Rogowski Coil Power Application

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Abstract - In this paper, a new application of the Rogowski coil, used to extract small amounts of power from HVAC transmission lines, is studied; the output voltage for single and three-phase is estimated. Practical results on a small-scale experimental model in the milliwatt range are presented which verify the theoretical studies.

I. INTRODUCTION

The access to electricity has become a normal human right in modern society. Although this is world wide recognized, there are still places where the people cannot use the electricity. One of the reasons for this inequity is due to the size of the country and impossibility to built small economical distribution systems in remote areas, which is the case for South Africa. A power supply of several kW would meet the needs for the people in the vicinity of the transmission line, as well as meeting the power requirements of telecommunication equipments.

As a general solution to address the problem is the use of renewable energy such as solar and wind. Some "non-conventional" methods to extract power from the high voltage transmission line were studied, such as capacitive coupling [1-3], inductive coupling [4]. In [5] is presented a power transfer method using high frequency aimed to reduce the volume of the solution while delivering a reasonable power of few kW.

This paper presents a tapping method based on a new power application of the Rogowski coil; it is to be mentioned that any large rectangular coil is a Rogowski coil [7] where the effect of round corners can be neglected. The present study addresses the design and development of a Rogowski/rectangular coil as a contactless power coupling system of sufficient power rating in order to provide 230 V, 50 Hz to a load.

II. AIR-CORE TRANSFORMER

The Rogowski coil is a well-known device used to measure currents. By placing a Rogowski coil under the transmission line an air-core transformer is created and a certain amount of power is transferred to a load. The model proposed is presented in the figure 1 and the equivalent diagram of air-core transformer is presented in figure 2.

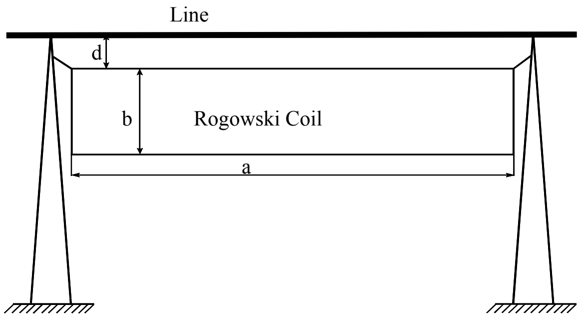

Fig.1 Rogowski Coil Power Application

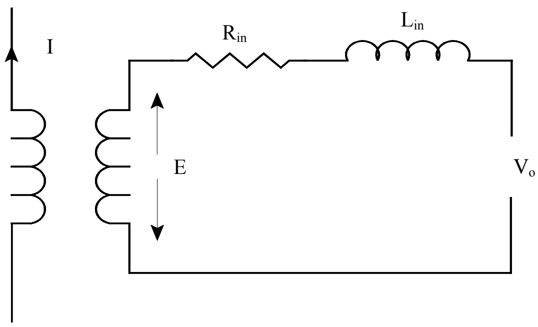

Fig.2 Air-core transformer, equivalent diagram

III. POWER TRANSFER

In order to estimate the power transferred through air-core transformer, the output voltage must be evaluated. The electromagnetic force (emf) induced by the magnetic field due to current ***i(t)*** through the line follows Faraday's law.

$$i(t) = I\sin(\omega t) \tag{1}$$

$$emf = -\frac{d\Phi}{dt} \tag{2}$$

Let's consider a more general relative position between line and coil: the line is parallel to the coil plan but ***D*** meters away as presented in the figure 3; the coordinate *y* is not shown.

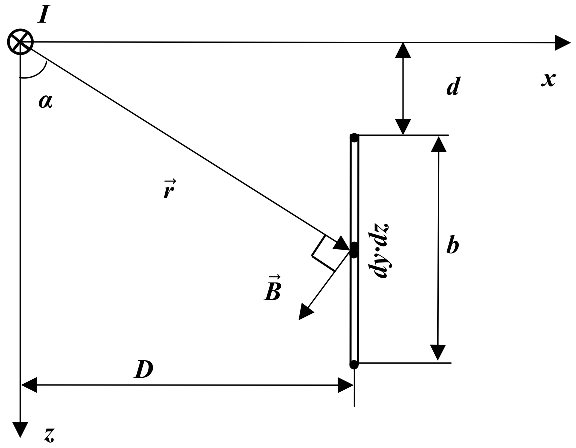

Fig.3 Single-phase system

$$\Phi = \int_S \frac{\mu_o \cdot i(t)\cdot N\cdot z}{2\pi\left(z^2+D^2\right)}dy\cdot dz \tag{3}$$

