Maximal ratio combining of two amplify-forward relay branches with individual links experiencing Nakagami fading

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***Abstract*-Relay based communication has gained considerable importance in the recent years. In this paper we find the end-to-end statistics of a two hop non-regenerative relay branch, with each hop being Nakagami-m distributed. Analytical expression for the density function of the signal envelope at the output of a maximal ratio combiner in the destination node is also derived and compared with the simulation results, assuming that the destination node receives the signal through two independent relay paths. These statistics are useful in evaluating the system performance.**

I. INTRODUCTION

Cooperative relaying based communication in wireless environment is emerging as an important technique in the recent times. Wireless relaying allows mobile terminals to participate in the transmission of information, themselves not being the initial source or the final destination [1][2]. Diversity is used to mitigate the effects of fading and therefore increasing the reliability of radio links in wireless networks. The main idea of cooperative diversity schemes is to use relay nodes as virtual antennas to facilitate the communication of one source-destination pair. Potential application areas of cooperation diversity include advanced cellular architectures, mobile wireless ad-hoc networks, wireless sensor networks and others hybrid networks in order to increase coverage, throughput, and capacity to transmit to the actual destination or next relay.

Relays can be broadly categorized as either non-regenerative or regenerative depending on their functionality [3]. For the non-regenerative type, a relay simply amplifies and forwards (A & F) the received signal, while in the latter the relay decodes, encodes, and forwards the received signal. The A & F mode puts less processing burden on the relay and, hence, is often preferable when complexity and/or latency issues are important as these relays are usual mobile terminals which are battery powered and has limited processing capacity. The signal reaching the destination from the source via non regenerative relays passes through N+1 cascaded channels, if there are N number of relays in the path. The signal received at the destination depends on the statistics of all these individual links. Proper utilization of the relay link from the system performance point of view requires a proper knowledge of the statistical behavior of the individual links and the end-to-end channel between the source and the destination via the relays. Analysis of the statistical behavior of relay channels has been a research area of considerable interest and recently some papers dealing with the methods of determining the statistics of such relay have appeared in the literature [3].

In this communication, the end-to-end statistics of a two-hop amplify forward type relay channels, where the individual link experiences Nakagami-m fading [4], have been evaluated. The Nakagami fading being a parameter based distribution makes a justified choice. Nakagami fading model provides the flexibility of changing the individual link statistics by changing the parameter m. For $m=1$, we get the conventional Rayleigh fading model while by taking $m>1$, the channel is made to behave more like a Rician channel. We first determine the density function of the signal received in the destination through such a relay link. While performing analysis we assume the relay to be an ideal noise free repeater with unity gain. Such assumptions set the upper bound on the system performance. In practical systems, the performance will degrade when noise is present at the relay. Next we consider the maximal ratio combining (MRC) of two such two-hop-amplify-forward relay channels where individual links are Nakagami-m faded. We derive the analytical expression for the density function of the received signal at the output of the MRC. The density function obtained analytically is compared for some specific m values with the simulated one.

This paper thus provides an analytical frame work for determining the statistics of a two-hop-amplify-forward relay channels and the MRC of two such relay channels when the individual links are Nakagami-m distributed. The remainder of the paper is organized as follows. Section II presents the end-to-end channel statistics of a relay branch. In section III the probability density function of the signal amplitude at the output of the MRC is derived and compared with the simulation results. Finally, Section IV summarizes the main results of the paper.

II. END-TO-END CHANNEL STATISTICS OF A RELAY BRANCH

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A typical relay based system without any diversity is shown in Fig.1. The signal reaches the destination (D) from the source (S) via a relay node (R). $h_1(t)$ and $h_2(t)$ are the channel statistics between the S-R and R-D link respectively. As mentioned, $h_1(t)$ and $h_2(t)$ are modeled as Nakagami-m distributed so that various fading scenarios such as Rayleigh and Rician can be generated as particular cases of the generalized model. The probability density function (pdf) of the amplitudes of $h_1(t)$ and $h_2(t)$ may be written as [4]

$$f_{R_i}(r_i) = 2\left(\frac{m_i}{\Omega_i}\right)^{m_i} \frac{r_i^{2m_i-1}}{\Gamma(m_i)} \exp\left(-\frac{m_i}{\Omega_i} r_i^2\right) \tag{1}$$

where, $r_i > 0$ & $i = 1,2$. $\Gamma(\bullet)$ is the gamma function,
m_i represents the m parameter of Nakagami-m distribution and $\Omega_i = E\left[r_i^2\right]$.

