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# Time and Angle of Arrival Statistics of Mobile-to-mobile Communication Channel Employing Dual Annular Strip Model

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## ABSTRACT

In this paper, a generalized channel model for mobile-to-mobile communication based on the single bounce geometry-based channel modeling techniques has been proposed and analyzed. The model assumes the scatterers to be present in annular strips around the transmitting and the receiving mobile stations. Time of arrival and angle of arrival statistics, being two important channel parameters, have been derived and verified through computer simulations.

### Keywords:

Angle of arrival, Channel modeling, Mobile-to-mobile channel, Time of arrival.

## 1. INTRODUCTION

Recently, with the advent and popularity of wireless ad hoc networks, advanced cellular networks and wireless sensor networks, studies on mobile-to-mobile (M2M) communication and relay-based communication have attracted significant attention of the scientific community [1-4]. In an M2M channel, both the transmitter and the receiver are surrounded by local scatterers, thus making the channel different from a conventional macrocellular channel.

The presence of scatterers around the transmitter modifies the statistics of the physical channel parameters. Moreover, in a wireless channel, multipath restricts the performance by introducing fading in narrow band channels and inter symbol interference (ISI) in wideband systems. The use of antenna arrays and/or directional antennas helps in improving the system performance. The process of beam steering and beam shaping by antenna arrays require prior knowledge of the angle of arrival (AOA) of the desired signal so that the main lobe of the beam can be steered to the direction of the desired signal and beam nulls can be formed in the direction of the interfering signals. The AOA, also known as directional of arrival (DOA), statistics can be obtained from the measured data or from site-specific channel models. The information of the time of arrival (TOA) statistics helps in determining the data rates and symbol periods so as to avoid ISI. Hence, there is a need to develop a statistical model which can characterize the distribution of the TOA and AOA of an M2M channel.

For macrocellular scenario, it is generally assumed that

the AOA at the MS is uniformly distributed in  $[0, 2\pi]$  as the scatterers are likely to surround the mobile from all directions. Moreover, in a macrocellular scenario, BSs are generally positioned on an elevated plane, e.g., hill tops, roof tops etc., which are devoid of scatterers. In the next section, the geometry-based single bounce one ring channel model representing the macrocellular scenario has been extended so as to model generalized M2M channels employing dual annular strip of scatterers. The proposed model also takes into account the scatterers surrounding the transmitter. Analytical expression for TOA and AOA probability density functions for the single bounce geometrical model have been derived and same is verified through simulation studies. Rest of article is arranged as follows: Section II elaborates on model under consideration. Analytical expressions for the TOA and AOA have been derived in Sections III and IV, respectively. Finally, conclusions are drawn in Section V.

## 2. MODEL DESCRIPTION

The M2M channel model under consideration may be considered as a geometry-based, single bounce model as shown in Figure 1.  $M1$  and  $M2$  denote the transmitting and receiving mobile communicating devices, respectively, separated by a distance  $D$ . In this model, scatterers are assumed to be distributed in two annular regions around  $M1$  and  $M2$ . In general, the distribution of scatterers may be arbitrary, and it will affect the TOA and AOA pdfs evaluated using the model. The analysis presented here has been carried out assuming uniform distribution of scatterers.  $R1$ ,  $R2$ ,  $R3$  and  $R4$  denote the inner and outer radii of the scattering

regions around  $M_2$  and  $M_1$ , respectively.  $N_1$  and  $N_2$  represent the number of scatterers at the transmitting and receiving ends. It has been assumed that a ray emanating from the transmitter reaches the receiver only after being scattered by a single scatterer either at the transmitter or at the receiver end. It has also been assumed that all scattered rays that reach the receiver have the same power. The rays reaching the receiver after multiple scattering have been assumed to have very little power compared to the rays reaching the receiver after single scattering. Hence, multiple scattering has not been taken into account. The separation between the transmitter and the receiver has been assumed to be large in comparison with the radii of the scattering regions. This assumption permits the application of geometrical optics and the waves can be represented as rays.

