \frac{(P_i - P_o)r_i^2 r_o^2}{(r_o^2 - r_i^2) r^2}
\]

\[
\sigma_a = \frac{Pr_i^2 - Pr_o^2}{(r_o^2 - r_i^2)}
\]

Stresses in thick cylinders due to an internal pressure only \((P_o = 0)\):

\[
\sigma_r = \frac{Pr_i^2}{(r_o^2 - r_i^2)} \left[ 1 - \frac{r_o^2}{r^2} \right]
\]

\[
\sigma_c = \frac{Pr_i^2}{(r_o^2 - r_i^2)} \left[ 1 + \frac{r_o^2}{r^2} \right]
\]

\[
\sigma_a = \frac{Pr_i^2}{(r_o^2 - r_i^2)}
\]

Stresses in thick cylinders due to an external pressure only \((P_i = 0)\):

\[
\sigma_r = \frac{-Pr_o^2}{(r_o^2 - r_i^2)} \left[ 1 - \frac{r_i^2}{r^2} \right]
\]

\[
\sigma_c = \frac{-Pr_o^2}{(r_o^2 - r_i^2)} \left[ 1 + \frac{r_i^2}{r^2} \right]
\]

\[
\sigma_a = \frac{-Pr_o^2}{(r_o^2 - r_i^2)}
\]

Shrinkage allowance for compound thick cylinder:

\[
s.a = 2r_{int} \left( \frac{1}{E_o} (\sigma_{c,0,int} + v_o P_{int}) - \frac{1}{E_l} (\sigma_{c,l,int} + v_l P_{int}) \right)
\]

Shrinkage allowance for shaft and hub:

\[
s.a = 2r_{int} \left( \frac{1}{E_o} (\sigma_{c,0,int} - v_o P_{int}) - \frac{1}{E_l} (-P_{int} + v_l P_{int}) \right)
\]

Torque transmitted by a shrink fit:

\[
T = 2\pi \mu r_{int}^2 LP
\]
7. **Failure theories**

<table>
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<tr>
<th>Ductile materials: Failure occurs when:</th>
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<tbody>
<tr>
<td>Maximum shear stress (Tresca): ( \sigma_1 - \sigma_3 \geq \frac{\sigma_y}{n} )</td>
</tr>
<tr>
<td>Maximum shear strain energy (von Mises):</td>
</tr>
<tr>
<td>( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2 \left( \frac{\sigma_y}{n} \right)^2 )</td>
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<table>
<thead>
<tr>
<th>Brittle materials: Failure occurs when:</th>
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</thead>
<tbody>
<tr>
<td>Maximum principal stress (Rankine):</td>
</tr>
<tr>
<td>( \sigma_1 \geq \frac{\sigma_{ut}}{n} ) (if ( \sigma_1 &gt; 0 )) or ( \sigma_3 \geq -\frac{\sigma_{ut}}{n} ) (if ( \sigma_3 &lt; 0 ))</td>
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<th>Modified Mohr:</th>
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<tr>
<td><strong>Quadrant 1:</strong> ( \sigma_1 \geq \frac{\sigma_{ut}}{n} )</td>
</tr>
<tr>
<td><strong>Quadrant 2:</strong> ( \frac{\sigma_2}{\sigma_{ut}} - \frac{\sigma_1}{\sigma_{uc}} \geq \frac{1}{n} )</td>
</tr>
<tr>
<td><strong>Quadrant 3:</strong> ( \sigma_2 \geq -\frac{\sigma_{uc}}{n} )</td>
</tr>
<tr>
<td><strong>Quadrant 4:</strong> ( \frac{\sigma_1}{\sigma_{ut}} - \frac{\sigma_2}{\sigma_{uc}} \geq \frac{1}{n} )</td>
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