Performance Analysis of Selection Combining In N Dual-hop Relay Branches With Individual Links Experiencing Nakaganmi-m Fading

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***Abstract*—In recent years relay based communication has gained attention researchers and the scientific community. In this paper the source-to-destination statistics of a two hop amplify-forward relay branch, with the channel fading statistics of each hop being Nakagami-m distributed has been evaluated. The expression for the statistics of the signal envelope at the output of a selection combiner in the destination node of a N-path dual-hop relay braches is derived and compared with the simulation results. The statistics are helpful in evaluating the system performance in terms of bit error rate**

***Keywords*—co-operative relaying, selection combining.**

I. Introduction

In modern wireless communication systems cooperative relaying is emerging as an important technique and has attracted the attention of researchers and scientists. In cooperative relaying the mobile terminals take part in the transmission of information, themselves not being the initial source or the final destination [1][2]. The reliability of the radio links are increased by implementing different diversity combining techniques at the receiver. In cooperative diversity schemes the relay nodes are used as virtual antennas to assist the communication between the source-destination pair. Cooperation diversity techniques are applied in mobile wireless ad-hoc networks, advanced cellular architectures, and hybrid networks in order to increase coverage, throughput, and capacity to transmit to the actual destination or next relay.

Cooperation diversity systems can be broadly categorized into two categories depending on the functionality of the relay, namely non-regenerative and regenerative [3]. In non-regenerative case, the relay just amplifies and forwards (A & F) the received signal, while for non-regenerative scenario the relay decodes, encodes, and forwards the received signal. A&F mode is often used when issue of complexity and/or latency needs to be addressed as the processing burden on the relay is less in the A & F mode. However, in a non-regenerative system, because of the presence of relay nodes between the source and the destination, the statistics of the signal received at the destination node depends on the channel statistics the signal experiences in the individual links. For properly analyzing the relay link in design of a system a good understanding of the statistical behavior of the channels is required. Analysis of statistical behavior of relay channel has been a research area of immense interest and recently some papers dealing with the methods of determining the statistics of such relay have appeared in the literature [3].

The statistics of two-hop amplify forward type relay channels where the individual link experiences Nakagami-m fading is considered first [4][6]. The versatility of the Nakagami fading model lies in its flexibility of changing the individual link statistics by changing the Nakagami parameter m. The conventional Rayleigh fading model is obtained for $m = 1$ while for $m > 1$, the channel is made to behave more like a Rician channel. The density function of the signal received in the destination through such a relay link is determined first. The relay is assumed to be an ideal noise free repeater with unity gain. Such assumptions set the upper bound on the system performance. In practical systems, the performance will degrade when noise is present at the relay. Next selection combining of N such two-hop-amplify-forward relay links where individual links are Nakagami-m faded has been considered. The analytical expression for the density function of the received signal at the output of the selection combiner has been derived. The density function obtained analytically is compared for some specific m values with those obtained from simulation studies.

This paper thus provides a mathematical analysis for determining the statistics of a two-hop-amplify-forward relay channels and the selection combining of N such relay channels when the individual links are Nakagami-m distributed.

The remainder of the paper is organized as follows. Section II presents the end-to-end channel statistics of a two hop relay branch, with individual hops experiencing Nakagami-m fading. In section III the probability density function of the signal amplitude at the output of the selection combiner for a N two-hop relay branches with individual links experiencing Nakiagami-m fading has been derived and compared with the simulation results. Finally, Section IV summarizes the main results and concludes the paper.

