

Cask Theory Based Parameter Optimization for Particle Swarm Optimization

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Abstract. To avoid the bored try and error method of finding a set of parameters of Particle Swarm Optimization (PSO) and achieve good optimization performance, it is desired to get an adaptive optimization method to search a good set of parameters. A nested optimization method is proposed in this paper and it can be used to search the tuned parameters such as inertia weight ω , acceleration coefficients c_1 and c_2 , and so on. This method considers the cask theory to achieve a better optimization performance. Several famous benchmarks were used to validate the proposed method and the simulation results showed the efficiency of the proposed method.

Keywords: PSO, Parameter Optimization, Try and Error method, Nested Optimization method, Cask theory.

1 Introduction

Particle Swarm Optimization (PSO) was developed by Kennedy and Eberhart [1]. This algorithm is inspired by the social behavior of a flock of migrating birds trying to reach an unknown destination. All members obey a set of simple rules that model the communication within the flock, between the flocks and the environment. Each solution is a “bird” in the flock and is referred to as a “particle”. PSO has attracted a lot of attention as it makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions [2, 4-7]. The formula of PSO is realized by two update functions:

$$V_i(t+1) = \omega V_i(t) + c_1 R_1 (P_i - X_i(t)) + c_2 R_2 (P_g - X_i(t)), \quad (1)$$

$$X_i(t+1) = X_i(t) + V_i(t+1). \quad (2)$$

Here $V_i = [v_i^1, v_i^2, \dots, v_i^n]$ is the velocity of particle i ; $X_i = [x_i^1, x_i^2, \dots, x_i^n]$ represents the position of particle i ; P_i represents the best previous position of particle i

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(indicating the best discoveries or previous experience of particle i); P_g represents the best previous position among all particles (indicating the best discovery or previous experience of the social swarm); ω is the inertia weight that controls the impact of the previous velocity of the particle on its current velocity and is sometimes adaptive. R_1 and R_2 are two random weights whose components r_1^j and r_2^j ($j=1,2,\dots,n$) are chosen uniformly within the interval $[0,1]$ which might not guarantee the convergence of the particle trajectory; c_1 and c_2 are the positive constant parameters. Generally the value of each component in V_i should be clamped to the range $[-v_{\max}, v_{\max}]$ to control excessive roaming of particles outside the search space.

The generalized procedure of applying standard PSO 2011 (SPSO 2011) [8] is

- 1) Initialize the swarm and assign a random position in the problem hyperspace to each particle and calculate the fitness function which is given by the optimization problem whose variables are corresponding to the elements of particle position coordinates; and set the topology of the whole particles.
- 2) The particles search the area according to equations (1) and (2); check the velocity and position of particles to find whether they violate the boundaries.
- 3) Evaluate the fitness function for each particle.
- 4) For each individual particle, compare the particle's fitness value with its previous best fitness value. For each individual particle, compare the particle's fitness value with its previous best fitness value. If the current value X_i is better than the previous best value P_i , then set P_i as X_i .
- 5) Change the topology if the optimization performance is not improved in a certain number of iterations.
- 6) Repeat steps 2)-5) until a stopping criterion is met (e.g., maximum number of iterations or a sufficiently good fitness value).

As can be seen from (1) and (2), there are several parameters which should be determined before PSO was applied. Similar as most of the evolutionary optimization algorithms, the parameters of PSO need to be chosen carefully to achieve good optimization performance. The parameters are often chosen based try and error method as different optimization problems have different characteristics and the parameters should not be same to achieve good optimization results. Hence, it is desired to find a suitable set of parameters of PSO without using the bored try and error method. For the evolutionary optimization algorithms, there are some methods optimizing the parameters of the optimization algorithms which are called meta-optimization. Meta-optimization is reported to have been used as early as in the late 1970s by Mercer and Sampson for finding optimal parameter settings of a genetic algorithm [9]. There are some meta-optimizations [10], [11], [12]. For different meta-optimizations, there are different performance indexes.

In this paper, an automatic parameters searching method is proposed based on the particle swarm optimization algorithm and the cask theory. The rest of this paper is arranged as follows: Section 2 presents the proposed algorithm with details. Simulations and comparison are given in Section 3. Finally, the concluding remarks appear in Section 4.