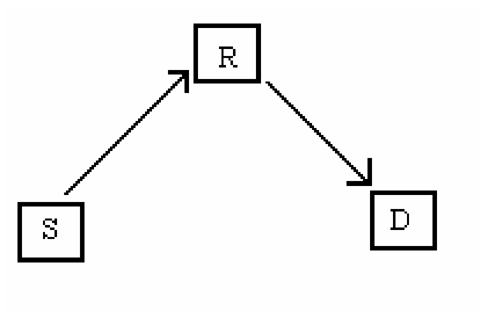

Fig. 1. A typical two hop relay based system.

Let R_1 and R_2 be the random variables representing the amplitudes of the two links S-R and R-D respectively. For the two hop system under consideration, m_1 and m_2 are the Nakagami-m parameters for the channels $h_1(t)$ and $h_2(t)$ respectively. In such a typical 2-hop cooperative relaying environment, the source transmits the information in a time slot $\frac{T}{2}$ and the relay amplifies and retransmits the same information in another time slot $\frac{T}{2}$.For the flat fading case the received signal at the destination node may be written as,

$$y(t) = A(t)h_1(t)h_2(t)x(t) + A(t)h_2(t)n_1(t) + n_2(t) \tag{2}$$

where,
$x(t)$ is the transmitted signal
$A(t)$ is the gain of the relay
$n_1(t)$ and $n_2(t)$ are the additive noise at the relay and the destination respectively.

As we assume $A(t)$ =1 and $n_1(t)$ =0, the received signal at the destination can be written as,

$$y(t) = h_1(t)h_2(t)x(t) + n_2(t) \tag{3}$$

The effective channel between the source and the destination can be represented by the product of two random variables and can be written as $Z = R_1 \cdot R_2$. The density function of the random variable Z gives the channel statistics between the source and the destination via a relay. It has been shown in [5], that the density function of the product of N Nakagami-m random variables Y is given as,

$$f_Y(y) = \frac{2/y}{\prod_{i=1}^{N}\Gamma(m_i)} G_{0,N}^{N,0}\left[y^2 \prod_{i=1}^{N} \frac{m_i}{\Omega_i} \Bigg|_{m_1, m_2}^{-} \right] \tag{4}$$

$G[\bullet]$ denotes the Meijer-G function and $\Gamma(\bullet)$ is the gamma function [6].

Specializing for a two hop system, from (4) it can be shown that the density function of the channel between the source and the destination is given by

$$f_Z(z) = \frac{4}{z\Gamma(m_1)\Gamma(m_2)} \left(\frac{z^2 m_1 m_2}{\Omega_1 \Omega_2}\right)^{\frac{m_1+m_2}{2}} K_{(m_1-m_2)}\left(2\sqrt{\frac{z^2 m_1 m_2}{\Omega_1\Omega_2}}\right) \tag{5}$$

where m_1 and m_2 are the Nakagami parameters of each hop and $K_\nu(\bullet)$ denotes the modified Bessel Function of second kind and order ν [6]. When individual hops are considered to be Rayleigh faded, $m_1 = m_2 = 1$, $f_Z(z)$ reduces to,

$$f_Z(z) = \frac{4z}{\Omega_1\Omega_2} K_0\left(2\sqrt{\frac{z^2}{\Omega_1\Omega_2}}\right)$$

which is same as the density function reported in (7) of [3]

Fig.2. shows a comparison between the density function obtained analytically from (5) with the density function obtained from simulation of Nakagami-m distribution by employing techniques reported in [7]. The Nakagami parameters for the links have been taken as m_1 =1, m_2 =2, Ω_1 =2 and Ω_2 =4. For the rest of the paper we assume unit noise power at the destination node.