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II. Channel Statistics For A Two Hop Relay Branch

A typical relay based system without any diversity is shown in Fig 1. The signal reaches the destination node (D) from the source node (S) via a relay node (R). $h_1(t)$ and $h_2(t)$ are the channel statistics between the source-relay and relay-destination link respectively. $h_1(t)$ and $h_2(t)$ are assumed to be Nakagami-m distributed so that fading scenarios corresponding to different physical environment can be generated as particular cases of the generalized model. Being Nakagami-m distributed the probability density function (pdf) of the amplitudes of $h_1(t)$ and $h_2(t)$ can be written as,

$$f_{R_i}(r_i) = 2\left(\frac{m_i}{\Omega_i}\right)^{m_i} \frac{r_i^{2m_i-1}}{\Gamma(m_i)} \exp\left(-\frac{m_i}{\Omega_i} r_i^2\right) \quad (1)$$

where, $r_i > 0$ & $i = 1,2$. $\Gamma(\square)$ is the gamma function, denotes the m parameter of Nakagami-m distribution and $\Omega_i = E\left[r_i^2\right]$

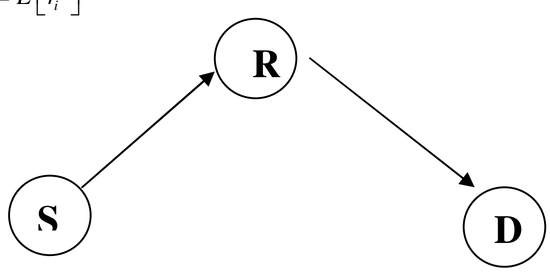

Fig. 1. A typical two-hop relay based system

R_i is the random variables representing the amplitudes of the links. $i = 1$ represents the S-R link whereas $i = 2$ represents the R-D link. m_1 and m_2 are the Nakagami-m parameters for the S-R and R-D channels respectively. In a typical 2-hop cooperative relaying environment with non-regenerative amplify and forward relays, the source transmits the information in a time slot $\frac{T}{2}$ and the relay amplifies and retransmits the same information in another time slot $\frac{T}{2}$. For the flat fading case the signal received by the destination node may be written as,

$$y(t) = A(t)h_1(t)h_2(t)x(t) + A(t)h_2(t)n_1(t) + n_2(t) \quad (2)$$

where, $x(t)$ is the transmitted signal, $A(t)$ is the gain of the relay, $n_1(t)$ and $n_2(t)$ are the additive noise at the relay and the destination nodes respectively.

It is assumed that $A(t) = 1$ and $n_1(t) = 0$. Considering the assumption the received signal at the destination can be written as,

$$y(t) = h_1(t)h_2(t)x(t) + n_2(t) \quad (3)$$

The effective channel between the source and the destination node is basically the product of two random variables and can be written as $Z = R_1 \cdot R_2$. The density function of the random variable Z gives the channel statistics between the source and the destination via a relay. The pdf of Z can be written in terms of the pdf of R_1 and R_2 as [5][8].

$$f_Z(z) = \int_{-\infty}^{\infty} f_{R_1}(r_1) f_{R_2}\left(\frac{z}{r_1}\right) \frac{1}{|r_1|} dr_1 \quad (4)$$

If R_1 and R_2 are Nakagami-m distributed then the random variable Z can be written as,

$$f_Z(z) = \frac{4}{z\Gamma(m_1)\Gamma(m_2)} \left(\frac{z^2 m_1 m_2}{\Omega_1 \Omega_2}\right)^{\frac{m_1+m_2}{2}} K_{m_1-m_2}\left(2\sqrt{\frac{z^2 m_1 m_2}{\Omega_1 \Omega_2}}\right) \quad (5)$$

Where, $K_v(\bullet)$ represents the modified Bessel function of second kind of order v.

III. N-Branch Selection Diversity

Different combining techniques like selection combining, maximal ratio combining, equal gain combining, etc are applied at the destination node for improving the performance of the wireless link. In this paper selection combining of signals at the destination has been performed, using the statistics of the channels described in the earlier section. Fig. 3. shows the system model, containing N relays. Here it has been assumed that the destination receives the signal only through the relays. The S-D link via the relays R_1, R_2, ….., R_N are assumed to be independent. $f_{Z_1}(z_1), f_{Z_2}(z_2), \ldots\ldots\ldots f_{Z_N}(z_N)$ are independent and gives the statistics of the S_D link via relay R_1, R_2, ….., R_N respectively. The signal from the N relays are assumed to reach the destination in N orthogonal time slots, which are buffered for post processing.

