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The Technology of Nonlinear Resonant Pole Converters

by

Arnoldus Stephanus de Beer

THESIS

submitted in partial fulfilment of the requirements for the degree

DOCTOR in ENGINEERING

in

ELECTRICAL and ELECTRONIC ENGINEERING

in the

FACULTY OF ENGINEERING

at the

RAND AFRIKAANS UNIVERSITY

SUPERVISOR : Prof. J.D. van Wyk
CO-SUPERVISOR : Prof. J.A. Ferreira

June 1994
Abstract

This study describes the technology of the Nonlinear Resonant Pole Converter. The NLRP is a soft switching topology that has different advantages over other power electronic converters. This study describes the evolution, analysis, application and design of the NLRP. An introduction and background section with operating principles are provided. Detail analysis with operational limits is given. A chapter is dedicated to control strategy and a novel control scheme is introduced. An overall design strategy with specific steps is proposed. A chapter is dedicated to components and lay-out. The nonlinear inductor and the reduction of its losses are of particular importance. Finally, details of a practical system, results and a conclusion are given.

Opsomming

Hierdie studie beskryf die tegnologie van die Nie-lineêre Resonante Fase-Arm Mutator. Die toepassing van hierdie topologie hou verskeie voordele bo ander drywings-elektroniese mutators in. Hierdie studie omvat die ontwikkeling, analise, toepassing en ontwerp van die Nie-lineêre Resonante Fase-Arm Mutator. 'n Inleiding en agtergrondstudie verskaf inligting met grondbeginsels van funksionering. 'n Volledige analyse word gebruik om funksioneringslimiete daar te stel. Beheer van die mutator word bespreek en 'n nuwe beheerstrategie word voorgestel. 'n Ontwerpstrategie, verdeel in stappe, word voorgestel. Komponente en die uitleg van die mutator word behandeld. Aandag word geskenk aan die vermindering van verliese in die nie-lineêre resonante induktor. Laastens word 'n eksperimentele stelsel beskryf met resultate en 'n opsomming.
The Technology of Nonlinear Resonant Pole Converters
"Ek het tot die insig gekom dat wat God doen, blywend is: jy kan daar niks byvoeg nie en jy kan daar niks van wegneem nie. God doen dit sodat die mens vir Hom ontsag sal hé."

Prediker 3:14
Acknowledgements

My supervisor, Professor Daan van Wyk, from whom I did not only learn academically and technically, but whose life, wisdom and enthusiasm motivated me and many other students.

My co-supervisor, Professor Braham Ferreira, who always had new ideas and whose pursuit of scientific excellence was an inspiration.

The Rand Afrikaans University and Foundation for Research Development, for financial support, without which this study would not have been possible.

My parents (both academics in their own right) for a life-time of commitment, love and support.

My wife, Nicolene, for love and care.
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List of Symbols

\( a \) : Radius of Litz wire bundle
\( A \) : Effective magnetic area
\( A_1 \) : Effective magnetic area of first section of core
\( A_2 \) : Effective magnetic area of second section of core
\( A_{sat} \) : Effective magnetic area of saturated part
\( A_{unsat} \) : Effective magnetic area of unsaturated part
\( B \) : Bandwidth
\( C_{load} \) : Equivalent load capacitance
\( C_{out} \) : Output filter capacitor
\( C_r \) : Resonant capacitor
\( C_{r1} \) : Top resonant capacitor
\( C_{r2} \) : Bottom resonant capacitor
\( C_s \) : Half-bridge source capacitance
\( D_1 \) : Top freewheeling diode
\( D_2 \) : Bottom freewheeling diode
\( f \) : Frequency
\( f_s(m) \) : Switching frequency as function of modulation depth
\( f_{s,max} \) : Maximum switching frequency
\( f_{s,min} \) : Minimum switching frequency
\( i_{c1}(t) \) : Current through top resonant capacitor
\( i_{c2}(t) \) : Current through bottom resonant capacitor
\( i_D \) : Freewheeling diode current
\( i_{err} \) : Error current
\( i_{load}(t) \) : Load current (variable)
\( i_{f1}, i_{f2}(t) \) : Current through resonant inductor (variable)
\( i_{out} \) : Output current
\( i_{ref} \) : Reference current
\( i_s(t) \) : Current through diode and switch
\( i_{switch} \) : Current through a switch
\( i_{load} \) : Load current (constant)
\( i_{load}(s) \) : Load current - frequency domain
\( i_{Lr\_average} \) : Average resonant inductor current
\( i_{on} \) : Current through switch in the "on"-state
\( i_{min} \) : Minimum threshold level for successful resonant transition
\( i_{max} \) : Maximum threshold level for successful resonant transition
\( i_{ref}(s) \) : Reference current - frequency domain
\( i_S \) : Switch current rating
\( i_{thresh} \) : Threshold current value of \( i_{Lr}(t) \) at which a switch is turned off
\( i_{thresh1} \) : Threshold current value of \( i_{Lr}(t) \) at which \( S_1 \) is turned off
\( i_{thresh2} \) : Threshold current value of \( i_{Lr}(t) \) at which \( S_2 \) is turned off
$k_{fb}$: Feedback winding ratio: $k_{fb} = n_{fb}/n_r$

$k_{sat}$: Saturation ratio for $L_r$; $k_{sat} = L_{sat}/L_{unsat}$

$K$: Open loop gain

$K_1$: Open loop gain when a nonlinear controller is present

$K_2$: Integrator gain

$l_1, l_2$: First and second section equivalent magnetic path lengths

$l_{sat}, l_{unsat}$: Equivalent magnetic path lengths of saturated and unsaturated parts

$L_{load}$: Equivalent load inductance

$L_{out}$: Output filter inductance

$L_r$: Resonant inductance

$L_{sat}$: Inductance value of $L_r$ in the saturated state

$L_{unsat}$: Inductance value of $L_r$ in the unsaturated state

$L_{stray}$: Stray inductance

$m, m(t)$: Modulation depth (or modulation index)

$\dot{m}$: Maximum modulation depth

$M$: Number of strands in Litz wire

$N, n$: Number of turns on a winding

$n_{fb}$: Number of turns on the feedback winding

$n_r$: Number of turns on the resonant inductor winding

$p$: Packing factor

$P_{conduction}$: Conduction losses

$P_{loss}$: Total losses in switches

$P_{switching}$: Switching losses

$P_{turn-off}$: Turn-off losses

$P_{turn-on}$: Turn-on losses

$r_s$: Radius of Litz wire strand

$R_{load}$: Equivalent load resistance

$R_s$: Stabilising resistance parallel to $C_s$ (or transfer function parameter)

$S_1$: Top switch of inverter leg

$S_2$: Bottom switch of inverter leg

$S_{3,4}$: Third and fourth switches in a topology

$S_{out}$: Output apparent power

$t_f$: Fall time

$t_{max-res}$: Maximum resonant transition time

$t_r$: Rise time

$t_{res}$: Duration of resonant switchover

$V_{on}$: Forward voltage drop in the "on"-state of a switch

$V_{out}$: Output voltage (DC)

$V_s$: Supply voltage

$v_{c}, v_c(t)$: PWM output voltage

$v_{cs}, v_{cs}(t)$: Source capacitor voltage

$v_{gate}$: Gate drive voltage
\( v_{Lr}(t) \): Voltage across the resonant inductor \( L_r \)

\( v_{out}, v_{out}(t) \): Output voltage (variable)

\( v_{ref} \): Voltage reference

\( v_{ST}(t) \): Voltage across top switch

\( v_{sense} \): Voltage used by gate drive to measure across switch

\( v_{switch} \): Voltage across a switch

\( \delta \): Skin depth

\( \phi_{load} \): Load phase angle

\( \lambda_{load} \): Load power factor

\( \mu_1 \): Permeability of first section of core

\( \mu_2 \): Permeability of second section of core

\( \mu_{sat} \): Permeability of saturated part of core

\( \mu_{unsat} \): Permeability of unsaturated part of core

\( \sigma \): Conductivity

\( \omega \): Angular frequency

\( \omega_r, \omega_{res} \): Resonant frequency

\( \omega_{load} \): Load frequency
Chapter 1

Introduction
Introduction

This study describes the technology of the Nonlinear Resonant Pole (NLRP) Converter. The NLRP is a topology which falls in the broader field of power electronics and electrical energy conversion.

Power electronics is a field of study resulting from a need to control electric power flow. Control of electric power is necessary in different fields of application. These include commerce, industry, transportation, utility systems, aerospace and telecommunications systems.

Electric energy conversion is effected using power semiconductor devices. These include bipolar junction transistors (BJTs), thyristors, metal oxide semiconductor field effect transistors (MOSFETs) and insulated gate bipolar transistors (IGBTs). Each of these devices has its specific range of application, which may vary according to the total power throughput (from a few watts to several hundred megawatts), switching frequency as well as different specified voltages and currents. As prices of power semiconductor devices are high in comparison to the total cost of converters, optimal utilisation of these devices is of concern.

During the electrical conversion processes, efficiency is of primary importance. There are two reasons for this. The first is the ever increasing cost of energy. The second is the management of heat, generated by the wasted energy. The last reason ties in with the utilisation of the devices. If the converter efficiency is low and energy is wasted by the devices, higher ratings in device specifications result. This leads to the use of more or larger devices, with an increase in cost. In this quest for optimal device utilisation and converter efficiency, the Nonlinear Resonant Pole has its place.

The elimination of switching losses and snubbers as well as the possibility of working at higher frequencies and power ratings are attractive characteristics of the NLRP. As with any other converter configuration, the NLRP also has negative points. Whether the NLRP will stand the test of time, depends on how the advantages compare with the weaker points. This can only be determined after thorough investigation and development.

This study is an investigation into the evolution, operating principles, control and design of the NLRP. A theoretical basis is established and practical implementations are described. The aim is to establish the limits of this technology.

The forerunner of the NLRP (the Linear Resonant Pole and its associated topologies) is described in several publications. This led to a solid knowledge base of its operational limits, control and application. This is in contrast with literature on the NLRP, which is limited to only a few aspects of the technology. Specific topics which need to be addressed are:
A general classification of the topology, distinguishing it from other resonant and soft switching configurations.

A description of the limiting factors of operation. These include the range of voltage and load current values for a given set of component values.

Control of the converter. Stability and the controllability of the converter at switching frequency have to be discussed.

Overall low frequency behaviour of the system. This is important for design purposes. If the converter is used as an inverter it supplies a low frequency sinusoidal voltage and current to the load.

Other factors and trade-offs which have to be taken in account in the design of the NLRP converter.

The practical aspects of the construction and design. Since the topology includes additional components (especially the nonlinear inductor) these aspects have to be addressed.

In view of the topics which need to be addressed, this study is divided into the following chapters:

Chapter 1: Introduction - This is the current chapter, stating the purpose and relevance of this study. Specific contributions are also discussed.

Chapter 2: Evolution of the Resonant Pole Converter - Classification of the resonant pole in the family of soft switching topologies. The principles of soft switching are discussed. Attention is given to the development of the load side and source side resonant poles as extensions of the zero voltage switching - clamped voltage topology. Usage of the resonant pole in different DC to DC and DC to AC configurations is shown. Finally, variations on the resonant pole are given, including the Nonlinear Resonant Pole with Feedback Winding - the topology used in this study.

Chapter 3: Operating Principles and Analysis - A qualitative and step by step description of operation is given. The second part is a mathematical analysis of the voltages and currents during one switching cycle. The principles and analysis of an average resonant inductor current - which is fundamental to operation - are given. The switching frequency (which varies with different operating conditions) is analysed. The conditions for successful resonant switchover are discussed and analysed, finally the ripple on the source capacitor voltage, an important parameter in successful operation, is considered.

Chapter 4: Control - Two methods of controlling the NLRP are discussed: current threshold control (which was previously used) and integral threshold control (which is newly introduced). Control of the total system can furthermore be divided into large signal (low frequency) and small signal (switching frequency) analysis. A linearised model for low frequency analysis is developed and used in the overall design. Some aspects of switching frequency analysis are addressed.
Chapter 5: **Dimensioning and Trade-offs** - The overall dimensioning process or design is divided into five aspects. These are output voltage, output power, resonant transition, switching frequency and dynamic response. Each of these subjects is discussed. The trade-offs in determining component values and parameters are shown. A step by step design procedure is given in the last section of this chapter.

Chapter 6: **Converter Structure and Components** - Two practical aspects of construction are discussed - firstly the design of the saturable resonant inductor and secondly the physical circuit layout. Minimisation of core and winding losses in the inductor is described. This is done by considering the constraints of current shape, frequency and saturation. A method of saturating using only part of the core is suggested. The last section shows how the circuit has to be laid out to reduce parasitic oscillations and voltage overshoots.

Chapter 7: **Simulation and Results** - Operation at switching frequency is simulated with a PSPICE model. It is based on a load-current and -voltage point. The experimental set-up is described and includes the control system, gate drives and start-up circuit. Different sets of measurements and results are given.

Chapter 8: **Conclusion** - This chapter is a discussion of results of this study. The limits of this technology are discussed. Topics for future work are given.

Four appendixes are given at the end of the study. They contain technical specifications of the experimental converter, mathematical manipulations, and a listing of the PSPICE simulation code. Several sheets of numerical calculations - done with the MathCAD™ computational package - are also included.

Specific contributions of this study are:
- The systematic classification of the NLRP and its evolution from zero voltage switching topologies (Chapter 2).
- An analytical description of currents and voltages during the resonant transition interval (Chapter 3-2-2).
- Estimation of the switching frequency - taking into account the effect of load current (Chapter 3-4-1).
- Description of the limits in load voltage and current for successful resonant switchover (Chapter 3-5).
- The development of an Integral Threshold Controller and a description of its advantages (Chapter 4-3).
- Development of a linearised low frequency model which is used in over-all dimensioning (Chapter 4-4).
- Description of the steps and trade-offs in the design of a NLRP converter (Chapter 5).
- The reduction of core losses by only saturating a part of the core (Chapter 6-1-2).
- The circuit simulation for operation at switching frequencies (Chapter 7-1).
The next section (chapter 2) deals with the classification and evolution of the NLRP.
Chapter 2

Evolution of the Resonant Pole Converter
Evolution of the Resonant Pole Converter

In this chapter the Nonlinear Resonant Pole (NLRP) is classified in the range of soft switching topologies. It is shown how the NLRP evolved. Different implementations, e.g. DC to DC and DC to AC converters, are discussed. Variations of the resonant pole are given, including the Nonlinear Resonant Pole with Feedback Winding - the topology used in this study.

2-1 Hard switching vs. soft switching

Power electronic converters convert one form of electrical energy into another. In this process losses occur. In power conditioning, attempts are made to reduce losses as far as possible. Soft switching converters find application in this quest for minimum losses.

Power semiconductor switches are used in the electrical energy conversion process. Examples are: thyristors, insulated gate bipolar transistors (IGBTs), bipolar transistors (BJTs) and field effect transistors (FETs).

These switches contribute largely to losses so that different switching schemes and topologies are used to optimise the conversion process.

Losses in switches can be divided into conduction \( P_{\text{conduction}} \) and switching losses \( P_{\text{switching}} \). Therefore:

\[
P_{\text{losses}} = P_{\text{conduction}} + P_{\text{switching}}
\]  

(2-1)

where \( P_{\text{losses}} \) is the total power loss of the switch, including all driving, snubbing and other protection circuits.

The difference between switching and conduction losses can be explained by looking at the switching waveforms of an ideal switch. Figure 2-1 shows such an ideal switch and figure 2-2 the switching waveforms.

In figure 2-2 it is seen that the switch is usually in an on or off state. During the off state the current through the switch is zero while a voltage drop \( V_s \) (say the supply voltage) exists across the switch. In the on state the switch will conduct a current \( I_{\text{on}} \). Practical switches have voltage drops across the device during conduction \( V_{\text{on}} \). These are usually in the range of a few millivolts to a few volts. This forward voltage drop during the on-state produces conduction losses.
Switching losses occur during switchover from the on or off state to another state. These losses are subdivided into turn-on ($P_{\text{turn-on}}$) and turn-off ($P_{\text{turn-off}}$) losses so that:

$$P_{\text{switching}} = P_{\text{turn-on}} + P_{\text{turn-off}}$$  \hspace{1cm} (2-2)

The time average of the voltage current product determines the losses or:

$$P_{\text{losses}} = \frac{1}{T} \int_0^T i_{\text{switch}}(t) \cdot v_{\text{switch}}(t) \, dt$$  \hspace{1cm} (2-3)

Figure 2-2 also shows a voltage current product curve. The shaded areas represent the losses.

In practical systems there are usually not much that can be done to prevent conduction losses. Conduction losses are primarily a function of the type of switch used (i.e. IGBTs, BJTs, etc.) and are therefore fixed.
Switching losses are reduced by employing different switching schemes, different topologies or by adding snubber circuits. The term soft switching is used in systems where switching losses have been reduced or eliminated to practically zero. This is opposed to hard switching where considerable switching losses occur.

Soft switching schemes are employed to increase overall converter efficiency. Another advantage is that the overall voltage and current stresses for the device are reduced.

For every switching device there is a safe operating area (SOA). The switching loci in the $v_{\text{switch}}(t)$-$i_{\text{switch}}(t)$ plane must be within the SOA. In figure 2-3 a SOA is given together with the switching loci of typical hard and soft switching devices [29]. If the switching stresses during hard switching exceed the SOA then soft switching must be employed if the same devices are to be used.

![Diagram of SOA and switching loci](https://example.com/diagram.png)

**Figure 2-3: Switching loci and safe operating area (SOA).**

Soft switching converters are classified as zero voltage switching (ZVS) or zero current switching (ZCS). With zero voltage switching, the voltage $V_s$ across the switch is zero during switching intervals. During zero current switching, the current is zero. If either the current or voltage is zero there are no switching losses.

The question now arises: how can soft switching be achieved? It seems that either snubber or resonant circuits has to be used.