### 3. TIME OF ARRIVAL STATISTICS FOR M2M CHANNELS

In this section, the time of arrival statistics for the M2M channel model described in Section 2 has been derived analytically and verified through computer simulations. The consideration of annular ring of scatterers surrounding the transmitting and the receiving mobile stations provides the flexibility to study models represented by circular rings of scatterers, annular rings of scatterers and disks of scatterers by simply varying the radial dimensions. Further, this model can be useful in the study of different channel statistics when the scatterers are uniformly distributed along the angular dimension but not so in the radial dimension with reference to a polar coordinate system. As shown in Figure 2, any scatterer lying inside or on the constant delay ellipse results in a TOA less than or equal to some time delay, say  $\tau$ . With reference to Figure 2, the hashed areas  $A_1(\tau)$  and  $A_2(\tau)$  contribute to the TOA cumulative distribution function (CDF). The TOA CDF may be written as

$$F_\tau(\tau) = \frac{\text{Number of scatterers contributing to the time delay less than or equal to } \tau}{\text{Total number of scatterers in the system}} \quad (1)$$

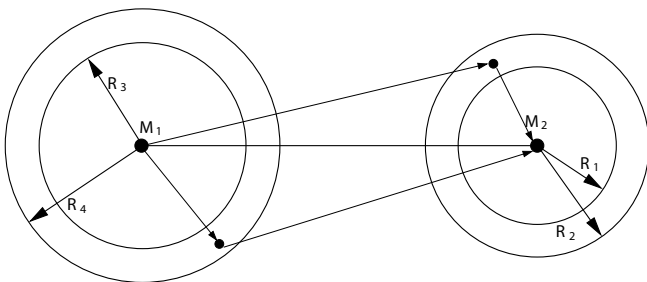


Figure 1: Uniformly distributed circular scattering regions surrounding the mobile nodes modeling M2M propagation environment.

As the scatterers are assumed to be uniformly distributed, the number of scatterers is proportional to the area in which they are present. Hence Equation (1) can be written as

$$F_\tau(\tau) = \frac{\text{Area contributing to the time delay less than or equal to } \tau}{\text{Total area over which the scatterers are present}} \quad (2)$$

With reference to Figure 2, Equation (2) can be written as

$$F_\tau(\tau) = \frac{A_1(\tau) + A_2(\tau)}{\pi(R_2^2 - R_1^2) + \pi(R_4^2 - R_3^2)} \quad (3)$$

The TOA pdf can be obtained on differentiating the TOA CDF (Equation (3)) with respect to  $\tau$

$$f_\tau(\tau) = \frac{1}{\pi(R_2^2 - R_1^2) + \pi(R_4^2 - R_3^2)} \frac{d}{d\tau} (A_1(\tau) + A_2(\tau)) \quad (4)$$

$$f_\tau(\tau) = \frac{1}{\pi(R_2^2 - R_1^2) + \pi(R_4^2 - R_3^2)} \times \frac{d}{d\tau} [\text{Area enclosed in } (ACIA - ABJA) + DFHD - EFG E)] \quad (5)$$

$$f_\tau(\tau) = \frac{1}{\pi(R_2^2 - R_1^2) + \pi(R_4^2 - R_3^2)} \left[ \begin{aligned} &\pi R_4^2 \cdot \left( \frac{1}{\pi R_4^2} \cdot \frac{d}{d\tau} (\text{Area in } ACIA) \right) \\ &- \pi R_3^2 \cdot \left( \frac{1}{\pi R_3^2} \cdot \frac{d}{d\tau} (\text{Area in } ABJA) \right) \\ &+ \pi R_2^2 \cdot \left( \frac{1}{\pi R_2^2} \cdot \frac{d}{d\tau} (\text{Area in } DFHD) \right) \\ &- \pi R_1^2 \cdot \left( \frac{1}{\pi R_1^2} \cdot \frac{d}{d\tau} (\text{Area in } EFG E) \right) \end{aligned} \right] \quad (6)$$

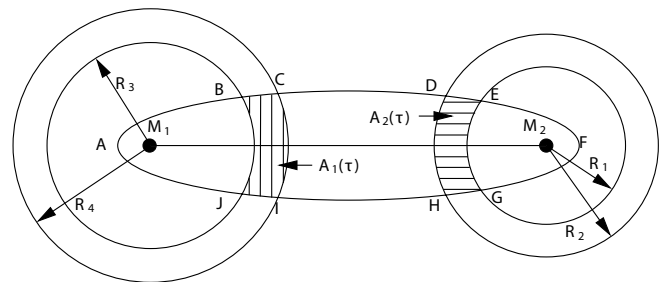


Figure 2: Shaded regions of scatterers for evaluating the TOA CDF for dual annular strip model.