The joint probability density function of N- independent relay channels is,

$$f_{Z_1Z_2\ldots\ldots Z_N}(z_1 z_2 \ldots\ldots z_N) = f_{Z_1}(z_1) \bullet f_{Z_2}(z_2) \ldots\ldots\ldots f_{Z_N}(z_N) \quad (6)$$

Combining equation (5) and (6)

$$f_{Z_1Z_2\ldots\ldots Z_N}(z_1 z_2 \ldots\ldots z_N) = (4)^N \prod_{i=1}^{N} \frac{\left(\frac{z_i^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}}\right)^{\frac{m_{2i-1}+m_{2i}}{2}} K_{m_{2i-1}-m_{2i}}\left(2\sqrt{\frac{z_i^2 m_{2i-1} m_{2i}}{\Omega_{2i-1}\Omega_{2i}}}\right)}{z_i \Gamma(m_{2i-1})\Gamma(m_{2i})} \quad (7)$$

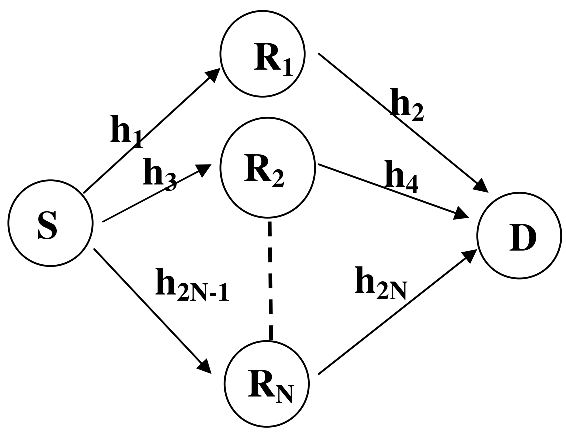

Fig. 2. N- branch dual-hop relay diversity links

In a selection diversity system the output has a signal-to-noise ratio equal to that of the strongest branch. The output signal-to-noise ratio will be equal to that of the i[th] branch, when it has the highest signal-to-noise ratio among the N branches. Under the condition that, all the N-1 branches have signal-to-noise ratio less than or equal to that of i[th] branch. Similarly, the output signal-to-noise ratio will be that of any other branch, when it has the highest signal-to-noise ratio and that of all the other branches is less than or equal to that particular branch. Assuming equal average branch powers, when the envelope of the signal at the output of the combiner is the value s , the envelope of any one of the branches among the N branches is also s and the envelope of the other branches is less than or equal to s . The occurrence that the envelope after selection combining is a value s , is the sum of all the occurrences over z_1, z_2 ……. z_N that produce s as output. The new probability density function $f_S(s)$ describes the distribution of the output envelope of s , and is derived from the distribution of z_1 , z_2 ……. z_N following the approach given in [8]. Mathematically it can be written as,

$$f_S(s)=\sum_{j=1}^{N}\int_0^s\int_0^s....\int_0^s f_{Z_1Z_2.....Z_N}(z_1z_2.....z_N)\Big|_{z_j=s}\,dz_1dz_2....dz_{(j-1)}dz_{(j+1)}....dz_N \quad (8$$

Combining equation (7) and (8) the pdf of the envelope at the output of the selection combiner is,

$$f_S(s)=\sum_{j=1}^{N}\int_0^s\int_0^s....\int_0^s (4)^N\prod_{i=1}^{N}\frac{\left(\frac{z_i^2m_{2i-1}m_{2i}}{\Omega_{2i-1}\Omega_{2i}}\right)^{\frac{m_{2i-1}+m_{2i}}{2}}K_{m_{2i-1}-m_{2i}}\left(2\sqrt{\frac{z_i^2m_{2i-1}m_{2i}}{\Omega_{2i-1}\Omega_{2i}}}\right)}{z_i\Gamma(m_{2i-1})\Gamma(m_{2i})}\Bigg|_{z_j=s}\bullet\; dz_1dz_2....dz_{(j-1)}dz_{(j+1)}....dz_N \quad (9)$$