2 Cask Theory Based Parameter Optimization

As the optimization performance depends on the optimization problems, the parameters of optimization algorithms should also depend on the optimization problems, which means different optimization problems should have different sets of parameters of optimization algorithms. As the optimization algorithms can find optimal or sub-optimal solution for the optimization problems, the optimization algorithms can also be used to find the optimal or sub-optimal parameters for PSO. Similar as the optimization procedure, the objective function or criteria related to the parameters of PSO must be defined firstly. There is an important theory is cask theory or barrel theory in Management Science [3]. The cask theory describes that the cubage of a cask is dependent on the shortest wood plate as shown in Fig. 1. This method takes the worst case as the performance criteria and it is possible to make the optimization performance not worse than the achieved one.



Fig. 1. Cask theory (www.baike.com)

The parameter optimization procedure is same with the standard one as mentioned in Section 1. The core of the parameter optimization is defining the objective function or criteria. The followings are the factors, which should be considered, when design the objective function for PSO parameter optimization:

- 1) Important parameters of PSO should be chosen and they will be the inputs of the objective function.
- 2) The optimization problem should be considered as the implicit objective as the parameters are used to achieve good optimization performance for the optimization problem.
- 3) The optimization performance should be stable when the obtained parameters are implemented.
- 4) The output of the objective function should follow cask theory to guarantee the worst optimization performance is not too bad.

Here, without loss of generality, the algorithm of the SPSO 2011 [8] is chosen as the optimization algorithm whose parameters (inertia weight ω , and acceleration coefficients c_1 and c_2) will be optimized and the SPSO 2011 with fixed parameter is used to optimize these parameters. Hence, for 1) the inputs of the objective function are the inertia weight ω , the acceleration coefficients c_1 and c_2 . For 2), the optimization problem will be the target of the SPSO 2011 with variant parameters (VSPSO 2011). For 3) and 4), the optimization problem should be optimized several runs by VSPSO to make sure the results are not stochastic; and the worst fitness value is chosen as the output of the objective function which follows the cask theory.

After the set of parameters are obtained, the normal procedure of PSO will be used to optimize the optimization problems.

3 Numerical Simulation

To demonstrate the efficiency of the proposed technique, eight well-known benchmarks are used to compare the proposed method and the standard PSO 2011 (Matlab version) [8]. The eight optimization problems were used as shown in Table 1. The parameters of these optimization problems are given in Table 2. These eight optimization problems are famous test functions for minimization methods and each of them has high dimension and several local minima. In the numerical simulation of SPSO 2011 with fixed parameters, the particle swarm population size is set $\text{floor}(10 + 2\sqrt{D})$. Here D is the dimension of the optimization problems and $\text{floor}(A)$ rounds the elements of float number A to the nearest integers less than or equal to A .

The rest of the parameters are as follows: inertia weight $\omega = \frac{1}{(2\log 2)} \approx 0.7213$,

learning rates $c_1 = c_2 = 0.5 + \log 2$, and velocity V_{\max} set to the dynamic range of the particle in each dimension. For VSPSO 2011, the inertia weight ω , the acceleration coefficients c_1 and c_2 are the parameters to be optimized and all the initial ranges of ω , c_1 and c_2 are $[0.2, 3]$. To reduce the run time, the maximum number of function evaluations is 500 with 10 independent runs. The maximum number of function evaluations is 500 for these VSPSO 2011 using the parameters obtained and SPSO 2011 with 100 independent runs.

The optimized parameters were given in Table 3. The optimization statistical analysis of proposed method and SPSO 2011 with fixed parameters was given in Table 4. As can be seen from Table 3, the parameters are totally different from the fixed parameters of SPSO 2011 and there are no rules to follow as the optimization problems are totally different. As can be seen from the Table 4, the optimization performance of VSPSO 2011 is more stable and it can guarantee the worst results are not worse than the worst results of SPSO 2011 as the proposed parameter optimization method is cask theory based parameter optimization method.