$$f_{\tau}(\tau) = \frac{1}{\pi(R_2^2 + R_4^2 - R_1^2 - R_3^2)} \cdot \left[ \begin{aligned} &\pi(R_4^2 \cdot f_{4\tau}(\tau) - \pi R_3^2 \cdot f_{3\tau}(\tau)) \\ &+ \pi R_2^2 \cdot f_{2\tau}(\tau) - \pi R_1^2 \cdot f_{1\tau}(\tau) \end{aligned} \right] \quad (7)$$

where,  $f_{i\tau}(\tau)$  represents the pdf of the TOA for the macrocellular scenario having  $R_i$  as the radius of the scattering circle and  $i$  can take any of integer values from 1 to 4.  $f_{i\tau}(\tau)$  is given as [5]

$$f_{i\tau}(\tau) = \frac{1}{\pi R^2} \frac{d\Delta A(\tau)}{d\tau} = \frac{c}{\pi R^2} \left[ \begin{aligned} &\frac{\pi\tau^2 c^2 k_2 - \tau c k_2^2 + \pi k_2 k_1^2 + \tau c k_1^2 - 2Rk_1^2}{4k_1 k_2} \\ &\left| \frac{\tau^2 c^2 k_0 k_4 + \tau c k_0 k_1^2}{2k_4^2 + 2k_0^2 k_1^2} \right| \frac{\tau^2 c^2 + k_1^2}{2k_1} \\ &\times \arctan\left(\frac{k_0 k_1}{k_4}\right) - \frac{R - \tau c}{(4R^2 D^2 - k_3^2)^{1/2}} \\ &\times \left( 2R^2 + \frac{\tau c k_1^2 k_4 (1 + k_0^2)}{(2k_4^2 + 2k_0^2 k_1^2)^{1/2}} \right) \end{aligned} \right] \quad (8)$$

where

$$\begin{aligned} k_0 &= \tan\left(\frac{1}{2} \arccos\left(\frac{-\tau^2 c^2 + 2R\tau c}{2RD}\right)\right) \\ k_1 &= \sqrt{\tau^2 c^2 - D^2} \\ k_2 &= \sqrt{D^2 - 4R^2 - \tau^2 c^2 + 4R\tau c} \\ k_3 &= -\tau^2 c^2 + D^2 + 2R\tau c \\ k_4 &= D - \tau c \end{aligned}$$

$c$  = velocity of light,

$R$  = radius of the circle containing scatterers around themobile station and

$D$  = distance between the base station and the mobile station.

Hence, Equation (7) can be written as

$$f_{\tau}(\tau) = \frac{c}{\pi(R_2^2 + R_4^2 - R_1^2 - R_3^2)} \cdot [Z_2 + Z_4 - Z_1 - Z_3] \quad (9)$$

where  $i = 1,2,3,4$ , and  $Z_i$  is given as

$$Z_{i=} \left[ \begin{aligned} &\frac{\pi\tau^2 c^2 k_{2i} - \tau c k_{2i}^2 + \pi k_{2i} k_{1i}^2 + \tau c k_{1i}^2 - 2R_i k_{1i}^2}{4k_{1i} k_{2i}} \\ &\frac{\tau^2 c^2 k_{0i} k_{4i} + \tau c k_{0i} k_{1i}^2}{2k_{4i}^2 + 2k_{0i}^2 k_{1i}^2} + \frac{\tau^2 c^2 + k_{1i}^2}{2k_{1i}} \\ &a \tan\left(\frac{k_{0i} k_{1i}}{k_{4i}}\right) - \frac{R_i - \tau c}{(4R_i^2 D^2 - k_{3i}^2)^{1/2}} \\ &\cdot \left( 2R_i^2 + \frac{\tau c k_{1i}^2 k_{4i} (1 + k_{0i}^2)}{2k_{4i}^2 2k_{0i}^2 k_{1i}^2} \right) \end{aligned} \right] \quad (10)$$

and

$$\begin{aligned} k_{0i} &= \tan\left(\frac{1}{2} a \cos\left(\frac{-\tau^2 c^2 + D^2 + 2R_i \tau c}{2RD}\right)\right) \\ k_{1i} &= \sqrt{\tau^2 c^2 - D^2} \\ k_{2i} &= \sqrt{D^2 - 4R_i^2 - \tau^2 c^2 + 4R_i \tau c} \\ k_{3i} &= -\tau^2 c^2 + D^2 + 2R_i \tau c \\ k_{4i} &= D - \tau c \end{aligned} \quad (11)$$

Simulations have been performed for different values of  $R_1, R_2, R_3$  and  $R_4$ .  $D$ , the distance between the transmitting and the receiving mobile stations has been kept fixed at 1000 m. Figures 3 and 4 show the plot of the simulated results and those obtained from Equation (9). It can be seen that the results are in close agreement, thus verifying the validity of Equation (9).