Where: S is the surface of the coil with

$$y = [0, a] \tag{4}$$

$$z = [d, d+b] \tag{5}$$

$$e(t) = \left[-\frac{di(t)}{dt}\right]\int_S \frac{\mu_0\cdot N\cdot z}{2\cdot\pi\cdot\left(z^2+D^2\right)}dy\cdot dz \tag{6}$$

$$e(t) = E\sin\left(\omega t + \frac{\pi}{2}\right) \tag{7}$$

where:

$$E = \omega\cdot I\cdot N\cdot a\cdot 10^{-7}\ln\frac{D^2+(b+d)^2}{D^2+d^2} \tag{8}$$

Given the model from figure 2 and using Matlab, the emf induced into the Rogowski coil for a primary current of 50 A is presented into figure 3.

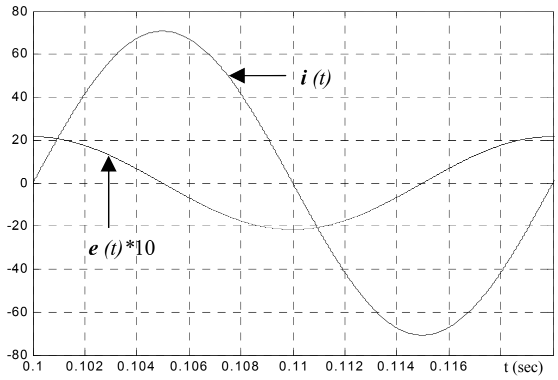

Fig. 4 Simulation parameters for single-phase system

The output power depends on the primary current ***I***, the number of turns in this coil (***N***), the size of the coil (***a, b*** and r_o – the radius of the coil's wire), the position relative to the line (***d*** and ***D***) and the internal impedance of coil R_{in}:

$$R_{in} = \frac{2\cdot(a+b)\cdot N\cdot\rho}{\pi\cdot r_o^2} \tag{9}$$

and L_{in} for low frequencies [6]:

$$L_{in} = 4\cdot 10^{-7}N^2\left[\begin{array}{l} a\ln\dfrac{2ab}{r_o\left(a+\sqrt{a^2+b^2}\right)} + 2\sqrt{a^2+b^2} + \\ b\ln\dfrac{2ab}{r_o\left(b+\sqrt{a^2+b^2}\right)} - \dfrac{7}{4}(a+b) \end{array}\right] \tag{10}$$

where:

$$a, b >> r_o \tag{11}$$

With a carefully choice of the number of turns (***N***), geometrical parameters (***a, b, d, D*** and r_o) and material (ρ) the desired power (1-3kW) can be achieved.

In practice, the most used configuration for transmission line is three-phase, therefore is necessary to find the emf induced into Rogowski coil in this situation. Let us consider the configuration presented in the figure 5, where D_L is the distance between phases, ***D*** is the distance between the lines plane and the coil plan.

Let us consider now a convenient three-phase current system:

$$i_1(t) = I\sin\left(\omega t - 120^0\right) \tag{12}$$

$$i_2(t) = I\sin(\omega t) \tag{13}$$

$$i_3(t) = I\sin\left(\omega t + 120^0\right) \tag{14}$$

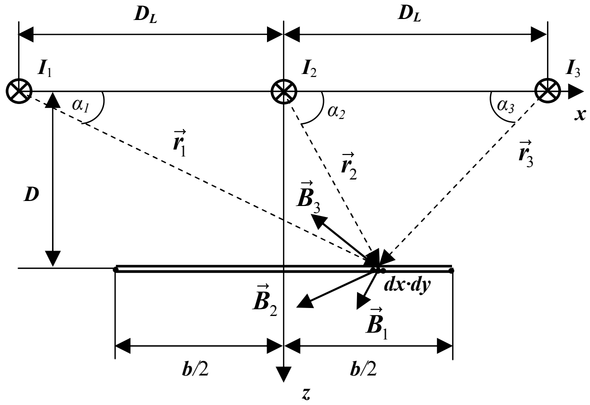

Fig.5 Rogowski coil under three-phase lines

The emfs induced by each of the phases are $e_1(t)$, $e_2(t)$ and $e_3(t)$.