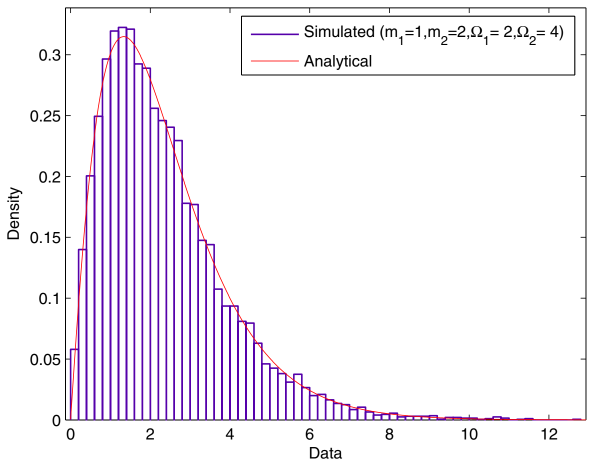

Fig.2. The S-D channel statistics of a two hop relay system

III. Two Branch Maximal Ratio Combining

The performance of a wireless link can be improved by applying combining techniques at the destination. Different combining techniques like selection combining, equal gain combining, maximal ratio combining, etc are generally used. MRC is the optimal technique as it uses each of the available diversity branches in a co-phased and weighted manner to get the highest achievable SNR at the destination. In the present study MRC of signals at the destination has been performed, using the statistics of the channels described in the earlier section. Fig. 3. shows the system model, containing two relays. Here it is assumed that the destination receives the signal only through the relays. The S-D link via relay R1 and that via R2 are assumed to be independent. $f_{Z_1}(z_1)$ and $f_{Z_2}(z_2)$ gives the statistics of the S-D link via relay R1 and R2 respectively. As seen to maintain orthogonality, we require at least three time slots. If in the first slot the source transmits to both the relays. The signal from the two relays reach the destination in two subsequent orthogonal time slots, which are buffered for later processing.

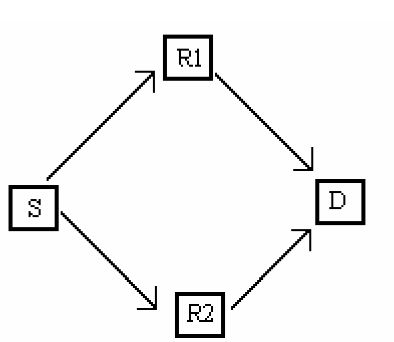

Fig. 3. Two branch dual hop relay diversity links.

The joint probability density function of two independent relay channels is

$$f_{Z_1Z_2}(z_1z_2) = \left[\frac{4}{z_1\Gamma(m_1)\Gamma(m_2)} \left(\frac{z_1^{\,2} m_1 m_2}{\Omega_1\Omega_2} \right)^{\frac{m_1+m_2}{2}} K_{(m_1-m_2)}\left(2\sqrt{\frac{z_1^{\,2} m_1 m_2}{\Omega_1\Omega_2}} \right) \right.$$
$$\left. \bullet \frac{4}{z_2\Gamma(m_3)\Gamma(m_4)} \left(\frac{z_2^{\,2} m_3 m_4}{\Omega_3\Omega_4} \right)^{\frac{m_3+m_4}{2}} K_{(m_3-m_4)}\left(2\sqrt{\frac{z_2^{\,2} m_3 m_4}{\Omega_3\Omega_4}} \right) \right] \quad (6)$$

Equation (6) needs to be integrated to find the probability density function at the output of the maximal ratio combiner. But in Cartesian coordinates the integral becomes very cumbersome. Changing into polar coordinates makes the computations easily tractable [8]. The variables z_1 and z_2 are written as,

$$z_1 = m\cos\phi$$
$$z_2 = m\sin\phi$$

as $z_1 \geq 0$ and $z_2 \geq 0$, so $0 \leq \phi \leq \frac{\pi}{2}$.

Hence the new probability density function can be deduced from (6), by change of variables and introduction of the Jacobian of the transformation. The new density function is defined as,

$$f_{M\Phi}(m,\phi) = \left|\tilde{J}\right| \cdot f_{Z_1Z_2}(z_1, z_2) \quad (7)$$

where $\tilde{J}$ represents the Jacobian.

$$\tilde{J} = \begin{vmatrix} \dfrac{\partial z_1}{\partial m} & \dfrac{\partial z_1}{\partial \phi} \\ \dfrac{\partial z_2}{\partial m} & \dfrac{\partial z_2}{\partial \phi} \end{vmatrix} \quad (8)$$

$$f_{M\Phi}(m,\phi) = m \cdot f_{Z_1Z_2}(z_1, z_2)\Big|_{\substack{z_1 = m\cos\phi \\ z_2 = m\sin\phi}} \quad (9)$$