#### 4. ANGLE OF ARRIVAL STATISTICS FOR M2M CHANNELS

The analysis of the AOA for the dual annular ring model for M2M channel has been derived analytically in this section. The hashed region in Figure 5 gives the area responsible for contributing to the AOA pdf between

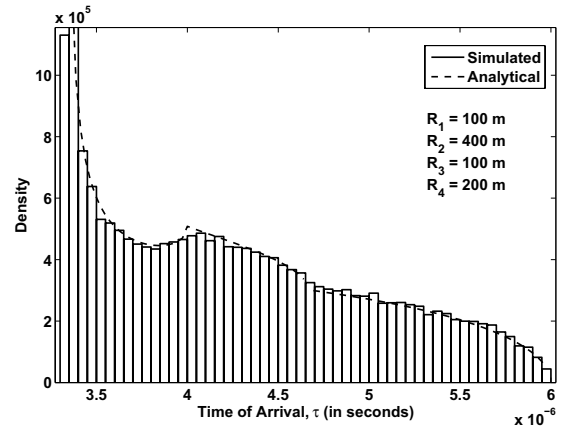
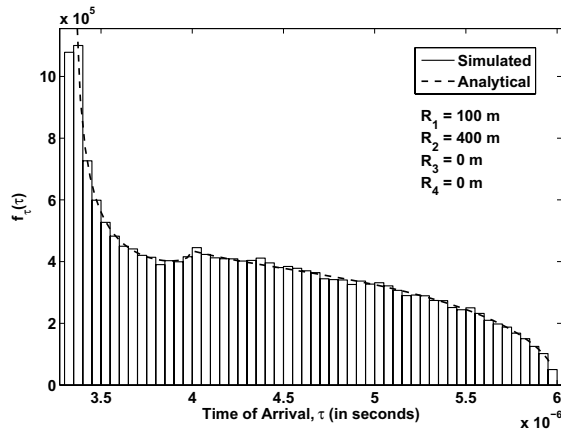


Figure 3: Plot of the theoretical and simulated probability density function of TOA, having annular ring of scatterers around the transmitter and the receiver.



**Figure 4: Plot of the theoretical and simulated probability density function of TOA, having an annular ring of scatterers around the receiver.**

$\theta$  and  $\theta + d\theta$ . The AOA ranges between  $[-\pi, \pi]$  and is symmetrical about  $\theta = 0$ . For the ease in analysis, the angular range is divided into different sections. The ranges are  $\alpha_2 < \theta < \pi$ ,  $\alpha_1 < \theta < \alpha_2$ ,  $-\alpha_1 < \theta < \alpha_1$ ,  $-\alpha_2 < \theta < -\alpha_1$  and  $-\pi < \theta < -\alpha_2$ .

The elementary area  $dA$  contributing to the elementary AOA  $d\theta$  having  $\alpha_2 < \theta < \pi$  is given in Equation (12). The contribution to the AOA is due to the scatterers around the mobile station  $M_1$ .

$$dA = \frac{1}{2}(R_4^2 - R_3^2)d\theta \tag{12}$$

Similarly, for the ranges  $\alpha_1 < \theta < \alpha_2$ ,  $-\alpha_1 < \theta < \alpha_1$ ,  $-\alpha_2 < \theta < -\alpha_1$  and  $-\pi < \theta < -\alpha_2$ , the incremental area  $dA$  contributing to the AOA between  $\theta$  and  $\theta + d\theta$  can be written as

