A new golden ratio local search based particle swarm optimization

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***Abstract*—At beginning of the search process of particle swarm optimization, one of the disadvantages is that PSO focuses on the global search while the local search is weaken. However, at the end of the search procedure, the PSO focuses on the local search as all most all the particles converge to small areas which may make the particle swarm trapped in the local minima if no particle find position near the minima at the beginning of search procedure. To improve the optimization performance, the local search is necessary for particle swarm optimization. In this paper, golden ratio is used to determine the size of the search area. Only two positions need to be checked to find whether there are local positions with lower fitness value around a certain particle position. This method is easy to use. It is also tested using several famous benchmarks with high dimensions and big search space to the efficiency of the proposed method.**

Keywords-Golden ratio; Particle swarm optimization; Local search

I. INTRODUCTION

Many systems require solving one or more nonlinear optimization problems. While analytical methods might suffer from slow convergence, and the curse of dimensionality, heuristics-based swarm intelligence can be an efficient alternative. Particle swarm optimization (PSO), part of the swarm intelligence family, is known to effectively solve large-scale nonlinear optimization problems [1][2][3][4]. More and more researchers have been interested in PSO methods which have been investigated from various perspectives such as in artificial life, social psychology, engineering, computer science and so forth [5].

PSO is an evolutionary computation technique developed by Kennedy and Eberhart [6] in 1995, which was inspired by the social behavior of bird flocking and fish schooling. In PSO, each solution is a "bird" in the flock and is referred to as a "particle". However, the original PSO algorithm [6] is easily trapped in local suboptimal points when applied to problems with many local extremes [7]. PSO has been developed for many years, one of the most famous one is the standard PSO which can be looked as a benchmark PSO to compare with other proposed variants of PSO [8]. The standard PSO has been developed for several versions, that is, SPSO 2006, 2007 and 2011 since the year of 2006. However, similar with most of the variants of PSO, the standard PSOs have a disadvantage that PSO focuses on the global search while the local search is weaken during the beginning of the search process of particle swarm optimization. And during the end of the search procedure, the standard PSO focus on the local search as all most all the particles converge to small areas which may make the particle swarm trapped in the local minima if no particle find position near the minima at the beginning of search procedure. To conquer or weaken this disadvantage, a simple local search technique is proposed in this paper.

The rest of this paper is structured as follows. Section 2 describes the preliminary knowledge about particle swarm optimization. The details of golden ratio local search technique are presented in Section 3. Section 4 gives a description of the experimental settings for the benchmarks and simulation results in comparison with the SPSO 2011. Finally, Section 5 gives some concluding remarks.

II. A BRIEF DESCRIPTION OF PSO

In general, single objective optimisation problems can be represented as the following optimisation problem [3]:

$$f(X_i) = \min(g(X_i)),\ X_i = [x_i^1, x_i^2, \cdots x_i^n],$$
$$\text{subject to } p_j(X_i) \le 0,\ j = 1, 2, \cdots, k. \quad (1)$$

Here $g(\cdot)$ is the objective function without constraints; $X_i(t)$ denotes the position vector of particle i consisting of n variables. Every candidate solution of $f(X_i)$ is called a "particle".

The standard particle swarm algorithm works by iteratively searching in a region and is concerned with the best previous success of each particle, the best previous success of the particle swarm, the current position and the velocity of each particle [3]. The particle searches the domain of the problem, according to

$$V_i(t+1) = \omega V_i(t) + c_1 R_1 (P_i - X_i(t)) + c_2 R_2 (P_g - X_i(t)), \quad (2)$$

$$X_i(t+1) = X_i(t) + V_i(t+1), \quad (3)$$

where $V_i = \left[v_i^1, v_i^2, \cdots v_i^n\right]$ is the velocity of particle i; $X_i = \left[x_i^1, x_i^2, \cdots x_i^n\right]$ represents the position of particle i; P_i represents the best previous position of particle i (indicating the best discoveries or previous experience of particle i); P_g

represents the best previous position among all particles (indicating the best discovery or previous experience of the social swarm); ω is the inertia weight that controls the impact of the previous velocity of the particle on its current velocity and is sometimes adaptive [9]; R_1 and R_2 are two random weights whose components r_1^j and r_2^j $(j = 1, 2\cdots, n,)$ are chosen uniformly within the interval [0,1] which might not guarantee the convergence of the particle trajectory; c_1 and c_2 are the positive constant parameters. Generally the value of each component in V_i should be clamped to the range $[-v_{\max}, v_{\max}]$ to control excessive roaming of particles outside the search space.