Snubbers are extra circuits added to the converter which can either be dissipative or regenerative. In dissipative snubbers, switching losses are "extracted" from the devices and dissipated in a resistance. In regenerative snubbers this "extracted" or "snubbed" energy is fed back into the system. Both these types of snubbers can be complex in terms of added circuitry [27].
Recent advances in soft switching include resonant circuits. In these converters extra circuits are kept to a minimum and soft switching is achieved by the resonance of either $v_{\text{switch}}(t)$ or $i_{\text{switch}}(t)$, employing inductances and capacitances. These types of converters are regenerative in nature. Only extra capacitors and inductors are added to the generic hard switching topologies.

According to Divan, Venkataramanan and de Doncker [11]: *Soft switched converters are characterised by intrinsic modes of operation which allow an automatic and lossless resetting of the snubber elements through an appropriate recirculation of trapped energy.*

A diagram summarising the evolution of hard switching to soft switching converters is given in figure 2-4.

2-2 General classification of soft switching converters

Converters are broadly categorised as hard switching or soft switching. This chapter deals with soft switching converters and specifically those which consist of phase arms or poles.

A phase arm or pole is a generic building block and consists of two switches and usually also two diodes. In figure 2-5, some converters are shown, making use of the generic converter leg (or pole). The points between the two switch/diode pairs, are switched between the positive and negative rail of the DC supply. Depending on the type of switching scheme employed, the average voltage output can be a variable DC voltage or a typical sine wave, variable in frequency and amplitude.
Figure 2-5: Converter topologies, incorporating the generic converter leg or "pole".

Figure 2-6: Classification of soft switching converters
From figure 2-6 it can be seen that soft switching converters are divided into four main groups. These include load resonant, resonant link, high frequency integral half cycle and resonant-switch converters [29]. The resonant pole is classified under resonant switch converters and will be dealt with later in this chapter. The other three categories of soft-switching inverters will be dealt with as background to the rest of this study.

Load-resonant converters use the load as a resonant tank. The converter switches change state when either the load current or load voltage passes through zero. Figure 2-7 shows a series-load resonant DC-DC converter [30] and figure 2-8 a parallel loaded resonant DC-DC converter [26]. In both these converters, resonance between $L_r$ and $C_r$ leads to soft switching.

Both converters usually have an isolation transformer before the rectifier, which is omitted here for simplicity. Capacitors $C_s$ are very large and used to complete the half-bridge configuration. The detailed description of these two converters are beyond the scope of this study. It would suffice to say that depending on the mode of operation, zero voltage or zero current switching, or both, occur during switchover from one diode/switch pair to another [34], [23].

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**Figure 2-7: Series-loaded Resonant DC-DC Converter**

**Figure 2-8: Parallel-loaded Resonant DC-DC Converter**
The group of resonant-link converters is often confused with the resonant-switch group under which the resonant pole is classified. Resonant-link converters achieve zero voltage switching by resonating the link voltage about a certain DC level. This differs from the resonant-switch converters in which resonance only takes place during switchover from one switch/diode pair to another. Figure 2-9 shows the simplest form of the resonant DC-link converter [10]. The DC link voltage that resonates because of $C_r$ and $L_r$ is also given.

![Resonant DC-link converter diagram]

Figure 2-9: The Resonant DC-link converter. Soft switching takes place when $v_f$ is zero.

The third group of soft-switching converters are the high-frequency link, integral half cycle converters [31]. Figure 2-10 shows a basic diagram of such a converter. Unlike the resonant-link converter [10] - where the input resonates between zero and two times the supply voltage - the input to these converters is high frequency AC. The converter incorporates a parallel resonant tank, so that higher current components need not be drawn from the HF supply. The six-pulse bridge is bi-directional, using bi-directional switches. Usually switches with enough blocking capacity are used in anti-parallel. Input and output waveforms are shown in figure 2-10. The low frequency output is synthesised from an integral number of half cycle input waveforms. Switching only takes place when $V_{in}$ is zero and therefore zero voltage switching is achieved.
The last group of soft switching converters are those of the resonant-switch variety. They are also known as quasi-resonant, pseudo-resonant or resonant-transition converters [29],[30],[21],[20]. The main feature that distinguishes this group is that resonance only take place during switching from one state to another.

Resonant transition converters are not only limited to inverter leg topologies. Single switch resonant transition topologies include variations of the buck, boost and other DC-DC converters [25]. These will not be considered, as the emphasis in this study is on "poles" or converter leg topologies (figure 2-5).

![Figure 2-10: High-frequency link integral-half-cycle converter.](image)

Figure 2-11 shows a typical voltage output of a PWM converter. If hard switching takes place, the voltage falls or rises directly from one voltage rail to another. With resonant transition converters, the voltage swings in a resonant fashion from the one rail to another. The resonant transition process will be dealt with in the next section.
2-3 Principles of resonant switching and the ZVS-CV topology

In this section the zero voltage switching-clamped voltage (ZVS-CV) topology is discussed. This generic topology is used to derive all the resonant pole converters.

In figure 2-5 hard switching takes place as no snubbers or resonant components are present.

In a ZVS-CV topology (figure 2-12) soft switching takes place. This is achieved by resonating the output voltage from the positive to negative rail or vice versa. Resonance only occurs during the transition from one switch/diode pair to another. These topologies are therefore referred to as pseudo- or quasi-resonant or resonant transition topologies.

The operation of the ZVS-CV DC to DC converter of figure 2-12 is described as follows:

- Assume that the output capacitor \( C_{\text{out}} \) and load resistor \( R_{\text{load}} \) can be substituted by a DC voltage source \( V_{\text{dc}} \) (figure 2-12). Furthermore assume that \( S_1 \) is conducting and that a positive output current \( i_{\text{out}} \) is flowing. Figure 2-13(a) summarises this situation. The full input voltage is across \( C_{r2} \). The voltage across \( C_{r1} \) is zero, as \( S_1 \) is conducting.
At a certain point, determined by the control system, $S_1$ is switched off. Zero voltage switching is achieved as the voltage across $S_1$ is practically zero. This situation is shown in figure 2-13(b). Not one of the devices $S_1$, $S_2$, $D_1$ or $D_2$ are conducting. The voltage $V_{\text{out}}$ now resonates between the positive and negative DC-rail. Resonance takes place between $C_{r1}$, $C_{r2}$ and $L_r$. By using a Thevinen equivalent circuit and recognising that one is now dealing with small signal AC performance, the resonance circuit can be simplified (figure 2-13(c)). In this arrangement the current through $L_r$ cannot change instantaneously. This current is now equally supplied by $C_{r1}$ and $C_{r2}$. Current flowing from $C_{r2}$ will decrease the voltage across $C_{r2}$, while current flowing from...
$C_{r1}$ will increase the voltage across $C_{r1}$. This trend will continue until the voltage across $C_{r2}$ is zero and the voltage across $C_{r1} = V_S$.

- As $V_{out}$ reaches the negative rail, $D_2$ will start conducting. This is again zero voltage switching as $V_{out}$ is practically zero when $D_2$ starts conducting. This situation is shown in figure 2-13(d). It is important to note that the voltage $V_{out}$ will never exceed or fall below the positive or negative rails. This is due to the clamping action of the diodes and hence the name "CV" or clamped voltage in the ZVS-CV topology.

- The moment $D_2$ starts conducting, $S_2$ is turned on. $S_2$ will conduct if $i_{out}$ is negative.

![Diagrams](image)

Figure 2-13: Various stages in the operation of a ZVS-V DC to DC converter

From the ZVS-CV topology described in this section, the evolution of the resonant pole converter follows.
2-4 Evolution of the resonant pole

In section 2-3 the principles of zero voltage switching were illustrated using a DC-DC converter. In figure 2-14 it is shown how the ZVS-CV DC-DC converter is extended to form a DC-AC converter.

The sinusoidal output waveforms (V_{out}) are produced using a PWM switching scheme. Typical frequencies for V_{out} would be ten to a few hundred Hz while the switching frequency would typically be in the kHz region.

2-4-1 Load side resonant pole

A disadvantage of the resonant pole shown in figure 2-14 is that the total load current must flow through L_r. In configurations where it is not necessary to filter the PWM voltage output, an added resonant inductor L_r can increase losses as well as costs.

These configurations are called load-side resonant poles, as the resonant components are between the switches and the load (figure 2-15).

Figure 2-14: Evolution of the resonant pole from a ZVS-CV DC-DC topology to a DC-AC converter
2-4-2 Source side resonant pole

The topology of the source side resonant pole was introduced by Patterson and Divan [30]. It was developed to reduce the rms rating of $L_r$ as well as the peak currents flowing through the switches. This has the advantage of fewer losses and lower costs. In the source side resonant pole, the resonating components are between the source and the switches.

The evolution of the resonant pole from power transfer through the resonating inductor (load side) to direct power transfer (source side) is shown in figure 2-15. The current flowing through $L_r$ is also shown. The resonant inductor for the load side pole will have a higher rms rating than that of the source side pole, because of the extra load current component.

2-4-3 Nonlinear resonant pole

The nonlinear resonant pole, which was introduced by Ferreira, Van Ross and Van Wyk [17], [16], is similar to the source side resonant pole of figure 2-16. The only difference is that the inductor saturates just before resonant switchover. This has the advantage of a resonant inductor with lower rms rating. Current stresses on the switches and losses are also reduced.

As the Nonlinear Resonant Pole is the subject of this study, a detailed description of its operation is given in chapter 3.

Figure 2-15: Evolution of the load side resonant pole to the source side resonant pole.
2-5 Basic Topologies

The purpose of this section is to illustrate the different uses of the resonant pole, spanning the range of generic DC-DC and DC-AC (single and three phase) topologies.

The basic topologies of the resonant pole are divided into two main categories. The first category is the fixed ratio of DC to DC converters or so called DC transformers. Instead of having two capacitors $C_S$ on the source side of the half bridge, these converters have two fixed sources of value $V_s/2$. The fixed ratio DC to DC converters are not able to produce PWM waveforms. This fact distinguishes them from the second category of variable DC-DC and DC-AC converters which are able to output a variable voltage by implementing a PWM scheme.

2-5-1 Fixed Ratio DC to DC converters

The resonant pole principle is used in the three main topologies of DC-DC converters. Figures 2-17, 2-18 and 2-19 show these configurations.

Figure 2-17 represents the half-bridge configuration, figure 2-18 the push-pull and figure 2-19 the full-bridge configuration.
2-5-2 Variable output DC to DC and DC to AC converters

In the group of fixed ratio DC-DC converters the duty cycle of the high frequency output is constant at 50%. To vary the duty cycle at the output (i.e. perform PWM), the voltage source $V_s$ is replaced by two capacitors $C_s$. Looking at the source side resonant pole (with the two resonant capacitors $C_r$ placed over the switches) of figure 2-15, it will theoretically be possible for the node between the two capacitors to be at any voltage smaller than $V_s$. In steady state operation there is a zero average voltage across the resonant inductor $L_r$. This implies that the average voltage output of the resonant pole always equals the voltage of the node between the two capacitors $C_s$. 

Figure 2-19: Full-bridge nonlinear fixed ratio DC-DC converter

Figure 2-20: Nonlinear Resonant pole DC-DC (variable output) converter
Resonant pole variable output DC to DC converters are of the kind shown in figure 2-20. In contrast to single switch DC to DC converters, the resonant pole version must consist of at least two fully controlled switches. There are certain applications at high power levels requiring reduction of EMI and switching losses where the resonant pole DC-DC topology would be a viable option, but usually single controlled switch DC-DC converters are used.

A typical application of the linear (or nonlinear) resonant pole would be a three phase inverter which is voltage fed from a six pulse rectifier. This concept can be extended to include AC-AC converters as shown in figure 2-21. The generic converter leg topology used in this illustration is that of the nonlinear resonant pole with feedback winding and is discussed in the following section. The three phase AC outputs are obtained by controlling the resonant pole so that the voltage nodes between the half-bridge capacitors vary with the desired average output.

Figure 2-21: Nonlinear resonant pole 3-phase AC-AC converter

2-6 Variations of the Resonant Pole Converter

In this section three variations on the resonant pole are described. Each of these variations includes extra components which are used to improve the overall efficiency of the inverter.
2-6-1 The Nonlinear Resonant Pole with Feedback Winding

This type of inverter is the same as described in section 2-4-3 (figure 2-16), except for the addition of a feedback winding on the load side. This feedback winding has the effect of adding a current component to the current through the resonant inductor.

Figure 2-22 shows a circuit diagram and a phase graph of the current through the resonant inductor. In the Nonlinear Resonant Pole without feedback winding, large negative and positive current peaks occur for $L_r$. With feedback winding the positive or negative peaks are reduced - depending on the direction of the load current. By reducing peaks in $i_{Lr}$, switching device stresses as well as winding losses are reduced.

Varying the voltage of the node between the two capacitors $C_s$ changes the average output voltage. Since the average voltage across the inductor equals zero, the average load voltage equals the voltage of the node between the two capacitors $C_s$. 

Page 2-18
This can be done by adding an external current source to this node [17] or switching $S_1$ and $S_2$ in such a way that a nett current flows into or out of the capacitors.

This topology is the subject of this study. A detailed description is given in chapter 3.

2-6-2 The Auxiliary Resonant Commutated Pole

The operation of the auxiliary resonant commutated pole (ARCP) [6] closely resembles that of the nonlinear resonant pole. The resonant inductor ($L_r$) is switched into the circuit, just before the resonant transition (figure 2-23). Either $S_3$ or $S_4$ is turned on, depending on the direction of the resonant inductor current. In the auxiliary commutated pole, the current between resonant intervals is zero. This is in contrast to other resonant poles where there is always a current through the inductor.

The advantage of the ARCP is that current stresses on the switching devices and resonant inductor are reduced. Since resonant switchover is not dependent on the saturation of the inductor, the switching frequency can be regulated. However, the added complexity of two extra switches and their additional conduction losses are a disadvantage.
The True PWM Zero Voltage Switching Pole

In both the nonlinear resonant pole and ARCP, capacitors \((C_s)\) form an artificial voltage source and complete a half bridge configuration. In the case of the True PWM Zero Voltage Switching Pole [2] (figure 2-24) these half bridge source capacitors are replaced by a circuit incorporating an autotransformer, switches and diodes. During the resonant interval the 2N part of the transformer is connected through a diode to the \(V_S\) supply. The autotransformer function therefore ensures that the point at the load side of \(L_r\) is kept at \(V_S/2\) as \(S_4\) is closed.

![Figure 2-24: Diagram of one phase arm of the "True PWM Zero Voltage Switching Pole"

The apparent advantage of this circuit is that no source capacitors are used. The inclusion of an autotransformer and extra switches are a disadvantage. Since some form of capacitance is always present in a voltage fed system it seems that the nonlinear resonant pole or ARCP would be a better choice.

Double and Triple Nonlinear Resonant Pole

It was recently shown [8],[7] that the substitution of nonlinear capacitors for \(C_r\) reduces parasitic effects and increases the overall performance of the inverter. This form of double nonlinear resonant pole is shown in figure 2-25.

Just as the inductance of a nonlinear inductor depends on the current through it, the capacitance of a nonlinear capacitor is a function of the voltage across the capacitor: the capacitance decreases with an increase in voltage. A nonlinear capacitor thus fits the characteristics of an ideal snubber capacitor much better. At turn-off, the voltage across the switch is kept low longer than with a linear capacitor. Turn-off losses are therefore reduced.
It was also suggested [7] that nonlinear capacitors be used for the half-bridge source capacitors $C_s$. This will lead to the formation of a triple nonlinear resonant pole (figure 2-26). It has the advantage of reduced parasitic effects. Added to this would be the reduced ripple of the voltage between the two source capacitors $C_s$ - an important factor in the successful operation of the NLRP.

In this chapter the evolution and categorisation of the NLRP were discussed. The principles of ZVS-CV were also given. Following this broad background, chapter 3 is a detailed description of the mechanisms and restrictions of NLRP operation.
Chapter 3

Operating Principles and Analysis
Operating Principles and Analysis

In chapter 2 the evolution of the resonant pole was discussed. A qualitative discussion of the operation of the generic topology and its variations was included. Chapter 3 deals in detail with the operating principles and analytical description of the NLRP.

The first section covers the operation from a qualitative point of view. In section two, solutions for the differential equations of the circuit are found. These are used to describe current and voltage as functions of time. Section three treats the resonant inductor current, its average value and associated parameters. This knowledge is essential, as control of the resonant inductor current is fundamental to the operation of the resonant pole. The variation in switching frequency (an important design criterion) is discussed in section four. Successful commutation and the restraints it places on operation are investigated in section five. The last section gives a description of the source capacitor voltage $v_{Cs}(t)$ and its variation during operation.

3-1 Operation

In this section a qualitative discussion of the operation of the NLRP with feedback winding (figure 3-1) is given. It is assumed that the switches are ideal and that the passive components are without losses. The resonant inductor $L_r$ saturates and two distinctive inductance values can be distinguished: in the unsaturated state $L_r = L_{unsat}$ and in the saturated state, $L_r = L_{sat}$.

![Figure 3-1: Single phase nonlinear resonant pole with feedback winding](image)
3-1-1 Operation at No Load and Zero Modulation

Fundamental to the operation of the nonlinear resonant pole is the commutation process where switchover from one diode/switch pair to another takes place. This is also called resonant transition or resonant switchover.

To understand the commutation process, it is necessary to look at the resonant pole without any load or feedback winding and at zero modulation depth. Zero modulation depth implies that $v_{CS}(t)=0$ V. Such a simplified circuit is shown in figure 3-2. The output voltage waveform ($v_{out}(t)$) and resonant inductor current ($i_{LR}(t)$) are shown in figure 3-3.