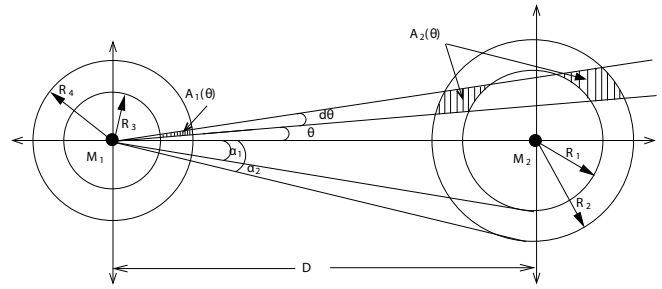
$$dA = 2D \cos(\theta) \sqrt{R_2^2 - D^2 \sin^2(\theta)} d\theta + \frac{1}{2}(R_4^2 - R_3^2) d\theta \quad \alpha_1 \leq \theta \leq \alpha_2 \tag{13}$$

$$dA = \left[ \begin{array}{l} 2D \cos(\theta) \sqrt{R_2^2 - D^2 \sin^2(\theta)} \\ -2D \cos(\theta) \sqrt{R_2^2 - D^2 \sin^2(\theta)} \end{array} \right] d\theta + \frac{1}{2}(R_4^2 - R_3^2) d\theta \quad -\alpha_1 \leq \theta \leq \alpha_1 \tag{14}$$

$$dA = 2D \cos(\theta) \sqrt{R_2^2 - D^2 \sin^2(\theta)} d\theta + \frac{1}{2}(R_4^2 - R_3^2) d\theta \quad -\alpha_2 \leq \theta \leq \alpha_2 \tag{15}$$

$$dA = \frac{1}{2}(R_4^2 - R_3^2) d\theta \quad -\pi \leq \theta \leq \alpha_2 \tag{16}$$

where the first term in Equations (13)-(15) is due to



**Figure 5: Dual annular strip model for determining the AOA pdf for a M2M channel.**

contributions from scatterers surrounding the mobile station  $M_2$ . The term in Equation (16) and the second term of Equations (13)-(15) is contributed by scatterers surrounding  $M_1$ .

The AOA pdf can be written as

$$f_{\theta}(\theta) = \frac{1}{A} \cdot \frac{d}{d\theta} (A_1(\theta) + A_2(\theta)) \tag{17}$$

Substituting the values of  $\frac{dA}{d\theta}$  in Equation (17) from Equations (12)-(16) for different ranges of  $\theta$ , the pdf of AOA can be obtained and is given in Equation (18). In Equation (17),  $A$  is the total area over which the scatterers are uniformly distributed. Thus,  $A = \pi(R_4^2 + R_2^2 - R_3^2 - R_1^2)$ .

$$f_{\theta}(\theta) = \left\{ \begin{array}{ll} \frac{0.5(R_4^2 - R_3^2)}{\pi(R_4^2 + R_2^2 - R_3^2 - R_1^2)} & \alpha_2 \leq \theta \leq \pi \\ \frac{2D \cos(\theta) \sqrt{R_2^2 - D^2 \sin^2(\theta)} + 0.5(R_4^2 - R_3^2)}{\pi(R_4^2 + R_2^2 - R_3^2 - R_1^2)} & \alpha_1 \leq \theta \leq \alpha_2 \\ \frac{2D \cos(\theta) \sqrt{R_2^2 - D^2 \sin^2(\theta)} - 2D \cos(\theta) \sqrt{R_1^2 - D^2 \sin^2(\theta)} + 0.5(R_4^2 - R_3^2)}{\pi(R_4^2 + R_2^2 - R_3^2 - R_1^2)} & -\alpha_1 \leq \theta \leq \alpha_1 \\ \frac{2D \cos(\theta) \sqrt{R_2^2 - D^2 \sin^2(\theta)} + 0.5(R_4^2 - R_3^2)}{\pi(R_4^2 + R_2^2 - R_3^2 - R_1^2)} & -\alpha_2 \leq \theta \leq \alpha_2 \\ \frac{0.5(R_4^2 - R_3^2)}{\pi(R_4^2 + R_2^2 - R_3^2 - R_1^2)} & -\pi \leq \theta \leq \alpha_2 \end{array} \right. \tag{18}$$

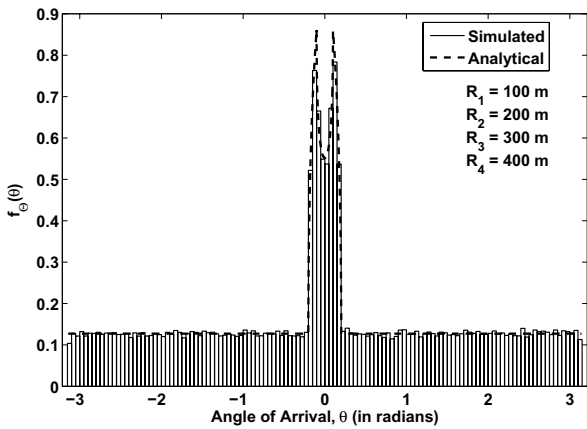


Figure 6: Plot of the theoretical and simulated probability density function of AOA, having annular ring of scatterers of equal width around the transmitter and the receiver.