There are some differences between the original PSO and standard PSO [8] about the parameter setting. For standard PSO 2011, the parameters are set as

$$\omega = \frac{1}{2\times\log(2)},$$

$$c_1 = 0.5 + \log(2),$$

$$c_2 = c_1$$

and the swarm size is $floor(10 + 2\sqrt{D})$.

Here, log() is the natural logarithm function, and $floor()$ is the round toward negative infinity function. Moreover, the random topology is used in SPSO 2011.

III. GOLDEN RATIO LOCAL SEARCH BASED PSO

As mentioned in Section I, PSO cannot achieve a good local research performance at beginning of search process which results in the particles trap into local minima especially for the optimization problems with a big problem space and a large number of minima. It is possible to improve the optimization performance if some local searching techniques are used to search the local area around the current particle positions during the whole searching process.

According to (2) and (3), PSO works by iteratively searching in a region which has the tendency to the best previous success of each particle and the best previous success of the particle swarm. Hence, it is more possible that the fitness value related to the current position is lower than the random search, and it can be expected that two neighbors, which are different from the neighbor in the topology, are located in the same 'cove' in the searching space. There are potential positions between the closest particle neighbors which have lower fitness value than these two particles. Here the golden ratio is used to determine the local search position as golden ratio has been widely used or shown in many areas, such as architecture [10][11], aesthetics [12], music [13], industrial design [14], and so on. Moreover, the golden ratio is also used in optimization [15] and achieves good efficiency. Here, only one position is chosen based on the golden ratio between the two neighbors as shown in Fig. 1.

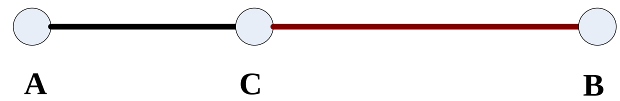

Figure 1 Local position determined by golden ratio

In Fig. 1 A and B are two particles. To realize the local search for particle A, the position C is determined by the nearest neighbor B and golden ratio. Here,

$$\frac{|\overrightarrow{AC}|}{|\overrightarrow{CB}|} = 0.618. \tag{4}$$

Here, $|\bullet|$ is the vector norm function. To extend the search space, the opposition based method is used. The opposition concept has been used in evolution optimization algorithm and a good optimization performance was obtained [16]. In this paper, the method of using opposition concept is rotating C with 180^o.

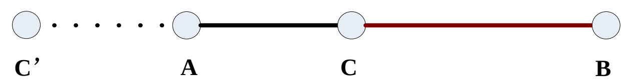

Figure 2 Local positions with opposition

To simplify the complexity of the proposed method, only two extra positions, which are C and C', are checked. To realize the proposed method, the following procedure can be used to implement the golden ratio local search based PSO:

1) *Initialize the parameters of PSO and swarm by assigning a random position in the problem hyperspace to each particle.*

2) *Evaluate the fitness function for each particle and set the fitness values as the best personal experience.*

3) *Randomize the topology if there is no improvment in the best swarm solution.*

4) *Update the positions of all the particles using (2)-(3).*

5) *Check for constraint violations. If the position is out of the range, the position is set to the border value and velocity is set to zero.*

6) *Evaluate the fitness function for each particle.*

7) *Golden ratio based local search for every particle. Find the best solution among position A, C and C' and take it as the current position.*

8) *Repeat steps 3)-7) until a stopping criterion is met (e.g., maximum number of iterations or a sufficiently good fitness value).*

To reduce the effect of different dimensions with different ranges, the optimisation problem variable interval must therefore be mapped to [-1, 1] and vice versa using

$$x^j = -1 + \frac{2(x_j - a_j)}{b_j - a_j},\ j = 1, 2, \cdots, n. \tag{5}$$

and

$$x_j = a_j + \frac{1}{2}(x^j + 1)(b_j - a_j),\ j = 1, 2, \cdots, n. \tag{6}$$

Here, a_j and b_j represent the lower boundary and the upper boundary of $x_j(t)$, respectively only one particle is analysed for simplicity.

IV. EXPERIMENTAL VARIFICAITON

To demonstrate the efficiency of the proposed technique, several famous benchmarks are used to compare the proposed method and SPSO 2011 (Matlab version) [8]. The parameters are set as SPSO 2011 and the particle swarm was run for 50 trials per function with a maximum iteration of 1000 for every trial.

The benchmarks are listed in Table 1. Five famous benchmark functions are chosen and every one will be tested for two different dimensions (30 and 100).