$$e(t) = e_1(t) + e_2(t) - e_3(t) \tag{15}$$

$$e_1(t) = -E \cdot \cos\left(\omega t - 120^0\right) \tag{16}$$

$$E = k \cdot \ln \frac{\left(D_L + b/2\right)^2 + D^2}{\left(D_L - b/2\right)^2 + D^2} \tag{17}$$

$$k = \omega \cdot I \cdot N \cdot a \cdot 10^{-7} \tag{18}$$

$$e_3(t) = -E \cdot \cos\left(\omega t + 120^0\right) \tag{19}$$

Due the symmetry:

$$e_2 = 0 \tag{20}$$

$$e(t) = -E\cos\left(\omega t - 120^0\right) + E\cos\left(\omega t + 120^0\right) \tag{21}$$

$$e(t) = \sqrt{3} \cdot E \cdot \sin\left(\omega t\right) \tag{22}$$

The emf induced into Rogowski coil by a three-phase system of 20 A is presented in figure 6.

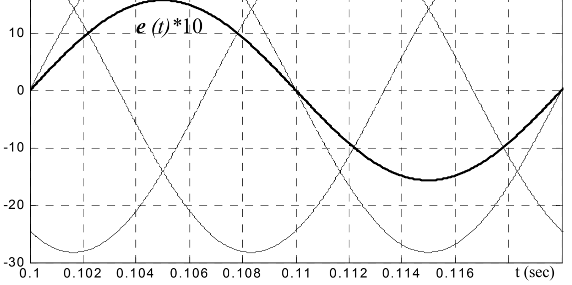

Fig. 6 Simulation parameters for three-phase system

IV. OPTIMAL POSITION

A.Single-Phase

Up to now, the vertical position of the coil was analysed; E_v is given by (8). And the maximum induced emf occurs when D = 0 which means one side of the coil is exactly below the line and d matching the clearance distance. Using the same procedure as for vertical positioning, the emf induced in the coil horizontal positioned for a single-phase line is:

$$E_h = \omega \cdot I \cdot N \cdot a \cdot 10^{-7} \cdot \ln \frac{D^2 + \left(d + b\right)^2}{D^2 + d^2} \tag{23}$$

For single-phase and taking in consideration the mechanical fitting, the optimal position is vertical directly below the line (D = 0) with d matching the clearance distance.

B. Three-phase

For the three-phase line, as in figure 5, the calculation of E_h/E_v was performed. The Rogowski coil was considered in two situations:

- *B.1* Under the line 1 or 3 (I_1 or I_3)
- *B.2* Under the line 2 (I_2)

In vertical position was considered the coil direct under the line and for horizontal position the axle of the coil - direct under the line.

B.1 Coil under line 1 or 3

Due to the symmetry the ratio E_h/E_v has the same value for either situation: coil under line 1 or line 3. Let us now consider the coil under the line1.

$$e_h(t) = e_{h1}(t) + e_{h2}(t) + e_{h3}(t) \tag{24}$$

Using the same derivation as for (20):

$$e_{h1}(t) = 0$$

Then:

$$e_{h2}(t) = -k \cdot \ln \frac{\left(D_L + b/2\right)^2 + D^2}{\left(D_L - b/2\right)^2 + D^2} \cos\left(\omega t\right) \tag{25}$$

$$e_{h3}(t) = -k \ln \frac{\left(2D_L + b/2\right)^2 + D^2}{\left(2D_L - b/2\right)^2 + D^2} \cos\left(\omega t + 120^0\right) \tag{26}$$

$$e_v(t) = e_{v1}(t) + e_{v2}(t) + e_{v3}(t) \tag{27}$$

$$e_{v1}(t) = -2 \cdot k \cdot \ln \frac{D + b/2}{D - b/2} \cos\left(\omega t - 120^0\right) \tag{28}$$

$$e_{v2}(t) = -k \cdot \ln \frac{D_L^2 + \left(D + b/2\right)^2}{D_L^2 + \left(D - b/2\right)^2} \cos\left(\omega t\right) \tag{29}$$

$$e_{v3}(t) = -k \ln \frac{\left(2D_L\right)^2 + \left(D + b/2\right)^2}{\left(2D_L\right)^2 + \left(D - b/2\right)^2} \cos\left(\omega t - 120^0\right) \tag{30}$$

By analysing (24) to (30), results an impossible straightforward analytical solution for E_h/E_v ratio.