The operation of the Nonlinear Resonant Pole without feedback winding (figures 3-2 and 3-3) is as follows:

- Assume that $S_1$ or $D_1$ is conducting. This situation is shown in figure 3-4 and corresponds with the intervals $[t_0; t_2]$ and $[t_8; t_{10}]$. At the instant $t_0$, the voltage across $L_r$ is $+V_S/2$, since $v_{CS}(t)=0$. The output voltage is also $+V_S/2$. The current rises positively through $L_r$ until saturation of the core has taken place. After saturation at $t_1$, the current rises sharply (the inductance is now smaller after saturation) until time $t_2$. At a specific current level which has been preset in the controller ($I_{thresh1}$ in figure 3-3), $S_1$ is switched off at $t_2$. This threshold at which $S_1$ is turned off must be at least of such a value that sufficient energy is stored in the inductor for complete commutation. Complete commutation can only take place if the inductive energy stored in $L_r$ is sufficient to let $v_{out}(t)$ swing all the way from $+V_S/2$ to $-V_S/2$ or vice versa. If this is not the case, the opposite diode will not start to conduct and zero voltage switching will be lost.
Figure 3-3: Voltage output and resonant inductor current for the nonlinear resonant pole without any load or feedback winding.

Figure 3-4: Configuration of the nonlinear resonant pole without load or feedback winding if switch S1 or diode D1 is conducting (Intervals [t₀;t₂] and [t₈;t₁₀] in figure 3-3).
At time $t_2$ (figure 3-3), resonant switchover starts. This configuration is shown in figure 3.5. During switchover neither of the switch/diode pairs are conducting. The output voltage swings between the positive and negative rails from $+V_{s}/2$ to $-V_{s}/2$. As it reaches $-V_{s}/2$, D2 will start conducting. This happens at $t_3$. $V_{out}(t)$ will swing all the way to $-V_{s}/2$ (complete commutation), provided that sufficient energy has been stored in $L_r$.

![Diagram](image)

Figure 3-5: Configuration of the nonlinear resonant pole without load or feedback winding during resonance (Intervals $[t_2; t_3]$ and $[t_7; t_8]$ in figure 3-3).

At time $t_3$ (figure 3-3), D2 starts to conduct the moment a large enough positive forward voltage is applied across the diode. Current now flows through the resonant inductor via D2. This configuration is shown in figure 3-6. As the current decreases, the resonant inductor desaturates at time $t_4$. Since D2 is conducting the voltage across S2 is almost zero. At this time the inductor current is still positive and D2 is conducting, S2 can now be turned on in anticipation of $i_{Lr}(t)$ becoming negative.

At time $t_5$, S2 takes over the current. This happens the moment the inductor current becomes negative. The inductor current decreases until saturation occurs at $t_6$. A resonant transition once again takes place when S2 is switched off at a predetermined level of the inductor current ($i_{threshold2}$). Resonance occurs during the interval $t_7$ to $t_8$. At $t_{8}$ D1 starts conducting after the output voltage has swung from $-V_{s}/2$ to $+V_{s}/2$. S1 is switched on while D1 conducts and there is zero voltage across the switch. After $t_8$, when S1 takes over the current from D1, the complete switching cycle is repeated.
3-1-2 Operation at No Load and Varying Modulation

The next concept to be discussed is the variation in duty cycle or modulation depth. Modulation depth is important for two reasons:
- It determines the pulse width of $v_{out}(t)$ and therefore the average output voltage.
- It determines the fundamental shape of the inductor current $i_{Lr}(t)$ during one switching cycle.

Assume that the modulation depth ($m$) is a parameter that varies between -1 and 1. Furthermore assume that the voltage $v_{CS}(t)$ can be written in terms of the modulation depth and the DC input voltage so that:

$$ v_{CS}(t) = m \cdot \frac{V_s}{2} $$

(3-1)

This means that as $m$ varies between -1 and 1, $v_{CS}(t)$ varies between the two rails $+V_s/2$ and $-V_s/2$.

As $v_{CS}(t)$ and $m$ vary, the voltage $v_{Lr}(t)$ across the resonant inductor during one switching cycle has two distinctive values. This is shown in figure 3-7. Since switchover is at a constant threshold value of $i_{Lr}(t)$, the output duty cycle varies in direct relationship to $m$. The average output voltage is directly proportional to the pulse width of $v_{out}(t)$ and therefore to $m$. This implies that the average output voltage equals $m \cdot V_s/2$. From equation 3-1 it follows that the average output voltage therefore equals $v_{CS}(t)$.

Another explanation is as follows: During steady state operation the average current over one switching cycle flowing through $L_r$ is zero. This implies a zero average...
voltage across \( L_r \) during one switching cycle. The average of \( v_{\text{out}}(t) \) therefore equals \( v_{Cs}(t) \).

In figure 3-7, the voltage across the resonant inductor \( v_{Lr}(t) \) is shown as a function of the modulation depth \( m \). If S1 or D1 is conducting, the voltage across \( L_r \) is \((1-m)V_s/2\). If S2 or D2 is conducting, the voltage across \( L_r \) is \(-(1+m)V_s/2\). Figure 3-7 clearly shows the points at which the inductor saturates, as well as the points where the switches are turned off (at \( +I_{\text{thresh}} \) and \( -I_{\text{thresh}} \)) to start the resonant transition period.

![Figure 3-7: Voltage across the resonant inductor and current through \( L_r \) for a modulation depth \( m \).](image)

The modulation depth is therefore an important parameter in describing the relationship between the output voltage \( v_{Cs}(t) \) and the supply voltage \( V_s \). From figure 3-7 it is clear that the modulation depth also determines the shape of \( i_{Lr}(t) \) during one switching cycle.

### 3-1-3 Operation with Load and without Feedback Winding

The load current is now taken into account, but the feedback winding is omitted. Such a circuit is shown in figure 3-8. During the on-state of the diodes or switches, operation with a load current flowing is similar to that of a normal PWM bridge. It is however during resonant switchover that the influence of the load current must be carefully considered. During commutation, the load current can either assist or work against successful resonant switch-over.
It is assumed that the output voltage is switched in such a fashion that a low frequency sinusoidal current flows through the load. The frequency of the load current is in practice much smaller than the switching frequency. This implies that one can assume that the output current will stay constant during one switching cycle. This is especially true if the load is largely inductive.

During the on-state of the diodes or switches, the load current is similar to that of a normal PWM bridge. If the load current is positive (as defined in figure 3-8) and if it is required by the control system that the output voltage must be +V_s/2, S1 will be conducting. The load current will flow from the positive rail of the supply via S1. If the controller determines that the output voltage must be -V_s/2, D2 will be supplying the load current. The same applies for S2 and D1 if the load current is negative.

During resonant switchover the influence of the load current can be divided into two categories. The first is where the load current assists in the resonant transition process. The second is where the load current works against the commutation process.

In the first case the threshold value of i_L(t) for which a switch is turned off, is not important. This is because the load current and not i_L(t) is responsible for swinging v_out(t) from one rail to another. In the second case, the threshold value of i_L(t) must carefully be chosen.

The difference between the load current assisting and working against commutation is explained with the aid of figure 3-9. Whether it is assisting or not is determined by the direction of the load current and whether a transition from S1/D1 to S2/D2 is
taking place or vice versa. The difference between these two cases (figure 3-9), is explained with S1 conducting.

First consider the case for which the load current assists with commutation or switchover (Figure 3-9(a)). Say S1 is conducting; the load current is constant and positive, and switchover has to take place so that D2 will start conducting. While S1 is conducting, the voltage across the top resonant capacitor is zero and a voltage $V_S$ is across the bottom $C_r$.

With switchover, the output voltage $v_{out}(t)$ swings from the top to the bottom DC rail. During this resonant process, currents $i_{Cr1}(t)$ and $i_{Cr2}(t)$ flow as indicated in figure 3-9(a). The load current is constant ($i_{load}(t) = I_{load}$) and flows in the same direction as $i_{Cr1}(t)$ and $i_{Cr2}(t)$. It therefore assists the resonant transition process. In this situation, the threshold value of inductor current $i_{Lr}(t)$ at which S1 is turned off, does not have a significant influence on the commutation process.

For the same situation of switchover but with $i_{load}(t)$ negative, the load current will work against the resonance process. This is illustrated in figure 3-9(b).

The load current is constant during the switchover period and has to flow through $L_r$. The resonant capacitor currents $i_{Cr1}(t)$ and $i_{Cr2}(t)$ must also flow through $L_r$. If this does not happen, $v_{out}(t)$ will not swing from the positive to negative DC rail and
resonant switchover will not be complete. The threshold value of $i_{LR}(t)$ at which $S_1$ is switched off must therefore be equal to the load current plus an extra value that will allow for complete resonant transition. If this condition is not met, the constant load current prevents $i_{Cr1}(t)$ and $i_{Cr2}(t)$ from flowing in the right direction and resonant switchover is not complete.

The opposite applies when $S_1$ or $D_2$ is conducting. If the load current is negative (figure 3-8) it will assist resonant transition from $S_2/D_2$ to $S_1/D_1$. If the load current is positive it will not assist resonant transition from $S_2/D_2$ to $S_1/D_1$.

The constraint on the inductor current, requiring it (under certain conditions) to be equal to the sum of the load current plus a value required for successful resonant switchover, is an important factor in the design of the control system. This will be discussed in more detail later on in this chapter.

3-1-4 Operation with Load and Feedback Winding

The feedback winding is a modification of the generic topology of the nonlinear resonant pole. The motivation for the added feedback winding is twofold. Firstly the peak current through the switches is reduced. Secondly the total rms current through, and therefore the losses in, the resonant inductor are reduced. By feeding the load current back via an extra winding, a shifting component is added to the resonant inductor current $i_{LR}(t)$.

![Figure 3-10: Typical resonant inductor current waveform for the nonlinear resonant pole with feedback winding (refer to figure 3-1).](image)
A typical resonant inductor waveform is shown in figure 3-10. The value of the component added to the usual inductor current is proportional to the feedback winding factor and load current. The feedback winding factor \( k_{fb} \) is determined by the ratio of the number of turns on the feedback winding and number of turns on the resonant inductor winding. In mathematical form:

\[
k_{fb} = \frac{n_{fb}}{n_r}
\]

where \( n_{fb} \) is the number of turns on the feedback winding and \( n_r \) the number of turns on the resonant winding.

It is important to note that the areas underneath the positive and negative sides of \( i_{Lr}(t) \) (figure 3-10) are equal. This implies a zero average value for the inductor current. This condition must be adhered to for steady state operation. If this does not happen, an average current not equal to zero will charge capacitors \( C_s \) (figure 3-1). This will lead to a shift in voltage \( v_{C_s}(t) \) and the converter will no longer be in a steady state condition.

The fact that capacitors \( C_s \) can be charged or discharged using the non-zero average of the inductor current will be explained in chapter 4 where the dynamic response of the converter will be dealt with. The conditions for which the average resonant inductor current equals zero are described in section 3-3.

### 3-2 Analysis

It is useful to describe the operation of the converter in analytical terms. Equations and results described in this section are used for detailed analysis and design.

In order to keep the analysis simple, but precise enough to describe fundamental behaviour, a few assumptions regarding the system are made.

**Assumptions for analysis:**
- All switches and diodes are assumed to be ideal, i.e. the voltage of all the diodes and switches during forward conduction are assumed to be zero.
- There is no reverse recovery of the diodes.
- Voltage sources are perfect and have a constant value of \( V_g/2 \).
- The load current \( I_{load} \) is constant during the switching cycle that is analysed.
- The inductor is ideal and there are no losses in the inductor and feedback winding.
- There are no losses in the capacitors.
- A step inductance is assumed when the resonant inductor saturates.
- The magnetic flux linkage does not change if \( L_r \) saturates.
For analysis, the operation of the converter is divided into three states. Each state corresponds to the change in topology due to the change in switches or diodes conducting. The three states are: S1 or D1 conducting, S2 or D2 conducting and when resonant switchover takes place. During switchover or commutation neither of the two switches nor the two diodes are conducting.

There are three variables that describe the operation of the converter. These are the resonant inductor current $i_{\text{r}}(t)$, voltage $v_{\text{C}}(t)$ and the output voltage $v_{\text{C}}(t)$ ($v_{\text{load}}(t)$ in figure 3-8). If one of the switches or diodes is conducting, then $v_{\text{C}}(t)$ is fixed at either $+V_s/2$ or $-V_s/2$.

The nonlinear inductor is modelled with a two step inductance of either $L_{\text{sat}}$ or $L_{\text{unsat}}$. The feedback winding is modelled as a current source proportional to the load current by a factor $k_{\text{fb}}$. The load current is assumed constant over one switching cycle. A complete description and derivation of the equations can be found in [7].

The following equations are an adaptation of that work, except for the equations during the resonant transition period. The equations in [7] and [33] do not include the effect of the feedback winding so that it is necessary to consider the complete derivation in the present analysis. The derivations of the equations during resonant transition mode can be found in appendix D.

### 3-2-1 S1 or D1 Conducting

In this analysis it is assumed that the load current is constant over one switching cycle. It is furthermore assumed that the effect of the feedback winding is to add an extra, constant current component $k_{\text{fb}}I_{\text{load}}$ to $i_{\text{r}}(t)$. The total magnetic component is modelled as a nonlinear inductor with an extra current source that injects a component $k_{\text{fb}}I_{\text{load}}$. This is shown as part of figure 3-11. The component that flows through the nonlinear inductor part is $i_{\text{l}}(t)$ so that:

$$i_{\text{r}}(t) = i_{\text{l}}(t) + k_{\text{fb}}I_{\text{load}}$$  \hspace{1cm} (3-3)

In sections 3-2-1 to 3-2-3 only the current $i_{\text{l}}(t)$ is given. This is in accordance with references [7] and [33]. The total resonant inductor current can be found from (3-3).

Consider the node A between $C_{\text{s1}}$ and $C_{\text{s2}}$ (figure 3-11). Applying Kirchhoff’s current and voltage loop rules, the following differential equations relate $v_{\text{C}}(t)$, $I_{\text{load}}$ and $I_{\text{l}}(t)$:

$$v_{\text{C}}(t) = \frac{V_s}{2} - L_{\text{r}} \frac{di_{\text{r}}(t)}{dt}$$

$$i_{\text{C1}}(t) = -C_{\text{s1}} \frac{dv_{\text{C1}}(t)}{dt}$$

$$i_{\text{C2}}(t) = C_{\text{s2}} \frac{dv_{\text{C2}}(t)}{dt}$$

$$I_{\text{l}}(t) = i_{\text{C2}}(t) - i_{\text{C1}}(t) - k_{\text{fb}}I_{\text{load}}$$  \hspace{1cm} (3-4)
When Laplace transforming equation (3-4) by substituting and performing an inverse Laplace transform, the following solutions for the differential equations of (3-4) are obtained:

\[ v_{C_1}(t) = [v_{C_1}(t_0) - \frac{V_s}{2}] \cos(\omega(t-t_0)) + \frac{k_p I_{load} + i_L(t_0)}{2C_s\omega} \sin(\omega(t-t_0)) + \frac{V_s}{2} \]  

(3-5)

\[ i_L(t) = [i_L(t_0) + k_p I_{load}] \cos(\omega(t-t_0)) - \frac{V_{cs}(t_0)}{\omega L_r} \sin(\omega(t-t_0)) - k_p I_{load} \]  

(3-6)

where: \( \omega^2 = \frac{1}{2C_s L_r} \)  

(3-7)

The resonant inductor value \( L_r \) can either be \( L_{unsat} \) or \( L_{sat} \). The complete derivation of the above equations together with the Laplace transformations can be found in appendix C of [7].

3-2-2 Resonant Transition

In the resonant period not one of the switches or diodes is conducting. The differential equations for the equivalent resonant circuit of figure 3-12 are:

\[ v_c(t) - v_{CS}(t) = L_r \frac{di_L(t)}{dt} \]

\[ i_{CS1}(t) = -C_{S1} \frac{dv_{CS}(t)}{dt} \]
Through Laplace transforms and inverse transforms the following time functions for $i_L(t)$, $v_{CS}(t)$ and $v_C(t)$ are obtained (see appendix D for details):

$$i_L(t) = \left\{ \begin{array}{l}
i_L(t_0) + I_{load} \left[ \frac{C_s}{C_s + C_r} + k_f \right] \cos[\omega(t - t_0)] \\
+ \frac{1}{\omega L_r} [v_c(t_0) - v_{Cs}(t_0)] \sin[\omega(t - t_0)] - I_{load} \left[ \frac{C_s}{C_s + C_r} + k_f \right] \end{array} \right.$$

(3-9)
where:

\[ \omega^2 = \frac{C_s + C_r}{2C_r C_s L_r} \]  

### 3-2-3 S₂ or D₂ Conducting

Considering the node A between \( C_{S1} \) and \( C_{S2} \) (figure 3-13) and applying Kirchhoff's current node and voltage loop rules, the following equations describing the relationship between \( v_{cs}(t) \), \( i_{load} \) and \( i_L(t) \) can be derived:

\[ v_{cs}(t) = \frac{V_s}{2r} \frac{dt}{dt} - \frac{L}{2} \frac{d}{dt} i_{L}(t) \]

\[ i_{C_{S1}}(t) = -C_{S1} \frac{dv_{cs}(t)}{dt} \]

\[ i_{C_{S2}}(t) = C_{S2} \frac{dv_{cs}(t)}{dt} \]

\[ i_L(t) = i_{C_{S2}}(t) - i_{C_{S1}}(t) - k_f b I_{load} \]  

Using Laplace transformations the following time equations for \( v_{CS}(t) \) and \( i_L(t) \) are obtained:

\[ v_{CS} = \left[ v_{CS}(t_0) + \frac{V_s}{2} \right] \cos[\omega(t - t_0)] + \frac{k_f b I_{load} + i_L(t_0)}{2C_{S2}\omega} \sin[\omega(t - t_0)] - \frac{V_s}{2} \]  

\[ i_L(t) = \left[ i_L(t_0) + k_f b I_{load} \right] \cos[\omega(t - t_0)] - \frac{v_{CS}(t_0) + \frac{V_s}{2}}{\omega L_r} \sin[\omega(t - t_0)] - k_f b I_{load} \]
where:

\[ \omega^2 = \frac{1}{2C_s L_r} \]  

(3-16)

Figure 3-13: Equivalent circuit for analysis of the system when S2 or D2 is conducting

The resonant inductor value \( L_r \) can be either \( L_{\text{unsat}} \) or \( L_{\text{sat}} \). The complete derivation of the above equations as well as the Laplace transformations can be found in appendix C of [7].