Table 1. Functions used to test the effects of the LGPSO method

Sphere	$f(x) = \sum_{i=1}^D x_i^2$
Rastrigin	$f(x) = 10D + \sum_{i=1}^D (x_i^2 - 10 \cos(2\pi x_i))$
Step	$f(x) = \sum_{i=1}^D [(x_i + 0.5)^2]$, [.] is rounding function
Rosenbrock	$f(x) = \sum_{i=1}^D (100(x_{i+1} - x_i^2)) + (x_i - 1)^2$
Ackley	$f(x) = 20 + e - 20e^{-\frac{1}{5}\sqrt{\frac{1}{D}\sum_{i=1}^D x_i^2}} - e^{-\frac{1}{D}\sum_{i=1}^D \cos(2\pi x_i)}$
Griewank	$f(x) = \frac{1}{4000} \sum_{i=1}^D (x_i - 100)^2 - \prod_{i=1}^D \cos(\frac{x_i - 100}{\sqrt{i}}) + 1$
Salomon	$f(x) = \cos(2\pi\sqrt{\sum_{i=1}^D x_i^2}) + 0.1\sqrt{\sum_{i=1}^D x_i^2} + 1$
Rotated hyper-ellipsoid	$f(x) = \sum_{i=1}^D (\sum_{j=1}^i x_j)^2$

Table 2. Functions parameters for the test problems

Functions	Dimension	Initial range
Sphere	30	±500
Rastrigin	30	±500
Step	30	±500
Rosenbrock	30	±500
Ackley	30	±500
Griewank	30	±500
Salomon	30	±500
Rotated hyper-ellipsoid	30	±500

Table 3. Optimized parameters for the test problems

Functions	Inertia weight ω , and	Acceleration coefficient c_1	Acceleration coefficient c_2
Sphere	0.5728	0.6336	0.8422
Rastrigin	0.5908	0.6726	0.9059
Step	0.6539	0.5442	0.6911
Rosenbrock	0.6392	1.2737	0.5954
Ackley	3.0000	3.0000	2.9441
Griewank	0.5901	0.9769	0.7857
Salomon	0.5424	0.3778	0.5264
Rotated hyper-ellipsoid	0.5360	0.8172	0.6147

Table 4. Comparison between standard PSO 2011 and VSPSO 2011

Problem	Method	best	Mean	Std.dev	Worst
Sphere	Standard PSO 2011	1.0482e+005	2.2438e+005	5.6333e+004	4.1521e+005
Sphere	VSPSO 2011	2.4199e+004	7.0329e+004	2.1287e+004	1.2790e+005
Rastrigin	Standard PSO 2011	1.1813e+005	2.2891e+005	4.7518e+004	3.5182e+005
Rastrigin	VSPSO 2011	3.1282e+004	7.6292e+004	2.2688e+004	1.3040e+005
Step	Standard PSO 2011	114834	2.1631e+005	4.3216e+004	345796
Step	VSPSO 2011	27158	7.2645e+004	2.2680e+004	151432
Rosenbrock	Standard PSO 2011	9.3528e+010	4.8421e+011	2.1182e+011	1.2005e+012
Rosenbrock	VSPSO 2011	4.8036e+009	3.5983e+010	2.4238e+010	1.4794e+011
Ackley	Standard PSO 2011	20	20.2424	0.1278	20.5651
Ackley	VSPSO 2011	20	20	0	20
Griewank	Standard PSO 2011	27.3732	57.4863	11.4453	90.7654
Griewank	VSPSO 2011	7.7910	16.8858	4.8584	35.0051
Salomon	Standard PSO 2011	37.5720	47.5551	4.7485	59.1266
Salomon	VSPSO 2011	17.2006	29.1404	4.0430	37.6110
Rotated hyper-ellipsoid	Standard PSO 2011	4.5205e+005	7.6140e+005	1.7838e+005	1.2665e+006
Rotated hyper-ellipsoid	VSPSO 2011	6.8559e+004	1.4094e+005	4.2328e+004	2.7582e+005

4 Conclusion

In this paper, a cask theory based parameter optimization based particle swarm optimization was proposed to find a good set of parameter of. This method can find sets of optimized parameters and using the obtained parameters can achieve better

optimization performance than the standard set of parameters. No prior experience is needed for this method. The simulations showed that the proposed method can achieve good optimization performance comparing with the SPSO 2011. Moreover, the simulations show that it can make sure the worst results are not worse than the worst results of SPSO 2011 as this is cask theory based parameter optimization method. This method can also be used to find the parameters of other optimization algorithms.

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