B.2 Coil under the line 2

For the horizontal positioning, the emf induced (E_h) is given by (17). For the vertical positioning:

$$e_v(t) = e_{v1}(t) + e_{v2}(t) + e_{v3}(t) \tag{31}$$

$$e_{v1}(t) = -k \ln \frac{D_L^2 + (D+b)^2}{D_L^2 + D^2} \cos\left(\omega t - 120^0\right) \tag{32}$$

$$e_{v2}(t) = -2 \cdot k \cdot \ln \frac{D + b/2}{D - b/2} \cos \omega t \tag{33}$$

$$e_{v3}(t) = -k \ln \frac{D_L^2 + (D+b)^2}{D_L^2 + D^2} \cos\left(\omega t + 120^0\right) \tag{34}$$

$$e_v(t) = -k \ln \left[\frac{\left(D + b/2\right)^2}{\left(D - b/2\right)^2} \cdot \frac{D_L^2 + \left(D - b/2\right)^2}{D_L^2 + \left(D + b\right)^2} \right] \cos\left(\omega t\right) \tag{35}$$

$$E_v = k \ln \left[\frac{\left(D + b/2\right)^2}{\left(D - b/2\right)^2} \cdot \frac{D_L^2 + \left(D - b/2\right)^2}{D_L^2 + \left(D + b\right)^2} \right] \tag{36}$$

Once the physical parameters of the system are known, then this analyse could be concluded. The horizontal position as close as possible to the lines plane could be recommended when safety and mechanical aspects are introduced in the analysis, such as what happened with our tapping system when one of main lines gets broken and falls down.

V. VOLTAGE REGULATION

As shown in equation (8), the voltage induced has a direct dependence on the line current I and therefore will fluctuate in amplitude according with the demand. The estimated fluctuation is from simple to double. For this reason a voltage regulation is introduced. The configuration studied for this model is presented in the figure 7.

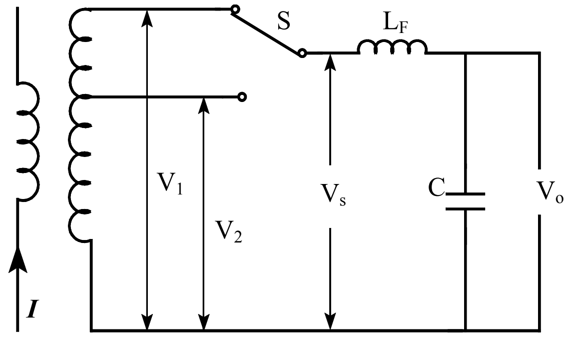

Fig. 7 Voltage regulation configuration

Using the changeover switch S as a pulse width modulator (PWM), the output voltage V_o can be controlled or regulated by means of duty cycle (δ), as presented in the Fourier series of $v_s(t)$:

$$v_s(t) = \left[\hat{V}_2 + \delta\left(\hat{V}_1 - \hat{V}_2\right)\right] \sin \omega_o t + \frac{\hat{V}_1 - V_2}{\pi} \times \sum_1^{\infty} \left\{ \frac{\sin n\delta\pi}{n} \times \left[\sin(\omega_o + n\omega_s)t + \sin(\omega_o - n\omega_s)t \right] \right\} \tag{37}$$

By means of an LC filter, the harmonics $\omega_o \pm n\omega_s$ are reduced significantly and the output voltage V_o depends only upon the duty cycle δ. To study the voltage regulation, it was taken in consideration that rms voltage V_1 is 400 V and V_2 is 200 V. The simulation results are presented in figure 8.

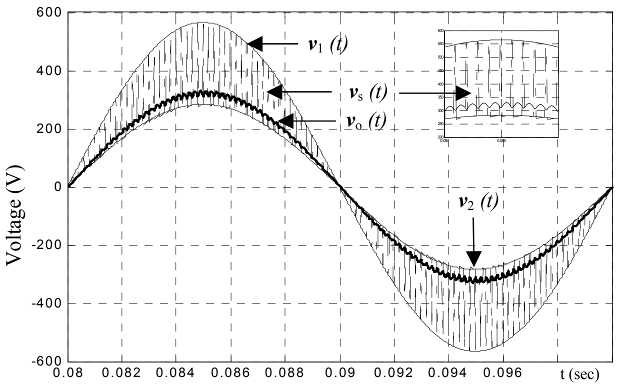

Fig.8 Simulation parameters for voltage regulation

V. EXPERIMENTAL RESULTS

Due to the difficulty of a full-scale model, the experiments have been done on reduced scale models. Two Rogowski coils, with characteristics presented in the table 1, were built.