3-3 Switching Frequency

The switching frequency of the NLRP can be calculated from the different time intervals of the resonant inductor current. In this study the effect of the load current (via the feedback winding) and threshold values are taken into account. This is an extension of the work done in [33].

3-3-1 Linear Approximation

The equations of section 3-2 lead to long and transient functions of the resonant inductor current. To obtain a simple analytical solution for the switching frequency, a linearised model (figure 3-14) is used. It is assumed that the inductor current is linear between the different threshold and saturation values. This would be true if the capacitors \( C_s \) are infinitely large and act as perfect DC voltage sources.
Figure 3-14: Linear approximation of the resonant inductor current for estimation of the switching frequency and average resonant inductor current.

To approximate the influence of the resonant transition time, a flat current crest of time length $t_{\text{res}}$ is specified. It is shown in the next section what influence $t_{\text{res}}$ has on the accuracy of the estimation.

The switching frequency is the inverse of the sum of the time intervals $t_0$ to $t_8$ in figure 3-14. The different time intervals can be calculated from $v_{Lr}(t) = \frac{L}{\Delta t}$, where $v_{Lr}(t)$ is the voltage across $L_r$, $\Delta I$ the difference in current and $\Delta t$ the length of the time interval. The lengths of the time intervals are:

\[
\begin{align*}
(t_1 - t_0) &= 2L_{\text{sat}} \frac{I_{\text{thresh}2} + k_{fb}I_{\text{load}} - I_{\text{sat}}}{(1-m)V_s} \\
(t_2 - t_1) &= 4L_{\text{unsat}} \frac{I_{\text{sat}}}{(1-m)V_s} \\
(t_3 - t_2) &= 2L_{\text{sat}} \frac{I_{\text{thresh}1} - k_{fb}I_{\text{load}} - I_{\text{sat}}}{(1-m)V_s} \\
(t_4 - t_3) &= t_{\text{res}} \\
(t_5 - t_4) &= 2L_{\text{sat}} \frac{I_{\text{thresh}1} - k_{fb}I_{\text{load}} - I_{\text{sat}}}{(1+m)V_s} \\
(t_6 - t_5) &= 4L_{\text{unsat}} \frac{I_{\text{sat}}}{(1+m)V_s} \\
(t_7 - t_6) &= 2L_{\text{sat}} \frac{I_{\text{thresh}2} + k_{fb}I_{\text{load}} - I_{\text{sat}}}{(1+m)V_s} \\
(t_8 - t_7) &= t_{\text{res}} \\
\end{align*}
\]
As shown in Appendix B-2 the switching frequency is:

\[
f_s(m) = \frac{1}{((t_1 - t_2) + (t_2 - t_1) + (t_3 - t_2) + (t_4 - t_3) + (t_5 - t_4) + (t_6 - t_5) + (t_7 - t_6) + (t_8 - t_7))}
\]

(3-18)

\[
f_s(m) = \frac{V_s(1 - m^2)}{8L_{sw}} \left[ k_{sat} \left( \frac{I_{thresh1}}{2} - \frac{I_{thresh2}}{2} - I_{sat} \right) + 2t_{res} V_s(1 - m^2) \right]
\]

(3-19)

The threshold levels \(I_{thresh1}\) and \(I_{thresh2}\) at which \(S1\) and \(S2\) are turned off are not the same. Two different levels have to be used as the current waveform \(i_{Lr}(t)\) is asymmetrical about the zero line. This is due to the effect of the feedback winding.

Also included in the approximation is the length of the resonant transition interval \(t_{res}\). This can be estimated from the expression for \(v_c(t)\), (equation (3-11)) and assuming \(C_r \rightarrow \infty\). As shown in figure 3-19, \(t_{res}\) is the time it will take \(v_c(t)\) to swing from the positive to the negative rail.

This resonant transition period can be expressed as:

\[
t_{res} = 2 \left[ \frac{1}{2 m V_s \omega_{res} C_r} \left( I_{load} - I_{thresh} \right) \left( I_{load} - I_{thresh1} \right) + 4 V_s^2 \omega_{res}^2 C_r m \right]
\]

(3-20)

for switchover from \(S1\) to \(D2\) and:

\[
t_{res} = 2 \left[ \frac{1}{2 m V_s \omega_{res} C_r} \left( I_{load} - I_{thresh2} \right) \left( I_{load} - I_{thresh2} \right) + 4 V_s^2 \omega_{res}^2 C_r^2 \right]
\]

(3-20)

for switchover from \(S2\) to \(D1\), where: \(\omega_{res} = \frac{1}{\sqrt{2 L_{sw} C_r}}\)

Figure 3-15 shows the switching frequency as function of modulation depth using equation 3-19 (see Appendix B-2). Curves are shown for different load currents. The parameters used are:

\[
V_s = 200 \text{ V} \quad I_{sat} = 0.8 \text{ A} \quad L_{sw} = 1.35 \text{ mH} \quad I_{thresh1} = 2 \text{ A} \quad k_{sat} = 65
\]

\[
C_s = 3.3 \mu \text{F} \quad C_r = 24 \text{ nF} \quad k_{fb} = 0.25
\]
From figure 3-15 it can be seen that the switching frequency is a parabolic function of modulation depth. The switching frequency is also dependent on the load current. If load current increases, the switching frequency drops.

The accuracy of this linear approximation is discussed in the next section.

3-3-2 Estimation Accuracy

In the above approximation of switching frequency a linear model of resonant inductor current was used. This differs from the shape of the true resonant inductor current. From the equations in section 3-2 it can be seen that the current is described by cosine and sine terms (and even there, the effects of losses were neglected). In this section it will be shown that the linear estimate differs from the analytical waveforms.

Figures 3-16 and 3-17 show the difference in the linearised and analytical models of the resonant inductor current. The analytical model was taken from equations in section 3-2. The graphs were prepared using a mathematical computer package. The listings of the programs used can be found in Appendix B.
The parameters used for figure 3-16 are:

\[ V_s = 200 \text{ V} \quad I_{sat} = 0.8 \text{ A} \quad L_{sat} = 1.35 \text{ mH} \quad I_{\text{thresh}} = 2 \text{ A} \quad k_{th} = 65 \]

\[ C_s = 3.3 \mu\text{F} \quad C_r = 24 \text{ nF} \quad k_{fr} = 0.25 \quad I_{\text{load}} = 0 \text{ A} \quad m = 0 \]

\[ t_{res} = \pi \sqrt{2L_{sat}C_r} = 3.1 \mu\text{S} \]

In figure 3-16 the waveforms are shown for zero load current and zero modulation depth. The period of the estimated waveform is longer than the analytical one. This can be attributed to the estimation of the resonant time. It therefore seems that at zero load and modulation depth the comparison between analytical and linear approximation is good.

In figure 3-16 the waveforms are shown for zero load current and zero modulation depth. The period of the estimated waveform is longer than the analytical one. This can be attributed to the estimation of the resonant time. It therefore seems that at zero load and modulation depth the comparison between analytical and linear approximation is good.

Figure 3-17 shows the situation where the converter is in deep modulation while supplying a load current. The parameters used for generating the waveforms of figure 3-17 are as follows (note here that the resonant time was chosen to be zero):

\[ V_s = 200 \text{ V} \quad I_{sat} = 0.8 \text{ A} \quad L_{sat} = 1.35 \text{ mH} \quad I_{\text{thresh}} = 12 \text{ A} \quad k_{th} = 65 \]

\[ C_s = 3.3 \mu\text{F} \quad C_r = 24 \text{ nF} \quad k_{fr} = 0.25 \quad I_{\text{load}} = 3 \text{ A} \quad m = 0.8 \quad t_{res} = 0 \]
From Figure 3-17 it can be seen that the periods of the analytical and linearly estimated resonant inductor waveforms differ. It was found that the period of the linear approximation is larger than the analytical period with increase in modulation depth as well as load current.

Figure 3-17: Difference in estimated and analytical waveforms of resonant inductor current with load current and large modulation depth (Current in A; t in sec)

Figure 3-18: Difference in the switching frequency as function of modulation depth (m) from a linear estimation and analytical point of view.
It is qualitatively concluded that the switching frequency (as function of modulation depth) is not parabolic but tends toward the shape shown in figure 3-18. The resonant transition time is omitted at deep modulation and large load current, because the linear approximation yields a switching period that is too long (lower switching frequency).

From figure 3-17 it can therefore be concluded that the switching frequencies as shown in figure 3-15 are only rough estimations and can only serve as broad design guidelines.

3-4 Average Resonant Inductor Current

Fundamental to the behaviour and control of the nonlinear resonant pole is the resonant inductor current. By controlling the resonant inductor current $i_{L_r}(t)$, the average output voltage $v_{CS}(t)$ and consequently the load current can be controlled.

This is done by controlling the current (and therefore the charge) that flows into the source capacitors $C_S$. Adding or subtracting charge from $C_S$ will vary the voltage $v_{CS}(t)$. If $v_{CS}(t)$ varies, the output current will vary. As the voltage $v_{CS}(t)$ equals the average output voltage.

The average output voltage $v_{CS}(t)$ is a function of the resonant inductor current or:

$$v_{CS}(t) = \frac{1}{2C_S} \int i_{L_r}(t) \, dt$$  \hspace{1cm} (3-21)

Equation 3-21 indicates that the voltage $v_{CS}(t)$ is dependent on the time integral of the resonant inductor current. If the integral of $i_{L_r}(t)$ is zero over one switching period then the average of $v_{CS}(t)$ will stay constant. If the integral of $i_{L_r}(t)$ over a switching period is positive, $v_{CS}(t)$ will rise and if negative, $v_{CS}(t)$ will drop.

The value of the integral in equation 3-21 is controlled by pre-setting the threshold levels at which $S1$ and $S2$ turn off. These threshold levels are named $I_{thresh1}$ and $I_{thresh2}$ for $S1$ and $S2$ respectively. In the absence of a feedback winding, the inductor current waveform is symmetrical about the zero line and the two threshold levels will equal each other. This type of waveform was shown in figure 3-7.

If a feedback winding is present the average inductor current depends on the feedback ratio $k_{fb}$, the size and direction of the load current and the threshold levels. The resonant inductor current, where a feedback winding is present, is shown in figure 3-10.

In figure 3-14, a linear approximation of $i_{L_r}(t)$ is given. This approximation is used to simplify the derivation of the average inductor current. It is used as opposed to the more complex trigonometric forms of equations 3-6, 3-9 and 3-15.
In the linear approximation it is assumed that the capacitors $C_s$ are infinitely large and that they act as perfect voltage sources. This leads to a linear function of time as opposed to the trigonometric forms of section 3-2.

The equation of the average resonant inductor current as a linear estimation (details of the derivation are found in appendix D) is:

$$I_{Lr\_average} = \frac{-4k fb I_{load} I_{sat} (L_{sat} - L_{unsat}) + t_{res} I_{thresh1} V_s \cdot (m^2 + 1)}{4 \cdot L_{sat} (I_{thresh1} - I_{thresh2}) + 6 \cdot L_{sat} (L_{unsat} - L_{sat}) + 2 \cdot t_{res} V_s \cdot (1 - m^2)}$$

In figure 3-14 it was assumed that the time for resonant transition is estimated by an arbitrary value $t_{res}$. It is shown in section 3-4-2 that the linear approximation is most accurate for $t_{res} = 0$. Substituting $t_{res} = 0$ in the above equation yields:

$$I_{Lr\_average} = \frac{L_{sat} \cdot (I_{thresh1} - I_{thresh2}) + 4 k fb I_{load} I_{sat} (L_{sat} - L_{unsat})}{2 \cdot L_{sat} (I_{thresh1} - I_{thresh2}) + 4 \cdot L_{sat} (L_{unsat} - L_{sat})}$$

Under steady state conditions the average inductor current equals zero. This can be used to derive expressions for $I_{thresh1}$ and $I_{thresh2}$. Setting $I_{Lr\_average} = 0$ in (3-23) yields:

$$I_{thresh1} = \frac{1}{L_{sat}} \sqrt{4 k fb I_{load} I_{sat} (L_{sat} - L_{unsat}) + L_{sat} (I_{thresh2})^2}$$

$$I_{thresh2} = \frac{-1}{L_{sat}} \sqrt{4 k fb I_{load} I_{sat} (L_{unsat} - L_{sat}) + L_{sat} (I_{thresh1})^2}$$

Appendix D-2 gives the derivation of equations 3-24 and 3-25.

### 3-5 Commutation Limits

The nonlinear resonant pole converter is a resonant transition topology. This means that it depends on a resonant process to switch over from one diode/switch pair to another. This switchover can also be called commutation, as current is commutated from one set of devices to another.
If the resonant transition process is not complete, zero voltage switching will not take place and in extreme cases the inverter will stop operating. The success of this commutation during the resonant transition period is dependent on a number of factors. Two factors, namely the load current and modulation depth, were briefly discussed in section 3-1.

This section deals with the constraints of the resonant transition process in more detail. Equations will be derived and used in the design description of chapter 5.

![Figure 3-19: Commutation between S1 and D2](image)

Commutation between switch S1 and diode D2 is successful if the output voltage $v_c(t)$ swings from $+V_s/2$ to $-V_s/2$. This is shown in figure 3-19. If the conditions of successful commutation is not met, then $v_c(t)$ will not reach the bottom DC-bus and D2 will not start to conduct. If D2 does not conduct, then zero voltage switching may no longer occur and depending on the control circuit the converter might stop operating. The same applies for commutation from S2 to D1.

The behaviour of the output voltage is investigated by reducing the total order of the circuit from three energy storage elements to two (i.e. a second order approximation of a third order system). This is done because the conditions for successful commutation are difficult to derive analytically from the function of output voltage $v_c(t)$ in (3-11). This equation is a transcendental function. An approximation is made by assuming that the source capacitors $C_s$ are much larger than the resonant capacitors $C_r$, or:

$$C_s \gg C_r \quad \text{or} \quad C_s \to \infty$$

From this approximation, the function of output voltage $v_c(t)$ in (3-11) simplifies to:
\[
\begin{align*}
  v_c(t) &= [v_c(t_0) - v_{cs}(t_0)] \cos[\omega(t-t_0)] - \left[\frac{(1+k_{PB})i_{\text{load}}(t_0) + i_{L}(t_0)}{2\omega C_r}\right] \sin[\omega(t-t_0)] + v_{cs}(t_0) \\
  \omega^2 &= \frac{1}{2L_{\text{cap}} C_r} 
\end{align*}
\]

where \(t_0\) is the start of the resonant interval, \(i_{\text{load}}(t_0)\) the load current at the start of the interval (assumed constant over the interval) and:

\[
  \omega^2 = \frac{1}{2L_{\text{cap}} C_r} 
\]

The approximation of \(v_c(t)\) in equation 3-26 implies that the capacitors \(C_s\) will act like voltage sources keeping \(v_{cs}(t)\) constant during the resonant period. This is a very good approximation since \(v_{cs}(t)\) rarely changes more than a few percent of the total supply voltage \(V_s\) during resonance. This is only true during resonance and not while any of the switches or diodes are conducting. The change in \(v_{cs}(t)\) during the conduction of one of the diode/switch pairs will be dealt with in section 3-6.

First consider the case of commutation from \(S_1\) to \(D_2\). For commutation to be successful, \(v_c(t)\) must be equal to or less than \(-V_s/2\) at the end of the resonant transition time (the negative peak must be at least equal to \(-V_s/2\) in figure 3-19). Rewriting from equation 3-26, this implies that:

\[
  -\frac{V_s}{2} \geq \sqrt{[v_c(t_0) - v_{cs}(t_0)]^2 + \left[\frac{i_{\text{load}}(t_0) + i_{L}(t_0)}{2\omega C_r}\right]^2 \cos[\omega(t-t_0) + \phi] + v_{cs}(t_0)} 
\]

where \(\phi = -\tan^{-1}\left[\frac{i_{\text{load}}(t_0) + i_{L}(t_0)}{v_c(t_0) - v_{cs}(t_0)}\right]\)

and

\[i_L(t_0) = i_{L}(t_0) - k_{PB}i_{\text{load}}(t_0)\]

The right hand side of equation 3-28 will be at a minimum for \(\cos[\omega(t-t_0) + \phi] = -1\). Furthermore, if the modulation index \(m(t_0)\) as well as the initial condition (at \(t = t_0\)) for \(v_c(t)\) are related by:

\[v_{cs}(t_0) = m(t_0) \frac{V_s}{2} \quad \text{and} \quad v_c(t_0) = + \frac{V_s}{2}\]

then the modulation index constraint for \(S_1\) commutating to \(D_2\) is given by:

\[
m(t_0) \leq \frac{1}{V_s^2} \frac{L_{\text{cap}}}{2C_r} \left[\frac{i_{\text{load}}(t_0) + i_{L}(t_0)}{2}\right]^2
\]

Page 3-24
This implies that for specific values inherent to the topology \( (V_s, C_r, L_{sat}, \text{and} \ k_{fb}) \) as well as for a given load current and initial resonant inductor current (set by the controller), there exists a maximum modulation index for stable operation.