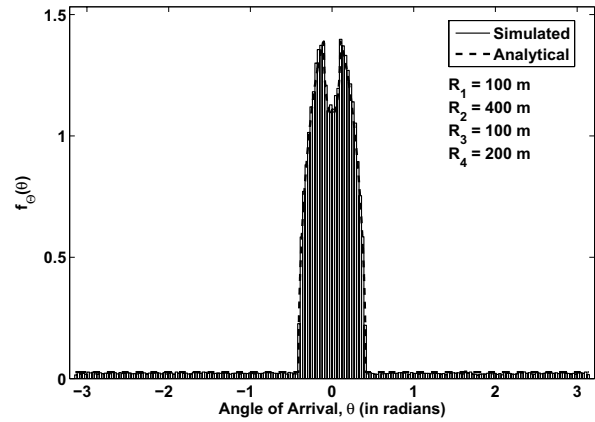


Figure 7: Plot of the theoretical and simulated probability density function of AOA, having annular ring of scatterers of different width around the transmitter and the receiver.

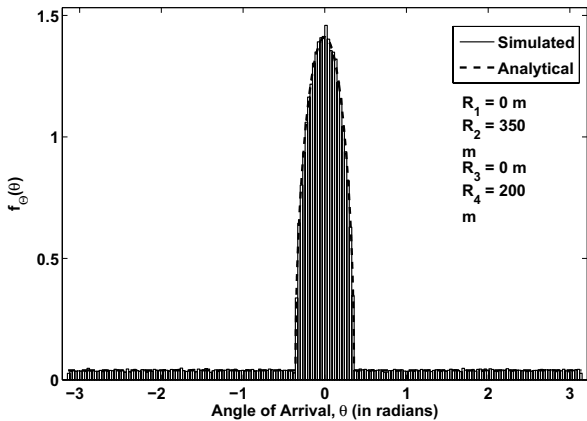


Figure 8: Plot of the theoretical and simulated probability density function of AOA, having a disc of scatterers around the transmitter and the receiver.

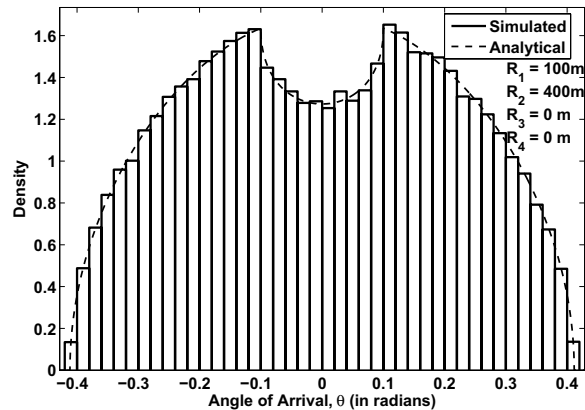


Figure 9: Plot of the theoretical and simulated probability density function of AOA, having an annular ring of scatterers around the receiver.

For simulation purposes, uniformly distributed scatterers have been generated. Plot of the results obtained theoretically from Equation (18) and those obtained through simulation have been shown in Figures 6-9 for different values of model parameters given in the legends of the respective figures. The agreement of the theoretical and the simulated results verifies the validity of Equation (18).

### 5. CONCLUSION

In this article, geometry-based, single bounce channel models for a M2M channel have been discussed. The scatterers have been assumed to be uniformly distributed in circular strips having the transmitter and the receiver located at their centers. The probability density functions for the TOA and the AOA for such M2M channel model have been derived. The derived density functions have been verified through computer simulations. The analytical expressions are useful for the design of M2M

communication systems. The uniform circular disk scattering model can be considered to be a special case of dual annular strip model. The dual annular strip model can also be used for circular scattering model with non uniform distribution of scatterers, for scenarios where scatterers are uniformly distributed along the angular dimension, but have non uniform distribution in the radial dimension, with reference to a polar coordinate system. For this case, the circular scattering region can be segmented into smaller annular strips and a circle of small radius over which the scatterer distribution can be assumed to be uniform.

### 6. ACKNOWLEDGMENT

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