Example: A three branch relay diversity system

For a three branch case the joint density function is given as,

$$f_{Z_1Z_2Z_3}(z_1z_2z_3)=f_{Z_1}(z_1)\,f_{Z_2}(z_2)\,f_{Z_3}(z_3) \quad (10)$$

Combining equation (7) and (10),

$$f_{Z_1Z_2Z_3}(z_1z_2z_3)=\frac{4}{z_1\Gamma(m_1)\Gamma(m_2)}\left(\frac{z_1^2m_1m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}}K_{m_1-m_2}\left(2\sqrt{\frac{z_1^2m_1m_2}{\Omega_1\Omega_2}}\right)$$
$$\square\frac{4}{z_2\Gamma(m_3)\Gamma(m_4)}\left(\frac{z_2^2m_3m_4}{\Omega_3\Omega_4}\right)^{\frac{m_3+m_4}{2}}K_{m_3-m_4}\left(2\sqrt{\frac{z_2^2m_3m_4}{\Omega_3\Omega_4}}\right)$$
$$\square\frac{4}{z_3\Gamma(m_5)\Gamma(m_6)}\left(\frac{z_3^2m_5m_6}{\Omega_5\Omega_6}\right)^{\frac{m_5+m_6}{2}}K_{m_5-m_6}\left(2\sqrt{\frac{z_3^2m_5m_6}{\Omega_5\Omega_6}}\right)$$

Or,

$$f_{Z_1Z_2Z_3}(z_1z_2z_3)=64\prod_{i=1}^{3}\frac{\left(\frac{z_i^2m_{2i-1}m_{2i}}{\Omega_{2i-1}\Omega_{2i}}\right)^{\frac{m_{2i-1}+m_{2i}}{2}}K_{m_{2i-1}-m_{2i}}\left(2\sqrt{\frac{z_i^2m_{2i-1}m_{2i}}{\Omega_{2i-1}\Omega_{2i}}}\right)}{z_i\Gamma(m_{2i-1})\Gamma(m_{2i})} \quad (11)$$

From equation (8) and (11), the distribution of the envelope at the output of the selection combiner can be written as,

$$f_S(s)=I_1+I_2+I_3 \quad (12)$$

Where,

$$I_1=\frac{64}{\prod_{i=1}^{3}\Gamma(m_{2i-1})\Gamma(m_{2i})}s^{m_1+m_2-1}\left(\frac{m_1m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}}\left(\frac{m_3m_4}{\Omega_3\Omega_4}\right)^{\frac{m_3+m_4}{2}}\left(\frac{m_5m_6}{\Omega_5\Omega_6}\right)^{\frac{m_5+m_6}{2}}\bullet$$
$$K_{(m_1-m_2)}\left(2\sqrt{\frac{s^2m_1m_2}{\Omega_1\Omega_2}}\right)\bullet\int_0^s z_2^{(m_3+m_4-1)}K_{(m_3-m_4)}\left(2\sqrt{\frac{z_2^2m_3m_4}{\Omega_3\Omega_4}}\right)dz_2$$
$$\bullet\int_0^s z_3^{(m_5+m_6-1)}K_{(m_5-m_6)}\left(2\sqrt{\frac{z_3^2m_5m_6}{\Omega_5\Omega_6}}\right)dz_3$$

$$I_2=\frac{64}{\prod_{i=1}^{3}\Gamma(m_{2i-1})\Gamma(m_{2i})}s^{m_3+m_4-1}\left(\frac{m_3m_4}{\Omega_3\Omega_4}\right)^{\frac{m_3+m_4}{2}}\left(\frac{m_1m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}}\left(\frac{m_5m_6}{\Omega_5\Omega_6}\right)^{\frac{m_5+m_6}{2}}\bullet$$
$$K_{(m_3-m_4)}\left(2\sqrt{\frac{s^2m_3m_4}{\Omega_3\Omega_4}}\right)\bullet\int_0^s z_1^{(m_1+m_2-1)}K_{(m_1-m_2)}\left(2\sqrt{\frac{z_1^2m_1m_2}{\Omega_1\Omega_2}}\right)dz_1$$
$$\bullet\int_0^s z_3^{(m_5+m_6-1)}K_{(m_5-m_6)}\left(2\sqrt{\frac{z_3^2m_5m_6}{\Omega_5\Omega_6}}\right)dz_3$$