Following the same argument as for commutation from S1 to D2, the next restriction can be obtained for commutation from S2 to D1:

\[
m(t_0) \geq \frac{1}{V_s^2} \frac{L_{sat}}{2C_r} \left[ i_{load}(t_0) + i_{Lr}(t_0) \right]^2
\]

Assuming that the average load voltage \( v_{load}(t) = v_{C_s}(t) \) and at the moment of switchover \( i_{Lr}(t_0) = I_{thresh1} \) (3-29) can be rewritten as:

\[
v_{load}(t) \leq \frac{1}{2V_s} \frac{L_{sat}}{2C_r} \left[ i_{load}(t) + I_{thresh1} \right]^2 \]

(3-31)

If it is assumed that for commutation from S2 to D1, \( i_{Lr}(t_0) = I_{thresh2} \) prior to switchover, then:

\[
v_{load}(t) \geq -\frac{1}{2V_s} \frac{L_{sat}}{2C_r} \left[ i_{load}(t) + I_{thresh2} \right]^2
\]

(3-32)

Inequalities (3-31) and (3-32) can be explained from two points of view:

- Given a fixed load voltage \( v_{load}(t) \) and load current \( i_{load}(t) \), a certain amount of current must flow through \( L_r \) for successful commutation. The value of this resonant inductor current is called the threshold value and is labelled \( I_{thresh1} \) for switchover from S1 to D2 and \( I_{thresh2} \) for switchover from S2 to D1.
- If on the other hand the threshold values \( I_{thresh1} \) and \( I_{thresh2} \) are fixed, there are limits to the load voltages and currents that will ensure successful commutation and stable NLRP operation.

The first interpretation of (3-31) and (3-32) is explained with the aid of figure 3-20. The minimum threshold value \( I_{thresh1} \) for successful commutation from S1 to D2 is given as a function of load voltage and current. If the NLRP is required to deliver a certain amount of current \( I_{thresh1} \) at a given load voltage \( V_{load}(t) \), then the minimum threshold value is given on the Z-axis. The following parameters were used (also see Appendix B-3):

\[
V_s = 300 \ V \quad L_{sat} = 10 \mu H \quad C_r = 10 \ nF
\]

When implementing a controller for stable operation, the minimum threshold current for successful commutation must be taken into account. This is done by setting a minimum value of \( i_{Lr}(t) \) before a switch is turned off for resonant switchover. This can be implemented in two ways: a fixed minimum can be set or it can vary for different load conditions (figure 3-20). A control system that varies the minimum threshold is more complex than one with a fixed minimum, but has added advantages in terms of component losses.
Figure 3-20: Minimum threshold current for successful resonant transition from S1 to D2 for values of load current and voltage.

The second interpretation of (3-31) and (3-32) can be stated as follows: Given a fixed value for the minimum threshold, what is the limits of load voltage and current that the NLRP will be able to supply? The answer can be found in plotting the commutation limits for a given threshold on the load voltage and current phase plane.

Figure 3-21 shows a phase plane for load voltage and current with different commutation limits. It is assumed that the load voltage and current are sinusoidal with frequency $\omega_{\text{load}}$. The load voltage is expressed as:

$$v_{\text{load}}(t) = \bar{m} \frac{V_s}{2} \cos(\omega_{\text{load}} t)$$ 

(3-33)

where $\omega_{\text{load}}$ is the angular frequency of the load current and voltage (usually around ten to a few hundred Hz) and $\bar{m}$ the peak modulation depth.

The load current is expressed as:

$$i_{\text{load}}(t) = \sqrt{2} I_{\text{load}} \cos(\omega_{\text{load}} t - \phi_{\text{load}})$$

(3-34)

where $I_{\text{load}}$ is the rms value of load current and $\phi_{\text{load}}$ the phase angle between load current and load voltage. Loads that are normally driven by an inverter usually
contain resistive as well as inductive elements so that the phase angle can be written as:

\[ \phi_{\text{load}} = \tan^{-1}\left(\frac{\omega_{\text{load}} L_{\text{load}}}{R_{\text{load}}}\right) \]  

(3-35)

In figure 3-21 the following parameters were used:

\[ V_v = 500 \text{V} \quad L_{\text{load}} = 10 \mu\text{H} \quad I_{\text{thresh}} = 35 \text{A} \quad C_r = 10 \text{nF} \quad I_{\text{load}} = 20 \text{A} \]

In the case of figure 3-21, it is evident that for a constant threshold of 45A, converter operation will be stable. If the threshold is reduced to 35A, unstable operation will occur for a load phase angle of \(\pi/2\) rad. (i.e. a totally inductive load).

Figure 3-21: Commutation limits in the load voltage and current phase plane for sinusoidal functions of \(v_{\text{load}}(t)\) and \(i_{\text{load}}(t)\). Threshold values are 35A and 45A, while load current and voltage are for phase angles of: 0, \(\pi/4\) and \(\pi/2\) rad.
3-6 Source Capacitor Voltage Constraint

In section 3-5 it was argued that if the resonant transition process is not complete, zero voltage switching is lost or the converter might stop operating. The constraints and modulation limits for successful commutation were also given. This section deals with another important factor that is fundamental to sustained operation, namely the ripple of the source capacitor voltage.

In the previous sections it was assumed that the source capacitor voltage \( v_C(t) \) is a perfect average of the load voltage. It was also assumed that this average does not contain any ripple or imperfections. In a practical resonant pole, this is not the case. Figure 3-22 shows a measured waveform for \( v_C(t) \). The parameters of operation for figure 3-22 were chosen to highlight the ripple effect and is clearly visible.

![Figure 3-22: Measured source capacitor voltage showing the ripple on \( v_C(t) \).](image)

The ripple of \( v_C(t) \) can stop the operation of the converter under zero voltage conditions or, depending on the control circuit, can stop operation altogether. This happens when the ripple at peak modulation comes too close to the positive or negative DC-bus.

It was explained in section 3-5 that for switchover a large enough current has to build up in the resonant inductor. This is to assure that there will be enough energy to swing the output voltage from one DC-bus to another. Consider the situation of figure 3-23. If \( S_1 \) is conducting and the ripple on \( v_C(t) \) comes too close to or reaches \( +V_S/2 \), the voltage across the resonant inductor becomes very small or even zero. This will stop the resonant inductor current from increasing. The moment the current
in the inductor cannot reach a specific threshold value, commutation will not be successful.

![Equivalent Circuit Diagram for S1 Conducting](image)

**Figure 3-23:** Equivalent circuit diagram for S1 conducting. If the voltage ripple on $v_{CS}(t)$ becomes too close to, or reaches $V_s/2$, the voltage across $L_r$ will drop, $i_{Lr}(t)$ will stop increasing and commutation will not be successful.

In the design of the nonlinear resonant pole converter, the ripple on $v_{CS}(t)$ must be treated as a design constraint. As a general rule it can be stated that the ripple will increase with modulation depth and will decrease if the source capacitors $C_s$ are made larger.

This can be seen from:

$$v_{cs}(t) = \frac{1}{2C_s} \int_{t_0}^{t} i_{Lr}(t) \, dt$$

The main difficulty with the ripple is to determine whether it will reach the positive or negative DC-bus.

From an analytical point of view it is difficult to determine the height of the ripple above the average value for $v_{cs}(t) = m \frac{V}{2}$. The capacitor source voltage over one switching cycle has to be divided into eight different time intervals. These are the same as in sections 3-3 and 3-4. Each time interval has a different initial condition and solution for the duration of that interval. This increases the amount of algebra to be done up to a point where it is impractical. Because of this, it is suggested that the analytical solution of $v_{CS}(t)$ is plotted to determine how close the ripple is to the DC-bus.

This method for calculating the ripple involves the plotting of the voltage $v_{CS}(t)$ at the peak modulation value using the analytical equations of section 3-2. Plots for $v_{CS}(t)$ under two different conditions are given in figures 3-24 and 3-25. The following parameters were used for figure 3-24:
In figure 3-25 the same parameters as in figure 3-24 were used. The only differences are: $I_{\text{load}} = 3 \, \text{A}$, $m = 0.8$ and $I_{\text{thresh1}} = 12 \, \text{A}$. It can be seen that the ripple increased appreciably with the increased modulation depth.

In figure 3-24, the voltage waveform is not symmetrical about zero volts. This is because the initial condition for the plot was taken as zero, irrespective of the average.

In conclusion it can be said that chapter 3 described the detailed behaviour of the nonlinear resonant pole from a qualitative as well as analytical point of view. Analytical equations were given describing the most important voltage and current time functions under any operating condition. The average resonant inductor current, as well as its parameters, was also described. The two main criteria for successful operation, namely successful commutation or resonant transition and the source capacitor voltage ripple, were addressed.
The theory discussed in chapter 3 can be incorporated into a design philosophy. This design philosophy is dealt with in chapter 5. Controlling the nonlinear resonant pole will be discussed in chapter 4.
Chapter 4

Control
Control

In this chapter the control of the Nonlinear Resonant Pole Converter is described. This includes a general background, descriptions of direct threshold and integral threshold control and large signal (low frequency) analysis. High or switching frequency analysis is briefly discussed.

Two methods of control are discussed in this study, namely direct threshold control and integral threshold control. Direct threshold current control was previously used ([4],[7],[8],[13],[14],[15],[16],[17] and [33]) while integral threshold control is introduced in this study. Integral threshold control will be discussed in section 4-3 and direct threshold control in 4-2. Section 4-1 is a background discussion and shows the difference between voltage loop and current loop control. Section 4-4 deals with the low frequency analysis of the total system. In section 4-5 it is shown how a high frequency analysis can be made.

4-1 Background

In chapter 3 the operating principles of the NLRP were given. A distinction was made between the physical power electronics part (converter) and the control system. In this chapter the NLRP is handled as an integrated system of control and power electronics.

The converter described in this study can either be viewed as a current or voltage controlled system. The most simple control of such a system is by introducing a proportional gain into the control loop. Diagrams of proportional voltage and current control systems are given in figure 4-1. The rest of this study is concerned with the current control strategy and it will be stated why this scheme is preferred to voltage control.

The difference between voltage and current control can be explained by first looking at current control. A representation of a current controlled system is given in figure 4-1. The input to the system is a current reference value. This reference value (i_ref) is the current value which the load current must follow. The load current is measured and fed back to the control system. The difference between load and reference values (or error current i_err) is fed through a proportional amplifier of gain K. The output of the amplifier stage is fed into the controller which commands the gate drive circuits of the power electronic switches. The switches in the physical converter are switched so that the output or load current will change according to the input demand. As was explained in chapter 3, the current in the resonant inductor is switched in such a way as to increase or decrease the voltage v_Cs(t) at the midpoint of the source capacitors. This voltage v_Cs(t) equals the average load voltage. As the applied load voltage changes with time, the load current changes in accordance to the load characteristics.

The voltage control loop of figure 4-1 is in many respects equivalent to the current control loop. Instead of comparing reference and load current values, reference and
load voltage values are compared. The similarity between the two loops is the use of switches to control the current flowing through the resonant inductor and therefore the voltage $v_{CS}(t)$. Voltage control only involves controlling the input to the load or load voltage and no consideration is given to the output of the load or load current.

Figure 4-1: Fundamental control loops for the Nonlinear Resonant Pole, including proportional current and voltage control.

It is also a possibility to control the NLRP by having an inner voltage loop and outer current loop. This may increase stability and controllability and is described in reference [33].

In this study it was decided to use the current control loop. Firstly, because the converter is intended for use where the load is inductive, for example electric machines. Since the load has a filtering effect on the current, it is convenient to measure load current (with a small ripple) as opposed to the output voltage which first has to be filtered. Secondly, in a high power converter it is better to make isolated measurements for inputs to the control circuitry. This is done to minimise electromagnetic interference. It was found that isolated current measurements (through Hall-effect transducers) are practically more viable than isolated voltage measurements. It is therefore convenient to measure the load and resonant inductor currents only as opposed to measuring any voltages.

Figure 4-2 shows more detail of a phase arm and current control system. The total system comprises the control loop. A reference input is given and load current is varied so that the error is minimised. The load current is controlled by switching $S_1$ and $S_2$. The switching signals for $S_1$ and $S_2$ are supplied from the controller. The controller receives an error signal which was amplified with a factor $K$. It compares the amplified error signal with the current through the resonant inductor $i_{LR}(t)$ and determines when $S_1$ and $S_2$ have to be switched.
In the following sections of this chapter, two types of controllers will be dealt with. The first type is the direct threshold controller. It directly compares $K_{\text{err}}$ and $i_{L_r}$ and switches $S_1$ and $S_2$. The second type is the integral threshold controller. It compares the integral of $i_{L_r}(t)$ to the amplified error signal. After this comparison, $S_1$ and $S_2$ are switched.

### 4-2 Direct Threshold Control

In this section the implementation of direct threshold current control will be discussed.

Direct threshold control refers to the method by which the current flowing through the resonant inductor $L_r$, is controlled. This type of control monitors the value of current in the resonant inductor and switches off either $S_1$ or $S_2$. 

---

**Figure 4-2: Representation of the total control system and physical NLRP phase arm.**
The main aim of this scheme is to control the average value of the resonant inductor current by controlling the levels or thresholds at which the switches are turned off. Variation in average inductor current causes $v_{CS}(t)$ to vary and therefore the load current also varies.

$Ki_{err} = I_{thresh1} - I_{min}$

Figure 4-3: Illustration of direct threshold control. The average value of $i_{Lr}(t)$ is controlled by turning off the switches at different threshold values.

In figure 4-3 an illustration of direct threshold control is given. A minimum current level is set. To ensure successful resonant transition, the switches will not be turned off below this level. The threshold level is set above the minimum in direct proportion to the amplified error signal. In the illustration of figure 4-3, the average value of the resonant inductor current will become larger if the threshold level is above $I_{min}$.

In figure 4-4, the effect of the load current via the feedback winding is included. As long as the added component is smaller than the minimum threshold level $I_{min}$, this type of controller can be used in systems with or without a feedback winding.

Figure 4-5 shows the total direct threshold control scheme. The current flowing through the resonant inductor is measured and compared to $I_{thresh1}$ and $I_{thresh2}$. These values set the levels where $S1$ or $S2$ is turned off. In switching $S1$ and $S2$ maximum and minimum levels are set so that the current flowing through the switches and other components does not exceed a safety limit. The minimum limit is set so that the current flowing through the resonant inductor adheres to the minimum value for successful resonant transition (these minimum values for $i_{Lr}(t)$ were discussed in chapter 3). The error signals are multiplied by a factor $K$, which is the proportional gain.
Figure 4-4: Control of the integral of resonant inductor current by adjusting the turn-off threshold level in direct proportion to the error signal. The effect of the load current via the feedback winding is also shown.

Step by step, control of the load current by means of the scheme shown in figure 4-5 is as follows:

- Assume a specific load current is flowing and that the reference signal $i_{\text{ref}}$ demands a higher load current to flow.
- The error signal $i_{\text{err}}$ will be positive as the signal $i_{\text{load}}$ is smaller than $i_{\text{ref}}$.
- The error level is set between the minimum and maximum threshold settings.
- For a positive error signal the threshold level for $S_1$ will be increased by a factor $K$.
- An increased threshold level for $S_1$ implies that the positive area underneath the $i_L(t)$ will be larger than the area underneath the zero line.
- Since the source capacitor voltage is related to the current flowing through the resonant inductor by $v_{C_S}(t) = \frac{1}{2C_s} \int_{0}^{t} i_L(t) \, dt$ an increase in the integral value (i.e. the area underneath $i_L(t)$ over one switching cycle) will increase the average value of $i_L(t)$ and therefore $v_{C_S}(t)$.
- The voltage $v_{C_S}(t)$ equals the load voltage so that an increase in $v_{C_S}(t)$ will increase the load current. The specific increase in load current for an increase in average load voltage is determined by the characteristics of the load.
- The load current will increase until the load current signal $i_{\text{load}}$ equals the reference value $i_{\text{ref}}$. At this point the error signal $i_{\text{err}}$ will be zero and the threshold values will be set at $I_{\text{min}}$.
- If the input demands that the load current has to decrease, the error signal will be negative, $I_{\text{thresh2}}$ will increase and $v_{C_S}(t)$ will decrease. A decrease in average load voltage will cause less load current to flow.
The following section deals with integral control. The integral of $i_{Lr}(t)$ (and therefore $v_{CS}(t)$) is not controlled by adjusting threshold levels, but by directly calculating the integral of $i_{Lr}(t)$ and comparing it to the error signal.

![Diagram](image)

Figure 4-5: Total control diagram for direct threshold control.

### 4-3 Integral Threshold Control

From figure 4-1 it can be seen that the source capacitor voltage $v_{CS}(t)$ is a fundamental parameter in the control of the Nonlinear Resonant Pole. This voltage equals the average load voltage. The voltage $v_{CS}(t)$ can either be controlled directly (voltage control) or the load current may be controlled with a current loop.

In section 4-2 direct threshold control was described. It was shown that the variation in capacitor source voltage $v_{CS}(t)$ is a direct consequence of varying the threshold levels of $i_{Lr}(t)$ at which the switches are turned off. It was stated that by controlling the threshold levels the right hand side integral of (4-1) is altered, where:

$$v_{CS}(t) = \frac{1}{2C_S} \int_0^t i_{Lr}(t) \, dt \quad (4-1)$$

Integral control is different from direct threshold control in so far as the voltage $v_{CS}(t)$ is not varied through threshold levels, but according to the calculated integral of $i_{Lr}(t)$. The advantage of this strategy is that the total control of $v_{CS}(t)$ becomes more linear. This strategy will be discussed further in section 4-4.
Figure 4-6 shows the resonant inductor current and its integral. Each time switchover takes place, the integrator is reset and starts at zero. Figure 4-7 shows an enlarged section at the time where S1 is turned off. If S1 is turned off where the integral is zero, the average of $v_{CS}(t)$ will not change. If S1 is turned off at a positive value of the integral, the average of $i_{Lr}(t)$ is larger and $v_{CS}(t)$ will rise.

$$K \cdot i_{err} = \int i_{Lr}(t) \, dt \geq 0$$

Figure 4-7: Switch S1 is turned off after the integral of $i_{Lr}(t)$ is compared with the amplified error signal.
Figure 4-8 shows a block diagram of the integral controller. An error signal is obtained by subtracting the load current signal from the reference value. Proportional control is achieved by amplifying the error signal with a factor K. The amplified error signal and integral of $i_{Lr}(t)$ are compared using comparators. The output signals of the comparators are compared with the minimum value of $i_{Lr}(t)$ which is necessary for successful resonant transition.