TABLE 1

Coil	a (m)	b (m)	r_o (mm)	N
1	2.01	0.46	0.75	100
2	2.01	0.86	0.75	100

Coil 1 was placed close to a single-phase "line" loaded with a current of 0 to 50 A. The emf induced into Rogowski coil as a function of the line current was determined; the coupling of the system was tight: $\boldsymbol{D}$ = 0 and $\boldsymbol{d}$ = 1 cm. The results are presented in the table 2.

TABLE 2

I (A)	10	20	30	40	50
E_{mes} (V)	0.318	0.626	0.939	1.249	1.562
E_{est} (V)	0.307	0.614	0.921	1.228	1.535
ΔE (V)	0.011	0.012	0.018	0.021	0.027

In order to validate (8), a variable position for coil 1 in relation with the 'line' was considered. The first set of measurements have been done keeping $\boldsymbol{d}$ = 1cm and varying $\boldsymbol{D}$. The results are presented in table 3.Then, the parameter $\boldsymbol{D}$ was kept fix at 1.5 cm and $\boldsymbol{d}$ was varied; the initial position for $\boldsymbol{d}$ = 0 was with the 'line' on the centre line of the coil. The estimation was done with an equation similar to (14). The results are presented in the table 4.

TABLE 3

$\boldsymbol{D}$ (cm)	0	5	10	15	20	25	30
$\boldsymbol{E_{mes}}$ (V)	1.562	1.011	0.723	0.561	0.451	0.367	0.308
$\boldsymbol{E_{est}}$ (V)	1.535	0.983	0.697	0.531	0.419	0.338	0.278
$\Delta\boldsymbol{E}$ (V)	0.027	0.028	0.026	0.030	0.032	0.029	0.030

TABLE 4

$\boldsymbol{d}$ (cm)	0	5	10	15	20	23	25	30
$\boldsymbol{E_{mes}}$ (V)	0.097	0.273	0.467	0.725	1.165	1.549	1.342	0.926
$\boldsymbol{E_{est}}$ (V)	0	0.195	0.411	0.685	1.133	1.520	1.312	0.889
$\Delta\boldsymbol{E}$ (V)	0.097	0.078	0.056	0.040	0.032	0.029	0.030	0.037

The analysis of the results shows errors than are not very small, but that could be explained by the surrounding electromagnetic field and the distortions. And the conclusion is that the measurements validate the model.

In figure 9 is presented the emf induced for an input current of 50 A. From this figure, it is noticeable the distortion of the induced emf. But due to the derivative nature of the process, any distortion in the line current is magnified into the induced emf

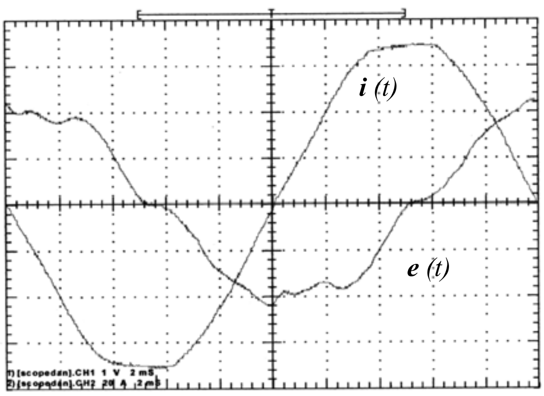

Fig. 9 Induced emf versus 'line current'

$$\boldsymbol{i}(\boldsymbol{t}) = \sum_{n=1}^{\infty} \boldsymbol{I_n} \sin\left(\boldsymbol{n} \cdot \boldsymbol{\omega t} + \varphi_n\right) \tag{38}$$

$$\boldsymbol{e}(\boldsymbol{t}) = -\boldsymbol{K}\frac{\boldsymbol{di}}{\boldsymbol{dt}} \tag{39}$$

where $\boldsymbol{K}$ is a constant depending on the system.

$$\boldsymbol{e}(\boldsymbol{t}) = -\boldsymbol{K}\omega\sum_{n=1}^{\infty} \boldsymbol{nI_n} \cos\left(\boldsymbol{n\omega t} + \varphi_n\right) \tag{40}$$

$$\frac{\boldsymbol{E_n}}{\boldsymbol{E_1}} = \boldsymbol{n}\frac{\boldsymbol{I_n}}{\boldsymbol{I_1}} \tag{41}$$

In figure 10 it is presented the total harmonic distortion (THD) for the input current and the output voltage. As one can noticed, the ratio between the THD of the output voltage and that for the current is five; the dominant harmonic of the current is also five, therefore (41) is validated. Due to this distorted output voltage, the control system for the full-scale model should include harmonic compensation.