TABLE I. FUNCTIONS USED TO TEST THE EFFECTS OF THE PROPOSED METHOD

Function Name	Formulation	Dim	Initial Range
Sphere function	$f_1(X)=\sum_{i=1}^{n}x_i^2$	10	±500
Rastrigin function	$f_2(X)=\sum_{i=1}^{n}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	10	±500
Rosenbrock function	$f_3(X)=\sum_{i=1}^{n}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	10	±500
Griewank function	$f_4(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2+\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	10	±500
Ackely function	$f_5(X)=-20\times\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	10	±500
Sphere function	$f_6(X)=\sum_{i=1}^{n}x_i^2$	30	±500
Rastrigin function	$f_7(X)=\sum_{i=1}^{n}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	30	±500
Rosenbrock function	$f_8(X)=\sum_{i=1}^{n}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	30	±500
Griewank function	$f_9(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2+\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	30	±500
Ackely function	$f_{10}(X)=-20\times\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	30	±500
Sphere function	$f_{11}(X)=\sum_{i=1}^{n}x_i^2$	100	±500
Rastrigin function	$f_{12}(X)=\sum_{i=1}^{n}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	100	±500
Rosenbrock function	$f_{13}(X)=\sum_{i=1}^{n}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	100	±500
Griewank function	$f_{14}(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2+\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	100	±500
Ackely function	$f_{15}(X)=-20\times\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	100	±500

The statistic results of the standard PSO 2011 and the proposed method are given in Table 2. As can be seen from Table 2, the proposed method can greatly improve the optimization performance and the stability of optimization when the dimensions are 10 and 30. However, there is a little improvement when the dimension is 100, which was caused by the improvement of the complication of the optimization problems. It also means that more local searches should be done when the number of dimension is big.

V. CONCLUSION

This paper proposed a golden ratio local search based particle swarm optimization. This method also used the opposition concept to improve the local search ability. This method was applied to several famous benchmarks and achieved a good optimization results especially the dimension of the searching space is not very big. Furthermore, this proposed technique can be easily embedded inside other evolutionary optimization algorithms. In the future, we will focus on improving the optimization performance when the dimension is big using some local searching strategies.

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TABLE II. RESULTS USING THE STANDARD PSO 2011 AND THE PROPOSED METHOD

Problem	Method	best	Mean	Worst	Std.dev
Sphere function f_1	Standard PSO 2011	0.2020	0.5903	1.2049	0.2199
	Proposed method	4.7832×10^{-4}	0.0142	0.0476	0.0123
Rastrigin function f_2	Standard PSO 2011	17.0480	40.4833	56.0410	9.2735
	Proposed method	8.8062	29.4936	50.4781	10.4372
Rosenbrock function f_3	Standard PSO 2011	23.0128	56.3832	108.4931	16.9555
	Proposed method	2.6188	9.0025	18.9887	3.4451
Griewank function f_4	Standard PSO 2011	0.0086	0.0684	0.1562	0.0324
	Proposed method	4.2879×10^{-4}	0.0041	0.0419	0.0061
Ackely function f_5	Standard PSO 2011	1.4301	2.3176	3.0554	0.3911
	Proposed method	0.0364	0.3413	1.2653	0.2762
Sphere function f_6	Standard PSO 2011	3.0458	6.9523	8.7257	0.9279
	Proposed method	1.9123	2.7449	3.9559	0.4840
Rastrigin function f_7	Standard PSO 2011	199.1225	233.9243	266.6047	15.7870
	Proposed method	109.5946	185.5221	229.6229	24.9829
Rosenbrock function f_8	Standard PSO 2011	260.4547	377.9755	461.0138	41.7120
	Proposed method	175.9785	218.4976	259.2466	21.8027
Griewank function f_9	Standard PSO 2011	0.1862	0.3066	0.3923	0.0434
	Proposed method	0.0897	0.1257	0.2074	0.0274
Ackely function f_{10}	Standard PSO 2011	3.1866	3.5137	3.7619	0.1336
	Proposed method	2.2784	2.8398	3.2952	0.2273
Sphere function f_{11}	Standard PSO 2011	23.8052	27.2318	29.4010	1.2801
	Proposed method	23.0614	27.2534	29.1615	1.2253
Rastrigin function f_{12}	Standard PSO 2011	823.2026	890.9073	947.3594	23.7985
	Proposed method	792.0004	881.0822	934.9773	35.2341
Rosenbrock function f_{13}	Standard PSO 2011	1340	1606	1828	99.9118
	Proposed method	1378	1602	1763	90.2874
Griewank function f_{14}	Standard PSO 2011	0.3364	0.4229	0.5061	0.0326
	Proposed method	0.3195	0.4242	0.4992	0.0303
Ackely function f_{15}	Standard PSO 2011	3.5200	3.6831	3.7948	0.0579
	Proposed method	3.5148	3.6709	3.7664	0.0551