![Block diagram of the integral controller](image)

**Figure 4-8: Block diagram showing the concept of integral control. S1 and S2 are switched in comparison to the integral of the current through $L_r$.**

Operation can be explained as follows:

- Assume that a certain load current is flowing and that the reference demands a higher load current.
- This will cause a positive error signal and a positive signal when amplified by K.
- The integral of $i_{Lr}(t)$ will be zero or negative. The output of the comparator driving S1 is high and the signal driving S2 is low.
- Switch S1 will stay on while the current $i_{Lr}(t)$ charges the source capacitors $C_S$ and increases the voltage $v_{CS}(t)$.
- The increase in $v_{CS}(t)$ will lead to a rise in load current and the error value will decline.
- The moment the value of $K_i_{err}$ drops below the output of the integrator, switch S1 can be turned off. S1 will only turn off if $i_{Lr}(t)$ is above the minimum value $l_{min}$. If both these conditions are met, S1 is turned off and S2 turned on. At the same time the integrator is reset.
• Switch S2 will conduct until the error signal becomes larger than the integrator output. When S2 is switched off, the integrator is reset, S1 will start conducting and the switching cycle starts again.
• If the reference input is kept constant, the load current will follow a zig-zag path about the reference value, similar to tolerance band current control ([29] p.147).

4-4 Large Signal Analysis

In this section of chapter 4, the large signal (low frequency) behaviour of the NLRP is described. This is done for two reasons. The first is to show the difference in performance between the direct threshold and integral threshold controllers. The second is to obtain a large signal transfer function that can be used to specify component values. Of particular importance here is the value of the source capacitor $C_s$, which largely determines the overall response from reference input to load current.

Low frequency is the frequency of the reference signal and load current as opposed to the switching frequency. In a typical NLRP the reference signal will have a frequency of 10 to a few hundred Hertz. The switching frequency typically varies from a few kHz to 100 kHz.

In sections 4-4-1 and 4-4-2 the differences in large signal behaviour between the direct threshold and integral threshold controllers are described. This is done by modelling the total system as shown in figure 4-9.

![Diagram of Low Frequency Model for Total System](image)

**Figure 4-9: Low frequency model for the total system.**

For the model in figure 4-9, the following assumptions were made:
• The model is only valid at low frequency, disregarding switching frequency effects.
• It is assumed that the average inductor current is a nonlinear function of the error signal multiplied with the open loop gain $K_1$.
• A low frequency average inductor current $I_{Lr \text{ average}}$ causes an output voltage $V_{C_s}$. 
When applied to the load, this output voltage $V_{CS}$ causes a load current $I_{load}$ to flow.

By studying the differences in the behaviour of the nonlinear block of figure 4-9 the advantages of integral threshold control as opposed to that of direct threshold control are shown. By describing the average inductor current that results from an amplified error signal the differences in the two control methods are shown.

In section 4-4-3, a linear estimation of the system in figure 4-9 is made and a transfer function obtained. This is done by assuming that the controller has a fixed gain and by modelling the phase arm as a network of linear elements. Although this approximation does not totally describe the large signal behaviour of the system, it is a useful tool for estimating system parameters. In particular $C_S$ and the total open loop gain $K$.

4-4-1 Direct Threshold Control

Fundamental to the behaviour of direct threshold control is the nonlinear input output relationship of the controller block in figure 4-9.

The relationship between the amplified error signal input ($K_i\text{err}$) and average inductor current output ($I_{Lr\_average}$) can be calculated from equation 3-19 where:

$$I_{Lr\_average} = \frac{L_{sat} \left( I_{thresh1}^2 - I_{thresh2}^2 \right) - 4k_{fb} I_{load} L_{sat} \left( L_{sat} - L_{unsat} \right)}{2L_{sat} \left( I_{thresh1} - I_{thresh2} \right) + 4L_{sat} \left( L_{unsat} - L_{sat} \right)}$$

Equation 4-2 describes the average inductor current as a function of the threshold values and load current. Although (4-2) is a linear approximation and does not take the resonant transition time into account, it can be used to approximate the characteristics of the nonlinear block of figure 4-9.

In figures 4-3 and 4-4 it can be seen that the average inductor current is increased by setting $I_{thresh1}$ above $I_{min}$ in direct proportion to the amplified error signal or:

$$I_{thresh1} = I_{min} + K_1 i_{err}$$

In (4-3) $K_1$ is the open loop gain. The threshold at which S2 is switched is kept at $I_{min}$ to ensure successful resonant transition. Substituting (4-3) and $I_{thresh2} = -I_{min}$ into (4-2), yields:

$$I_{Lr\_average} \left( I_{err}, I_{load}, K_1 \right) = \frac{L_{sat} \left( (K_1 i_{err} + I_{min})^2 - I_{min}^2 \right) - 4k_{fb} I_{load} L_{sat} \left( L_{sat} - L_{unsat} \right)}{2L_{sat} \left( K_1 i_{err} + 2I_{min} \right) + 4L_{sat} \left( L_{unsat} - L_{sat} \right)}$$

(4-4)
In figure 4-10, a plot is given for the average inductor current as function of the input error signal, load current and loop gain. Derivation and code used are found in Appendix B-4. The following parameters were used:

\[ V_s = 200V \quad C_r = 24\, nF \quad L_{sat} = 20.8\, \mu H \]
\[ k_p = 0.25 \quad I_{min} = 2.0\, A \quad I_{sat} = 0.8\, A \]

The effects of the load current and loop gain on the transfer function of error signal and average inductor current (as shown in figure 4-10) are summarised in figure 4-11. As \( K_1 \) increases, the average inductor current becomes exponentially larger for a given error signal. As the load current increases, the average inductor current becomes a discontinued function of the error signal.

Of particular importance in figure 4-11 is the sudden increase of inductor current about a zero error signal. This discontinuity has the effect of rapidly increasing or decreasing the average inductor current for a non-zero positive or negative error. The sudden change of current flowing into the capacitors \( C_s \) will cause a rapid variation of voltage \( v_{CS}(t) \), increasing the ripple on \( v_{CS}(t) \). As explained in chapter 3, an increased ripple on \( v_{CS}(t) \) has a negative influence on the stability of operation.
From the trends shown in figures 4-10 and 4-11, a second conclusion can be drawn. For a large error signal, and especially if $K_1$ is large, the exponential relationship may cause an extremely large inductor current to flow. This may again lead to a large ripple on $v_{CS}(t)$ and instability.

It can be concluded that the direct threshold controller has limits as to the way in which the average inductor current can be controlled effectively. With this control strategy, the ripple on $v_{CS}(t)$ may worsen and cause unsuccessful resonant transition or shut-down of the converter.

In the next section it will be shown that the integral threshold controller has a more favourable transfer function from error signal to average inductor current.

### 4-4-2 Integral Threshold Control

As in section 4-4-1, the transfer function of error signal to average current provides information about the low frequency behaviour of the total system. To calculate this transfer function a linear estimation of the average inductor current for a given error input is made.

In figure 4-12 it is assumed that $S_1$ will be turned off at $i_{\text{switch}}$ after the current through the resonant inductor has saturated, $i_{\text{min}}$ has been reached and the current has passed $i_{\text{thresh1}}$. At the point where $S_1$ is turned off:
\[ K_1 i_{err} = K_2 \int_{t \at Switch}^{t \at Threshold} i_L(t) \, dt \] (4-5)

where \( K_1 \) is the loop gain and \( K_2 \) the gain of the integrator.

At \( I_{\text{Threshold}} \) the integral of \( i_L(t) \) equals zero and therefore the average value of \( i_L(t) \) equals zero. The value of \( i_{Lr\_average} \) is the average current that is "added" between the \( I_{\text{Threshold}} \) and \( I_{\text{Switch}} \) point. The average current is:

\[ i_{Lr\_average} = \frac{I_{\text{Switch}} + I_{\text{Threshold}}}{2} \] (4-6)

To write \( i_{Lr\_average} \) of (4-6) in terms of the error signal and other known parameters, \( I_{\text{Switch}} \) can be found by linearising the current between \( I_{\text{Threshold}} \) and \( I_{\text{Switch}} \) and calculating the area underneath the trapezium. From (4-5):

\[ K_1 i_{err} = K_2 \frac{I_{\text{Switch}} + I_{\text{Threshold}}}{2} (t_{\text{Switch}} - t_{\text{Threshold}}) = K_2 \frac{i_{sat}}{(1-m)\frac{V_s}{L_{sat}}} \left( I_{\text{Switch}}^2 - I_{\text{Threshold}}^2 \right) \] (4-7)

Calculating \( I_{\text{Switch}} \) from (4-7) and taking the value for \( I_{\text{Threshold}} \) at steady state (from (3-24)) as:
an expression for $I_{Lr\_average}$ is obtained:

$$I_{Lr\_average} = \sqrt{1 - m \sqrt{K_1 I_{\text{err}} V_s (m - 1) + 4 k_f b I_{\text{load}} K_2 I_{\text{sat}} (L_{\text{unsat}} - L_{\text{sat}}) - K_2 L_{\text{sat}} I_{\text{min}}^2}}$$

$$+ \frac{1}{2 \sqrt{L_{\text{sat}}}} \sqrt{4 k_f b I_{\text{load}} I_{\text{sat}} (L_{\text{sat}} - L_{\text{unsat}}) + L_{\text{sat}} I_{\text{min}}^2}$$

(4-9)

Equation 4-9 can now be used to obtain the nonlinear transfer function of error signal to average inductor current.

Figure 4-13: Average resonant inductor current as function of the error input for the integral threshold controller. $I_{Lr\_average} = I_{Lr\_average}(i_{\text{err}}, m, K_1, K_2)$

In figure 4-13 the average resonant inductor current is plotted for different values of the modulation index $m$, the loop gain $K_1$ and integrator gain $K_2$. Derivation and code used are found in Appendix B-4. Different values of load current have no significant effect on the plot of figure 4-13. The following parameters were used:
The effects of different parameters on the transfer function are summarised in figure 4-14.

![Diagram showing the effects of parameters K₁, K₂ and m on the transfer function of error signal to average inductor current for the integral threshold controller.](image-url)

**Figure 4-14:** Effects of the parameters K₁, K₂ and m on the transfer function of error signal to average inductor current for the integral threshold controller.

An important characteristic of the integral threshold controller is that the transfer function is independent of load current. Unlike the discontinuity with the direct threshold controller, this transfer function is continuous in zero. This fact is an important advantage over the direct threshold controller.

Another favourable feature of this controller is the behaviour of the transfer function with change in modulation depth m. It can be seen from figures 4-13 and 4-14 that the average inductor current decreases for a given error signal if m is increased. Since the ripple on $v_{CS}(t)$ increases with m, this controller counters the increasing ripple by reducing the average current flowing into capacitors $C_S$.

From the above it can be seen that the integral threshold controller has definite advantages over the direct threshold controller. This will lead to improved overall performance and reduction of the ripple on $v_{CS}(t)$.
4-4-3 Transfer Function

In the design of the NLRP it is important to know the difference in low frequency amplitude and phase shift of the output (load current) from the input reference signal. This type of information can only be obtained after deriving the large signal transfer function of the total system.

A transfer function of the NLRP with integral threshold control can be derived from the block diagram of figure 4-15. The calculation of such a transfer function is complicated by the nonlinear block, representing the controller. In this section it will be shown how a linear approximation of the system can be derived. From the measured results in chapter 7, it is shown that the linear approximation closely resembles total transfer characteristics. This transfer function is used in chapter 5 to aid in calculating system parameters.

Figure 4-15: Block diagram showing the large signal transfer function of a NLRP with integral threshold controller.

In deriving a linear model for the NLRP with integral threshold control (figure 4-16), the following steps were taken:
- It is assumed that the switching frequency plays no role in the large signal model.
- Components are ideal and linear as shown in figure 4-16.
- The controller is linearised and it is assumed that the average resonant inductor current is a constant $K$ multiplied with the error signal. This factor $K$ incorporates the open loop gain as well as the integrator gain.
- It is assumed that by controlling the average resonant inductor current, the voltage $V_{CS}(s)$ is controlled. This in turn is responsible for the current flowing through the load which is a function of the load impedance.
- The voltage $V_{CS}(s)$ equals the load voltage.
- Losses in the system are modelled by a resistance $R_s$ parallel to the source capacitor $C_s$. 
Figure 4-16: Linearised model of the NLRP with integral control that is used for transfer function analysis.

Figure 4-17: Magnitude plot for the linearised transfer function of the NLRP with integral threshold controller. Plots are for different values of K where: $|I_{load}(s)/I_{ref}(s)| = |F_{total}(j2\pi f, K)|$, (X-axis in Hz).
From the model in figure 4-16 the following transfer function of reference to load current is obtained:

\[
\frac{I_{\text{load}}(s)}{I_{\text{ref}}(s)} = \frac{KR_s}{2s^2C_sR_sL_{\text{load}} - 2C_sR_sR_{\text{load}} - 2L_{\text{load}} - 2R_{\text{load}} - K R_s} \tag{4-10}
\]

The function of reference current to load voltage is:

\[
\frac{V_{Cs}(s)}{I_{\text{ref}}(s)} = \frac{KR_{\text{load}} - sL_{\text{load}} - R_s}{2s^2C_sR_sL_{\text{load}} - 2C_sR_sR_{\text{load}} - 2L_{\text{load}} - 2R_{\text{load}} - K R_s} \tag{4-11}
\]

The derivation of equations 4-10 and 4-11 can be found in Appendix B-5.

Figure 4-18: Phase plot for the linearised transfer function of the NLRP with integral threshold controller. Plots are for different values of K where:
\[
\text{Phase} \left( \frac{I_{\text{load}}(s)}{I_{\text{ref}}(s)} \right) = \Phi \left( F_{\text{load}}(j2\pi f, K) \right), \text{ (X-axis in Hz)}
\]

To determine the values used in the model the following steps are taken:
- The values of $R_{\text{load}}$, $L_{\text{load}}$ and $C_s$ are taken as the component values of the physical phase arm and load.
- The value of K is taken as an average between the open loop gain and integrator gain. It is adjusted to fit the measured transfer function.
- The value of $R_s$ is estimated to fit the measured transfer function.
In chapter 7 it will be shown how results from this model correlate with measured values. Typical results of a magnitude and phase plot are shown in figures 4-17 and 4-18. The following parameters were used:

\[ C_s = 12 \mu F \quad R_s = 70 \Omega \quad L_{\text{load}} = 5mH \quad R_{\text{load}} = 1.5 \Omega \]

### 4-5 Switching Frequency Analysis

Section 4-4 dealt with the low frequency analysis of the system. In this section suggestions are made for analysing the response of the system at high or switching frequencies. Performance at low frequency is important for applications where a fast response with minimum phase shift is required. High frequency performance is important for the internal operation of the converter.

If values of components or gain factors are chosen to favour low frequency performance, it may on the other hand cause operating conditions that limit modulation depth or even zero voltage switching. In extreme cases it may even happen that the total system becomes unstable and the converter has to be shut down.

In figure 4-19 a block diagram is shown which can be used to determine the high frequency response of the system. The nonlinear inductance is modelled with a two step linear approximation. In a high frequency analysis a function describing the nonlinear inductance model as well as the controller has to be derived.

**Figure 4-19:** Block diagram that can be used to determine the high or switching frequency response of the total system.
The most difficult part of a switching frequency analysis is the modelling of the controller. The controllers of sections 4-2 and 4-3 have to be modelled for their high frequency behaviour.

Figure 4-20: Total system for high frequency analysis, incorporating integral threshold control.

High frequency analysis of switching converters is usually done by describing the behaviour of the system about an operating point. All the nonlinearities in the system are modelled with describing functions. In the case of the NLRP converter, an input reference current, output load current and modulation depth will be calculated. The high frequency behaviour will be analysed about this operating point.

Figure 4-20 shows a model which can be used for the high frequency analysis of the total system with integral threshold control. The control system directly compares the integral of \( i_{LR}(t) \) with the amplified error signal. As a result of this comparison the output voltage \( v_{CS}(t) \) will be switched between the negative and positive DC-bus.

One of the major concerns in the design of a NLRP converter is the ripple of the voltage \( v_{CS}(t) \). The frequency of the ripple equals the switching frequency and therefore a high frequency model will be able to provide information about the behaviour of the ripple.

To keep the high frequency behaviour of the converter inside permissible limits a sliding mode control approach should be followed. With sliding mode control the parameters of operation (for example peak resonant inductor current, voltage \( v_{CS}(t) \) and the switching frequency) can be kept within limits. If the switching frequency drops too much, the controller operation can be kept at a minimum value. By monitoring the ripple on \( v_{CS}(t) \), switching frequency or the switching thresholds could be altered if the ripple causes operation at a too low or high modulation depth.
Sliding mode control and its benefits can only be implemented after a high frequency analysis has been done. After limits have been set, it could be matched against values from a model and alterations to the control system could be done. Figure 4-21 gives a simplistic graphical representation of sliding mode control for the NLRP converter.

Together with a low frequency analysis, a switching frequency analysis can be used to optimise the overall performance of the NLRP converter. This would especially be true if a sliding mode control approach is used.

In this chapter a new approach to controlling the NLRP, namely integral threshold control, was introduced. It was shown that this controller has more favourable characteristics than the direct threshold controller. A linearised model for large signal transfer function analysis was derived and switching frequency analysis was briefly discussed.
Chapter 5

Dimensioning and Trade-offs
**Dimensioning and Trade-offs**

This chapter deals with the dimensioning of components of the nonlinear resonant pole converter. The trade-offs involved in choosing component specifications and values are discussed.