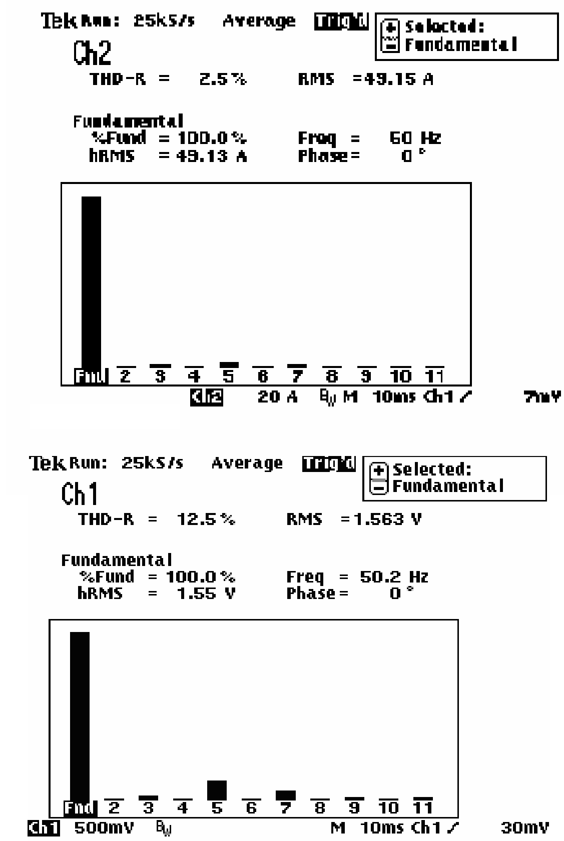

Fig. 10 Harmonic content of the current/voltage

The second coil was used to study the emf induced by a three-phase system into a Rogowski coil. The distance between 'lines' was 50 cm and the rms value of the current system was 20 A. The output voltage was 1.213

V compared with and estimated value of 1.109 V. In figure 11 is presented the output voltage due to the three-phase system and the reference current $\boldsymbol{i}_2$ (t).

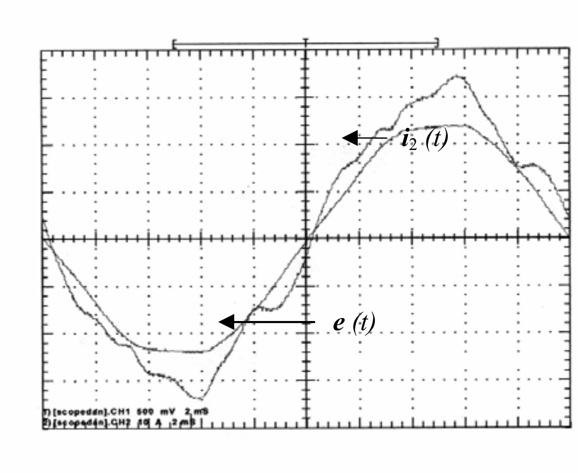

Fig. 11 Induced emf versus reference current

Another set of experiments has been done under a double/parallel 440 kV transmission line with coil 2 placed 8.5 m below. The distance has been telemetric determined.

In order to eliminate the uncertainty of the line current, the ratio $\boldsymbol{E}_{h(rms)}/\boldsymbol{E}_{v(rms)}$ has been measured and estimated. The result of the measurement under the extreme right line was 0.564 and the estimated was 0.689. The relatively big difference could be explained due to the fact that the measurements for vertical and horizontal position have not been done simultaneously.

VI. CONCLUSIONS

A new application of the Rogowski coil as a contactless power coupling system has been studied. The model with simulation parameters has been presented. The experimental results for single-phase and three-phase application validate the theoretical model with good approximation.

Based on simulated and experimental results a full-scale model is in preparation. Other aspects that should be studied on full-scale model will be the switching frequency in order to optimise the output filter and the influence of the very high electrostatic potential induced in the system.

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