$$I_3 = \frac{64}{\prod_{i=1}^{3}\Gamma(m_{2i-1})\Gamma(m_{2i})} s^{m_5+m_6-1}\left(\frac{m_5m_6}{\Omega_5\Omega_6}\right)^{\frac{m_5+m_6}{2}}\left(\frac{m_1m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}}\left(\frac{m_3m_4}{\Omega_3\Omega_4}\right)^{\frac{m_3+m_4}{2}} \bullet$$

$$K_{(m_5-m_6)}\left(2\sqrt{\frac{s^2m_5m_6}{\Omega_5\Omega_6}}\right)\bullet\int_0^s z_1^{(m_1+m_2-1)}K_{(m_1-m_2)}\left(2\sqrt{\frac{z_1^{2}m_1m_2}{\Omega_1\Omega_2}}\right)dz_1$$

$$\bullet\int_0^s z_2^{(m_3+m_4-1)}K_{(m_3-m_4)}\left(2\sqrt{\frac{z_2^{2}m_3m_4}{\Omega_3\Omega_4}}\right)dz_2$$

The density functions at the output of the selection combiner for a three branch diversity system given in equation (12) as well for the density function obtained through simulation for the same parameters has been plotted in Fig. 3. The m-parameters of all the six links, of the three branches, were set to unity. $\Omega_1,\Omega_2,\Omega_3,\Omega_4,\Omega_5$ and Ω_6 were taken to be 2. The simulation studies were carried out in MATLAB. The Nakagami-m distributed random variables were generated following the procedure given in [7]. Selection at the receiver was done based on the signal strengths. Fig. 4 plots the bit error rates against signal to noise ratio for two to five branch systems. The improvement of the ber can be on increasing the number of relay branches.

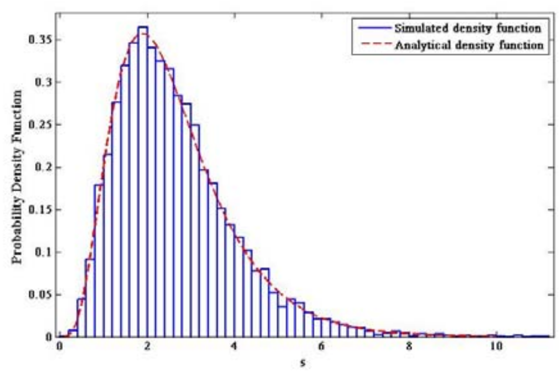

Fig. 3. Analytical and simulated density functions at the output of selection combiner for a three diversity branch system.

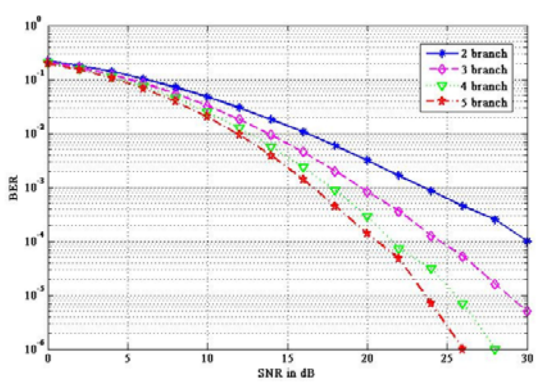

Fig. 4. The bit error rate Vs SNR plot for different number of diversity branches.

IV. Conclusion

In this paper the signal envelope at the output of a selection combiner, having signals from N independent relay channels as inputs has been presented. The validity of the analysis were verified for a three branch diversity case. The channel statistics of the individual hops of a relay diversity branch were assumed to be Nakagami-m distributed. The end-to-end density function of a two hop relay branch, with each hop being Nakagami-m distributed has been evaluated. The analytical results were verified through simulation for particular cases.

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