During the design of the resonant pole, values for the resonant inductance $L_r$, source capacitance $C_s$ and resonant capacitors $C_r$ have to be found. The ratings of the switches and diodes must also be specified. This is done by applying the theory and background of chapters 3 and 4.

Dimensioning and design can take place after four important specifications are considered. These are:

- Input and output voltage
- Apparent power and power factor
- Switching frequency
- Dynamic response

These four factors are the input to a design algorithm or over-all dimensioning strategy. In this process, trade-offs will be made so that component values and sizes can be specified. During this process it may also be necessary to redefine the input parameters. A graphical representation of this strategy is shown in figure 5-1.

Five aspects that govern converter behaviour, as well as their associated parameters, are discussed in the first five sections of this chapter.

![Figure 5-1: Overall dimensioning strategy.](image)

The first section deals with the magnitude of the output voltage in relation to the DC supply. The power rating of the converter as well as switch ratings are discussed in the second part. Section 3 deals with values and trade-offs that determine resonant transition behaviour. Switching frequency as a design parameter and its variation are discussed in section 4. Section 5 deals with the overall response of the system.
from reference input to load current output. In the last section of this chapter all the above factors are integrated into a system design methodology.

5-1 Output Voltage

This section deals with the magnitude of the output voltage relative to the input DC value.

![Half bridge NLRP with feedback winding](image)

In the NLRP the output amplitude of the voltage is controlled or determined by the maximum modulation depth $\hat{m}$. In the single phase half bridge configuration (figure 5-2), the maximum rms output voltage ($V_{load}$) is:

$$V_{load} = \frac{1}{\sqrt{2}} \hat{m} \frac{V_s}{2}$$  \hspace{1cm} (5-1)

From figure 5-3 the output voltages for a three phase PWM inverter can be calculated. It is assumed that the peak output for each phase arm in a three phase system is $\frac{V_s}{2}$. Just as in (5-1) the per phase rms voltage would be:

$$V_{an} = V_{bn} = V_{cn} = \frac{1}{\sqrt{2}} \hat{m} \frac{V_s}{2}$$  \hspace{1cm} (5-2)
In a balanced three phase system, the rms output line voltage $V_{\text{load}}$ is given as:

$$V_{ab} = V_{bc} = V_{ca} = V_{\text{load}} = m \frac{\sqrt{3}}{2} V_s$$  \hspace{1cm} (5-3)

### 5-2 Output Power

In this section, the output power rating is discussed. The sizes and ratings of the switches and diodes are most affected by the total kVA requirement of the converter. The second important factor is the power factor. The load can typically vary from totally resistive to totally inductive. This has an influence on converter operation and performance, as discussed in section 5-2-2.

#### 5-2-1 Switch Rating

The total apparent power or kVA of a three phase Y-connected inverter is related to the per phase load current ($I_{\text{load}}$) and line to line output voltage ($V_{\text{load}}$) by:

$$S_{out} = \sqrt{3} V_{\text{load}} I_{\text{load}}$$  \hspace{1cm} (5-4)
If the power rating and the output voltage of the inverter are specified, the per phase output current can be calculated from (5-4). Knowing the input voltage (which determines $V_{\text{load}}$) and the output current the switch and diode ratings can be obtained.

The two main criteria in the specification of semiconductor switches and diodes, are:
- The maximum breakdown voltage. This is the maximum voltage the device can withstand before breakdown occurs.
- Rated rms or average current. This is the value of rms or average current at which the device is specified to work at.

In practical converters, a safety margin is incorporated so that the ratings of the devices are chosen to be higher than the maximum requirements of the converter.

Theoretically, the maximum voltage that a device in an inverter (such as the one of figure 5-3) must be able to withstand is the DC input voltage. If one of the switches in any of the phase arms is on, the total input voltage is across the device. Due to parasitic effects and overshoot during transients, the device voltage rating in a practical system can be up to two times the input DC voltage.

The minimum rms rating of a device can be calculated from the low frequency output current. In figure 5-4, it is shown that each of the switch/diode pairs in a phase arm carries load current for half a cycle.

From figure 5-4 it can be seen that $i_{\text{load}} = i_{S1} - i_{S2}$. Assume that the rms rating of one of the switch/diode currents ($i_{S1}$ or $i_{S2}$) is given by $I_s$. The rms load current ($I_{\text{load}}$) and switch current rating are related by [9]:

\[
I_{\text{load}} = I_s.
\]
\[ I_S = \sqrt{\frac{1}{T} \int_0^T i_{S_{1,2}}^2(t) \, dt} = \sqrt{\frac{1}{2T} \int_0^T i_{\text{load}}^2(t) \, dt} = \frac{I_{\text{load}}}{\sqrt{2}} \]  

(5-5)

If the load is totally resistive, all the current will flow through the switches. If it is totally inductive, the load current will flow through the diodes. This implies that the current rating for both a switch and a diode must be \( I_S \).

From (5-5), (5-4) and (5-3) the absolute maximum rating of the inverter in terms of the device parameters is:

\[ S_{\text{out}} = \frac{3}{2} \hat{m} V_s I_s \]  

(5-6)

where it is assumed that the switch can withstand the input DC-voltage.

Equation 5-6 is only a rough estimation linking load current and output power to the switch ratings. In a practical NLRP converter, the switches have to conduct additional current peaks which are not included in (5-6). Voltage and current safety factors must also be allowed for.

### 5-2-2 Power Factor

The load characteristic can typically vary between resistive and inductive. If the converter is used as machine drive the load can have a large inductive component. In terms of the load phase angle \( \phi_{\text{load}} \), the total active power can be written as:

\[ P_{\text{out}} = \sqrt{3} V_{\text{load}} I_{\text{load}} \cos(\phi_{\text{load}}) \]  

(5-7)

with: \( 0^\circ \leq \phi_{\text{load}} \leq 90^\circ \)

for sinusoidal \( V_{\text{load}} \) and \( I_{\text{load}} \).

The load power factor is:

\[ \eta_{\text{load}} = \frac{P_{\text{out}}}{S_{\text{out}}} = \cos \phi_{\text{load}} \]  

(5-8)

In the design of the NLRP the load power factor has an influence on two criteria. The first is the maximum obtainable modulation depth. In chapter 3 it was stated that the maximum obtainable modulation depth decreases as the load phase angle increases. As the modulation depth decreases the rms output voltage will also decline.
The second influence that the load phase angle has is the change in dynamic response at low frequencies. In general it can be said that the more inductive the load is, the more difficult it would be to suddenly change the flow of load current. A largely inductive load will therefore decrease the dynamic response of the system. This was shown in chapter 4.

Considering these factors it is important to take the power factor of the load into account. Figure 5-5 shows the general trend in maximum modulation depth (and therefore output voltage), as well as low frequency dynamic response with increase in load phase angle.

5-3 Resonant Transition

During resonant transition, neither of the switches nor the diodes are conducting. Energy is exchanged between the resonant capacitors $C_r$ and resonant inductor $L_r$. Values of $C_r$ and $L_{sat}$ which will be necessary for successful resonant transition have to be determined.

$C_r$ and $L_{sat}$ are determined by two factors. The first is the resonant transition time, which must be long enough for the switches to turn off and commutation to take place. The second is the limit of modulation depth. For each set of values of $C_r$, $L_{sat}$ and a given threshold current value, there is a maximum obtainable modulation depth.
It will be shown that choosing values for the resonant components is a trade-off between the length of the transition time and the modulation depth at which the converter is operating.

5-3-1 Transition time

The resonant transition time is the time it takes for a switch to turn off till the opposite diode starts conducting. This is shown in figure 5-6. Also shown is the fall time of the switch (which is presumed to be linear).

The resonant transition time should be long enough for the switch to turn off. If the resonant transition time is shorter than the fall time, current through the switch will keep the output from swinging to the opposite DC rail. Zero voltage switching will therefore not be achieved.

From equation (3-24) the resonant transition time for switchover from S1 to D2 can be determined from the linear estimation as:

\[
\tau_{res} = \frac{2 \arctan \left( \frac{1}{2mV_s \omega r C_r} \left[ I_{load} + I_{thresh} + \sqrt{(I_{load} + I_{thresh})^2 - 4V_s^2 \omega_r^2 C_r^2 m} \right] \right)}{\omega_r}
\]

where:
In equation 5-9 there are three variables which determine the value of the resonant transition time. These are the load current, modulation depth and threshold current value. Since all three of these can change within one cycle of load current it would be convenient to have an estimation of the transition time that is independent of these variables.

An indication of the order of resonant transition time can be made from the resonant frequency $\omega_r$. From figure 5-6 one can deduce that the maximum resonant transition time is one half of a period of a sinusoidal waveform of frequency $\omega_r$. The maximum resonant transition time $t_{\text{max, res}}$ is given by:

$$t_{\text{max, res}} = \frac{2\pi}{2 \omega_r} = \pi \sqrt{2L_{\text{sat}}C_r}$$

(5-11)

If a maximum transition time of two to three times the switch current fall time $t_f$ is chosen, a value of $\sqrt{L_{\text{sat}}C_r}$ can be estimated. From this product and from the $L_{\text{sat}}/2C_r$ fraction described in the following section, separate values of $L_r$ and $C_r$ can be calculated.

5-3-2 Modulation Constraints

In this section it will be shown that from the modulation constraints a value for $L_{\text{sat}}/2C_r$ can be obtained. Using this ratio and the $\sqrt{L_{\text{sat}}C_r}$ product of (5-11), individual values for $L_{\text{sat}}$ and $C_r$ can be calculated.

From chapter 3 the two modulation constraints for successful resonant transition at a given time $t_0$ are given by:

From S1 to D2: \[ m(t_0) \leq \frac{1}{V_s^2} \frac{L_{\text{sat}}}{2C_r} [i_{\text{load}}(t_0) + i_{L_r}(t_0)]^2 \] (5-12)

From S2 to D1: \[ m(t_0) \geq \frac{1}{V_s^2} \frac{L_{\text{sat}}}{2C_r} [i_{\text{load}}(t_0) + i_{L_r}(t_0)]^2 \] (5-13)

Inequalities 5-12 and 5-13 relate different parameters for successful resonant transition. If it is assumed that the supply voltage $V_s$ is constant, that a given maximum modulation depth has to be obtained and that a given load current will flow, the threshold value $i_{L_r}(t_0)$ and fraction $L_{\text{sat}}/2C_r$ remain variable parameters.

Ideally, the threshold value must be chosen as small as possible. This will keep winding and magnetic losses to a minimum. Losses in the inductor are a major concern in the NLRP and are described in chapter 6. Another reason for keeping the
threshold levels small, is that excessive current peaks place additional stresses on the switching devices and reduce their lifetime.

After deciding on the threshold levels, a value for $L_{\text{sat}}/2C_r$ must be selected so that (5-12) and (5-13) are always valid.

Figure 5-7 shows the influence that $L_{\text{sat}}/2C_r$ has on different parameters. Choosing a too small value of $L_{\text{sat}}/2C_r$ will result in a small maximum modulation depth - which directly results in a small output voltage. Making $L_{\text{sat}}/2C_r$ large implies that low threshold levels can be used, but also that the switching frequency may drop to unacceptable levels.

The selection of a value for $L_{\text{sat}}/2C_r$ is therefore a trade-off between maximum modulation depth, minimum threshold levels and switching frequency.

Once $\sqrt{L_{\text{sat}}C_r}$ and $L_{\text{sat}}/2C_r$ are known, specific values for $L_{\text{sat}}$ and $C_r$ can be calculated.
5-4 Switching Frequency

In chapter 3 it was shown that a varying switching frequency is a characteristic of the NLRP converter. The switching frequency band within which the converter operates, can be determined by component values and operating parameters. In this section it will be shown how the converter can be dimensioned to operate within a specified range of switching frequencies.

There are different reasons why switching frequency is an important design parameter. These include:

- Switching losses in the devices as well as losses in the magnetic component.
- Electro-magnetic interference (EMI) resulting from radiated and conducted emissions.
- The relationship between the high switching frequency and low frequency operation.

The nonlinear inductor is especially prone to high frequency losses. These losses can be subdivided into winding and core losses. As a rule it can be said that the higher the switching frequency the higher the losses in the magnetic component will be. The losses in the nonlinear inductor are addressed in chapter 6.

Electro-magnetic interference (EMI) is the result of radiated and conducted emissions. Although switching transitions are the main cause of EMI, it may be necessary to restrict the switching operation within a certain frequency band. If a
switching frequency band is specified (figure 5-8), the NLRP must be dimensioned for a maximum (\( f_{s\_{\text{max}}} \)) as well as minimum switching frequency (\( f_{s\_{\text{min}}} \)).

The last consideration is the difference in switching frequency and modulation or load frequency \( \omega_{\text{load}} \). The switching frequency must be high enough not to interfere with the low frequency behaviour of the system. The modulation frequency would typically be in the order of a few Hertz to a few hundred Hertz. The switching frequency can be from a few kHz to about 100 kHz. If the switching frequency is too low the response of the system from input reference to output load current may be affected and the output load current would not be able to respond to sudden changes in the input.

![Dynamic low frequency response and Component losses](image)

Figure 5-9: General trend of higher component losses with increased switching frequency as opposed to slower overall dynamic response with lower switching frequency.

Considering these factors, the switching frequency band is a trade-off between component losses and overall system response. The general trend of higher losses with increased frequency and slower dynamic response with lower frequencies is illustrated in figure 5-9.

From figure 5-8 it can be seen that the maximum as well as minimum switching frequency must be estimated to determine the switching frequency band.

From equation 3-23 the switching frequency can be estimated as:
The maximum switching frequency occurs at \( m=0 \) or:
\[
f_s(m) = \frac{V_s(1-m^2)}{8L_{unsat} \left[ \frac{1}{k_{sat}} \left( \frac{I_{\text{thresh}1}}{2} - \frac{I_{\text{thresh}2}}{2} - I_{sat} \right) + I_{sat} \right]}
\]  
(5-14)

The maximum switching frequency occurs at \( m=0 \) or:
\[
f_{s_{\text{max}}} = \frac{V_s}{8L_{unsat} \left[ \frac{1}{k_{sat}} \left( \frac{I_{\text{thresh}1}}{2} - \frac{I_{\text{thresh}2}}{2} - I_{sat} \right) + I_{sat} \right]}
\]  
(5-15)

The minimum switching frequency occurs at \( m = \hat{m} \) and can be estimated as:
\[
f_{s_{\text{min}}} = \frac{V_s(1-\hat{m}^2)}{8L_{unsat} \left[ \frac{1}{k_{sat}} \left( \frac{I_{\text{thresh}1}}{2} - \frac{I_{\text{thresh}2}}{2} - I_{sat} \right) + I_{sat} \right]}
\]  
(5-16)

The values for \( L_{sat}, L_{unsat} \) (and therefore \( k_{sat} \)) as well as \( I_{sat} \) can be determined after the design of the nonlinear inductor has been completed. The specific magnetic design of the nonlinear inductor is dealt with in chapter 6.

### 5-5 Dynamic Response

In this section the parameters and trade-offs involved in total system response are dealt with. The dynamic response of the total system depends on the transfer function of input reference signal to output load current. Of particular importance is the value of the source capacitance \( C_s \).

In chapter 4-4, a linearised low frequency model for the total system was developed. The transfer function of the system includes the open loop gain \( (K) \), load inductance and resistance \( (L_{\text{load}} \text{ and } R_{\text{load}}) \), source capacitance \( C_s \) and a resistive parameter \( R_s \). The transfer function is:
\[
\frac{I_{\text{load}}(s)}{I_{\text{ref}}(s)} = \frac{R_s \cdot K}{2s^2C_s \cdot R_s \cdot L_{\text{load}} - (2L_{\text{load}} - 2C_s \cdot R_s \cdot R_{\text{load}}) \cdot s + K \cdot R_s - 2R_{\text{load}}}
\]  
(5-17)

Depending on the application, the difference in phase angle and amplitude of input and output are important to know. If the inverter is supplying a sinusoidal output at \( \omega_{\text{load}} \), that frequency must be checked against the total transfer function (figure 5-10). At \( \omega_{\text{load}} \) there is a phase shift and amplitude difference that depends on the transfer function characteristic. The trade-off in this section therefore lies in choosing a characteristic (① or ② in figure 5-10) that will suit the converter application.
If the NLRIP is not used as a single frequency inverter, for example an active power filter [1], a different type of response specification may be necessary. In such a situation, a dynamic step response specification will be set. An illustration of the output current in response to a step function input is given in figure 5-11. Typical parameters here include rise time ($t_r$), peak time ($t_p$), overshoot ($M_p$) and settling time ($t_s$). These parameters can be related to the second order estimation of equation 5-17 [19].

The parameters of (5-17) would have to be adjusted to satisfy these design requirements. Except for $C_s$ and to a lesser extent $K$, all the parameters in (5-17) are already fixed. $R_{load}$ and $L_{load}$ are specified from the load characteristics, while $R_s$ is related to the losses in the converter. The open loop gain $K$ may be varied, but will largely be fixed by the characteristics of the control system. Varying the overall response of the system implies that different values of $C_s$ have to be chosen.
Choosing the correct value for $C_s$ is a trade-off between two factors. The first involves the response of the system and the second the source capacitor voltage ($v_{CS}(t)$) stability. From chapter 3-6 it was seen that the ripple on $v_{CS}(t)$ increases as the value of $C_s$ decreases. The ripple on the source capacitor voltage can lead to a severe restriction in modulation depth. A large $C_s$ will increase the maximum obtainable modulation depth.