$$f_M(m) = \int_0^{\pi/2} f_{M\Phi}(m,\phi)\, d\phi \quad (10)$$

$$f_M(m) = C \cdot m^{m_1+m_2+m_3+m_4-1} \int_0^{\pi/2} \frac{1}{\sin\phi\cos\phi} \cdot \cos^{m_1+m_2}(\phi) \cdot \sin^{m_3+m_4}(\phi) \cdot$$
$$K_{(m_1-m_2)}\left(2\sqrt{\frac{m^2\cos^2(\phi) m_1 m_2}{\Omega_1\Omega_2}} \right) \cdot K_{(m_3-m_4)}\left(2\sqrt{\frac{m^2\sin^2(\phi) m_3 m_4}{\Omega_3\Omega_4}} \right) d\phi \quad (11)$$

where,

$$C = \frac{16}{\prod_{i=1}^{4}\Gamma(m_i)}\left(\frac{m_1 m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}}\left(\frac{m_3 m_4}{\Omega_3\Omega_4}\right)^{\frac{m_3+m_4}{2}}$$

The density function at the output of the MRC given by (11) requires to be evaluated numerically for specific values of m_i & Ω_i, where $i = 1,2,..,4$.

For specific case when $m_i = 1$ & $\Omega_i = 2$ for $i = 1...4$, (11) reduces to

$$f_M(m) = C\cdot m^3 \cdot \int_0^{\pi/2} \cos(\phi)\cdot\sin(\phi)\cdot K_0(m\cos(\phi))\cdot K_0(m\sin(\phi))\,d\phi \tag{12}$$

The above integral has been computed numerically to obtain the final density function at the output of maximal ratio combiner.

Fig. 4. gives the plot of the density functions at the output of the maximal ratio combiner for the equation given in (12) as well for the density function obtained through simulation for the same parameters. The simulations were done in MATLAB. The Nakagami-m distributed random variables were generated following the procedure given in [7]. A close match can be seen in Fig. 4.

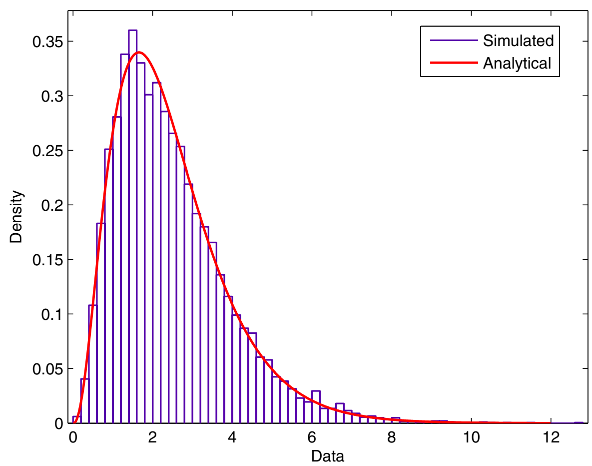

Fig. 4. Analytical and simulated density functions at the output of maximal ratio combiner

The variations of bit error rate (BER) with signal to noise ratio (SNR) are shown in Fig. 5. The channel gains for simulating the BER plot were obtained by two different methods. Rejection method [9] was used to generate random variables satisfying (12). In the second method, the channel gains were obtained directly by separately generating nakagami distributed random variables [7] for each link. The BER performances obtained adopting the two approaches show close match.

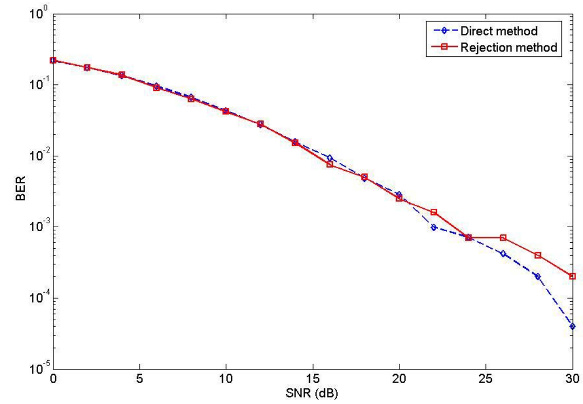

Fig. 5. BER variation with SNR.

IV. Conclusion

In this communication we present the expression for the probability density function of the signal envelope at the output of a maximal ratio combiner, having signals from two independent relay channels as inputs. The channel statistics of the individual hops of a relay diversity branch were assumed to be Nakagami-m distributed. We also evaluate the end-to-end density function of a two hop relay branch, with each hop being Nakagami-m distributed. The analytical results were verified through simulation for particular cases.

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