**Figure 5-11: Step response specifications.**

**Figure 5-12: Trade-off in the choice of $C_s$ between overall low frequency response and stability of $v_{CS}(t)$ which determines maximum modulation depth $\hat{m}$.**
The dynamic response of the system is largely dependent on $C_s$. The larger $C_s$ becomes, the longer the output will take to respond to changes in the input. If the converter is designed to produce a single frequency output, a larger $C_s$ will increase the phase shift between $i_{\text{ref}}$ and $i_{\text{load}}$. If the converter is designed to the step response specifications of figure 5-11, the rise time $t_r$ and settling time $t_s$ will both increase.

Choosing a value for $C_s$ is therefore a trade-off between overall dynamic response and the stability of $v_{CS}(t)$. The general dependency between the overall frequency response and stability of $v_{CS}(t)$ on $C_s$ is illustrated in figure 5-12.

5-6 Overall Design

The overall design and dimensioning process involves the transformation of pre-set parameters to component values that can be implemented in the NLRP. During this process, some trade-offs are made. This section is a summary of processes and trade-offs that constitute the overall design. It may be necessary to make trade-offs to the input parameters to obtain a satisfying design.

The input parameters necessary to obtain a design include the input DC and output voltages, the load characteristics, the apparent power and power factor of the load, the overall frequency response of the system and the desired switching frequency.

With the overall dimensioning process completed, the following will be known:

- The values of the source capacitors $C_s$.
- The values of the resonant capacitors $C_r$.
- The magnetic parameters of the nonlinear inductor, including the saturated and unsaturated inductance.
- The voltage and current ratings for the switches.
- The type of control system to be used.

In the first five parts of this chapter, dimensioning was divided into sections that describe a certain part of converter operation. These included:

- The specification of voltages in relation to the modulation depth.
- The dimensioning of the switches to handle total power throughput.
- The resonant transition process with the associated transition time and modulation constraint.
- The variation in switching frequency.
- The total transfer function and low frequency response from input reference signal to output load current.

Each of the above can be viewed as parts of the overall design algorithm. In each case component values or parameters are obtained with another set of parameters or component values as input. In figure 5-13, the five aspects of converter operation are shown as characteristic processes together with inputs and outputs.
Dimensioning of the converter using the aspects of converter operation, as shown in figure 5-13, cannot be done in isolation. The reason is that the values obtained from one set of calculations may be the input to the next. The overall dimensioning process is therefore recursive, and has to be repeated until all initial design parameters are satisfied.

From figures 5-13 and 5-14, the overall dimensioning process can be described as follows:

Figure 5-13: The five aspects that describe converter operation, which are used for the dimensioning of the converter.
Step 1: The first step is to estimate a maximum modulation depth that will result in the specified output voltage (Equations 5-1 and 5-3). A good estimate for \( \tilde{m} \) is between 0.75 and 0.9. If the output voltage is too low, a higher input DC voltage must be used. During the overall dimensioning process it may also be necessary to choose a different \( \tilde{m} \) to match other parameters against given specifications. If this is the case the process will have to start at step 1 again.

Step 2: The next step is to determine the switch and diode voltage and current ratings. The voltage rating of the switches and diodes must at least be equal to the input DC voltage \( V_S \). Usually a safety factor is incorporated, making the specified device breakdown voltages up to 2 times \( V_S \). From equation (5-5) the diode and switch current ratings can be obtained. Again a safety factor must be allowed, since the peak current that is drawn by the nonlinear inductor is not taken into account by (5-5). Output current for determining device current ratings is calculated from the specified apparent power output (Equations (5-4) and (5-6)).

Step 3: Following the choice of switching devices and diodes, parameters for the nonlinear magnetic component can be derived from the resonant transition or switch-over time. From (5-11), an estimation of switch-over time will result in a value for \( \sqrt{L_{sat} C_r} \). An estimation of switch-over time must be made in conjunction with the fall time of the switching devices. If the transition or switch-over time is less than the fall time, the switches will not switch off in time and zero voltage switching will be lost. If the transition time is chosen too long (i.e. the \( \sqrt{L_{sat} C_r} \) product too large) the output voltage will be reduced.

Step 4: After a value for \( \sqrt{L_{sat} C_r} \) is determined a value for \( L_{sat}/2C_r \) can be calculated from the modulation constraint. Having two equations containing \( L_{sat} \) and \( C_r \), the individual component values can be calculated. From figure 5-14 it can be seen that the modulation constraint involves several design parameters. These include the maximum modulation depth \( \tilde{m} \), DC input voltage \( V_S \), load current \( i_{load}(t) \), current threshold values, the load phase angle \( \phi_{load} \), saturated inductance \( L_{sat} \) and resonant capacitor value \( C_r \). Since \( \tilde{m} \), \( V_S \), \( i_{load}(t) \) and \( \phi_{load} \) are fixed, the modulation constraint results in the matching of a \( L_{sat}/2C_r \) value to a threshold value or values (depending on the control system). From figure 5-7 it can be seen that increasing the factor \( L_{sat}/2C_r \) reduces the threshold current needed for complete resonant transition. It may also reduce the switching frequency to below the specified range so that increasing the threshold current values may be the only option left. This has to be done with caution as excessive threshold levels increase current peaks through the switches and the nonlinear inductor. Coping with large current peaks may down size the available output of the converter for a given set of switching devices and passive components.

Step 5: Following the calculation of \( L_{sat} \) and knowing the peak current that is going to flow through \( L_r \), a magnetic design of the nonlinear resonant inductor can be made. The magnetic design will provide values for the unsaturated inductance \( (L_{unsat}) \) as well as the current point where the core saturates (\( L_{sat} \)). In the
magnetic design, the losses in the core and in the windings will be taken into account. For a particular magnetic material, the ratio between the saturated and unsaturated inductance is a constant $k_{\text{sat}}$. Once $L_{\text{sat}}$ is fixed, $L_{\text{unsat}}$ is also fixed. Since the switching frequency is largely dependent on $L_{\text{unsat}}$ and fixed to a specified range, it may be necessary to recursively change $L_{\text{sat}}$.

Figure 5-14: Overall dimensioning with steps 1 to 7 shown.

Step 6: The switching frequency is dependent on a number of factors. These include the maximum modulation depth $\bar{m}$, $V_S$, $I_{\text{sat}}$, $L_{\text{unsat}}$ (and therefore $L_{\text{sat}}$) and the current threshold values. Since all these parameters are already fixed, the switching frequency band from (5-15) and (5-16) must be checked against the desired range. If the calculated frequency does not match the design range, $L_{\text{unsat}}$ would most likely be changed. This change would affect $L_{\text{sat}}$ and therefore the commutation constraint as well as the resonant transition time. Steps 3, 4 and 5
would therefore be repeated with the new value for $L_{\text{sat}}$, until a satisfactory switching frequency band is obtained.

**Step 7:** The last step involves the choice of $C_S$ from the low frequency transfer function characteristics. From chapter 4 it was seen that $C_S$ is an important parameter in the overall dynamics of the system. $R_{\text{load}}$ and $L_{\text{load}}$ are fixed by the choice of load and the open loop gain is predominantly determined by the type of control system used. This implies that the only parameter left to vary the low frequency dynamic response is $C_S$. Making the value of $C_S$ smaller will increase response and reduce phase shift between $i_{\text{ref}}$ and $i_{\text{load}}$. If $C_S$ is made too small, the ripple on $v_{CS}(t)$ will increase and the obtainable modulation depth will decrease. These trade-offs were graphically illustrated in figure 5-12. The size of the ripple on $v_{CS}(t)$ as function of $C_S$ has to be determined numerically as discussed in chapter 3-6. After $L_{\text{load}}$, $R_{\text{load}}$ and the open loop gain have been established, $C_S$ can be chosen to satisfy the required phase angle shift and magnitude difference between $i_{\text{ref}}$ and $i_{\text{load}}$.

In this chapter the dimensioning of the NLRP was discussed. The chapter was divided into six different sections. In each of the first five sections, an operational process characteristic to the NLRP was described. From these five aspects, component values and design parameters were calculated. In the last section seven steps were given to describe the overall dimensioning process.

During the overall dimensioning process, trade-offs must be made to satisfy initial requirements. It may even be necessary to redefine the primary design specifications to accommodate some of the operational parameters. These trade-offs and their effects on operation were shown and discussed.
Chapter 6

Converter Structure and Components
Converter Structure and Components

In this chapter, two main subjects will be dealt with, namely the design of the saturable magnetic inductor and the physical layout of the converter. With the design of the resonant inductor, reduction in core as well as winding losses is the main concern. These are dealt with in two sections. In the section covering physical layout, it will be shown what current loops have to be minimised. This minimisation reduces current and voltage overshoots as well as high frequency parasitic oscillations.

A number of power electronic applications rely on the behaviour of nonlinear or saturable magnetic components. Examples of this include the Royer and Jensen magnetic oscillators, where saturation of a transformer causes switch over [22]. Magnetic amplifiers rely on the nonlinear characteristics of inductors and transformers, while a family of nonlinear resonant converters were recently introduced [15]. The fact that magnetic amplifiers are used in many computer power supplies to stabilise secondary voltage supplies is an important practical application of the technology of saturating magnetics [3].

As technology advances, the power handling capability and switching frequency of converters are increasing. This places higher demands on magnetic components with switching frequencies rarely more than 40 kHz for saturable components in the kW range.

The major drawback of these components in the high frequency, high power range is heat dissipation. Losses in the windings as well as core losses are too high to be dissipated by a relatively small volume of core material. In this chapter it is shown how core losses as well as winding losses can be reduced.

6-1 Core Losses

Two fundamental constraints important in heat transfer and cooling of the components are discussed in this section. One relates to the winding losses while the other relates to core losses. Winding losses can only be minimised by using a fixed geometry and optimal window height. This means that it is difficult to totally reduce core losses as a certain amount of core material has to be present for an optimal window size. Winding as well as core losses become especially high if the magnetic component is saturated. It is proposed in this section that the core losses of nonlinear magnetic components (which are left after the winding losses have been optimised) can in fact be reduced or even be optimised by only saturating part of the core.

Experimental results obtained from a 200 VA saturable inductor are given. It will be shown that the trade-off between nonlinearity and core losses can be made in such a way that the effect on the operation of the resonant pole inverter will be minimal.
6-1-1 Fundamental Constraints

Losses in transformers and inductors can be divided into core losses and winding losses so that:

\[ P_{\text{total}} = P_{\text{core}} + P_{\text{windings}} \]  \hspace{1cm} (6-1)

In a typical 50 Hz transformer the design has to be optimised for \( P_{\text{core}} = P_{\text{windings}} \) so that core and winding materials are used effectively. At higher frequencies and with ferrite material, a relatively small core is used so that \( P_{\text{core}} \) and \( P_{\text{windings}} \) are kept to a minimum, regardless of their relative sizes. In order to keep both \( P_{\text{core}} \) and \( P_{\text{windings}} \) to a minimum (and therefore an optimal design) there are however, some physical constraints.

Winding geometry constraint

It is not possible to reduce winding losses below a minimum value for a given frequency. Due to the skin and proximity effects an optimal window height exists. If the height is either increased or decreased the losses will increase.

Consider the foil windings in the E-core of figure 6-1. Ideally there exists a certain optimum height \( "b" \) for each of these conductor windings for which the AC resistance is a minimum. The ideal one dimensional height (for the winding window) in a \( N \) turn winding stack which has zero magnetic field below the first turn, can be approximated by the following equation of Snelling [32]:

\[ h_{id} = N \left[ \frac{15}{5N^2 - 1} \right] \delta^{\frac{1}{4}} \]  \hspace{1cm} (6-2)

where \( N \) is the number of turns and \( \delta \) the skin depth. This optimum height of a winding for minimum AC resistance is shown for 20 kHz and 100 kHz in figure 6-2.
Assume that one needs N turns for a specific application and that the window area has to be filled to reduce stray inductance. By taking into account the optimum height for minimum AC resistance there will be an optimum window height as shown in figure 6-3. This window height corresponds to "b" in figure 1, while the window length "a" is a function of the maximum current through the component. This implies that for a certain winding configuration and VA rating the window size of the core is fixed to achieve minimum winding losses.

Figure 6-3: There exists an optimal window height for which a number of windings will have a minimum of winding losses.
Core geometry constraint

Core losses can be divided into hysteresis and eddy current losses. Heat dissipation due to eddy currents in ferrite material is usually so small in comparison to other sources, that it can be ruled out.

Hysteresis losses are proportional to the area enclosed by the characteristic B-H curve. These losses are also proportional to the effective magnetic volume ($V_e$) of core material subject to the applied excitation as well as the number of times the B-H hysteresis loop is completed (or frequency) so that:

$$P_{\text{core}} = \text{constant} \cdot f \cdot V_e$$

(6-3)

where $f$ is the frequency of excitation. For a given frequency, the core losses are therefore directly proportional to $V_e$. The following question now arises: How small must the core be to minimise $P_{\text{core}}$?

The physical core size (represented by $V_e$) is limited by the following factors:

- a certain effective magnetic core area ($A$) must be present to achieve a specified level of flux density
- a certain window size must be present to fit the optimal size and number of windings

Under these physical constraints a specific core size must be used and $P_{\text{core}}$ is therefore a fixed power that must be dissipated. The problem is that under high frequency - high power conditions the core is not thermally capable of dissipating all this power, even if a heat sink is added. The solution therefore would therefore be to reduce $P_{\text{core}}$ without altering the nonlinear properties of the magnetic component.

6-1-2 Core losses

In this section it will be shown how the core losses can be reduced by saturating only a part of the core. This is done by reducing the effective magnetic core area ($A_1$) in a part of the core.

Hysteresis losses are a function of the operating frequency and maximum flux density. For a given frequency the losses per unit volume increase drastically with an increase in maximum flux density. These values can be obtained from core material data books.

If one assumes that the frequency and flux in the core remain constant, the core losses per unit volume as function of $A$ can be represented by figure 6-4.
Power dissipation per unit volume

\[ \frac{W}{m^3} \]

High B
Saturation

\[ \phi = BA = \text{constant} \]

low B

\[ m^2 \]

\[ A \]

Figure 6-4: Power dissipation per unit volume of core material at different values of flux density B.

Figure 6-5: Saturated and unsaturated B-H curves, representing different losses (i.e. area's enclosed).

If for a given core and constant flux, the effective magnetic cross section \( A_1 \) in a certain part of the core is reduced, losses per unit volume will be much higher in that part in comparison with the wider section \( A_2 \).
Figure 6-5 shows two B-H curves with different losses (i.e. enclosed areas) representing the saturating and unsaturated part of the magnetic core. Since the area enclosed by the hysteresis curve of the unsaturated part is smaller than that of the saturated part, losses will be considerably less than with the whole core saturating.

If the inductance of a saturable inductor can be approximated by two values $L_{\text{sat}}$ and $L_{\text{unsat}}$, it will be shown that a component with reduced area for partial saturation complies with a two-step nonlinear inductance. Consider a toroidal core with a number of turns $N$ of which a part is narrowed to reduce the effective magnetic core area in that part. Such a toroid is shown in figure 6-6, with two path lengths $l_1$ and $l_2$ and effective areas $A_1$ and $A_2$.

If there is no flux leakage, then the inductance $L$ of the core can be calculated as:

$$L = \frac{N^2}{\mu_1 A_1 l_1 + \mu_2 A_2 l_2}$$  \hspace{1cm} (6-4)

with $\mu_2$ the permeability of the wider and longer section and $\mu_1$ that of the smaller section, which is meant to saturate first. If it is furthermore assumed that for the unsaturated state $\mu_1 = \mu_2 = \mu_{\text{unsat}}$, and for the saturated state $\mu_2 = \mu_{\text{sat}}$, the inductance of the inductor in the unsaturated state is given by:

$$L_{\text{unsat}} = \frac{\mu_{\text{unsat}} N^2}{\left(\frac{l_1}{A_1}_{\text{unsat}} + \frac{l_2}{A_2}_{\text{unsat}}\right)}$$  \hspace{1cm} (6-5)

In the saturated state and under the practical constraint of:

$$\mu_{\text{sat}} \ll \mu_{\text{unsat}}$$

then:

$$L_{\text{sat}} = \frac{\mu_{\text{sat}} N^2 A_{\text{sat}}}{l_{1_{\text{sat}}}}$$  \hspace{1cm} (6-6)

The toroidal inductor therefore has two distinct inductance states, fitting the requirement of a two step nonlinear inductor.

Since the power dissipation per unit volume (as shown in figure 6-4) is much lower for $A_2$ than $A_1$, the total $P_{\text{core}}$ would be considerably less than what it would have been if the whole core was saturated.
Figure 6-6: Toroidal core with two path lengths $l_1$ and $l_2$ and two effective areas $A_1$ and $A_2$.

For optimal performance in the NLRP, the ratio of the unsaturated to saturated inductance $L_{\text{unsat}}/L_{\text{sat}}$ must be as high as possible. These two criteria, namely a maximum nonlinear ratio and a smallest possible saturating part are now used to obtain the geometrical size of the small part of the core (which is meant to saturate) in relation to the larger part. Once this relation of the geometrical parameters $l_1, A_1, l_2$ and $A_2$ (see figure 6-6) have been established, the total magnetic core design can be completed.

The first point to discuss is the optimisation of the ratio $L_{\text{unsat}}/L_{\text{sat}}$. As it will further on become evident, the maximum value for $L_{\text{unsat}}/L_{\text{sat}}$ or $(L_{\text{unsat}}/L_{\text{sat}})_{\text{max}}$ is achieved if the core has a single and uniform effective magnetic path length $l$ and magnetic cross area $A$. In other words, the core is to be saturated uniformly without one section saturating before the other. The inductance of a uniform core can be described by:

$$L = \frac{\mu N^2 A}{l}$$

(6-7)

with $N$ the number of turns in the winding, $\mu = \mu_r \mu_0$ the permeability of the core material and $A$ and $l$ the effective magnetic area and path length respectively.

If, however, this uniform core is to saturate and it is assumed that the maximum ratio of inductance can be obtained under this condition, then:
In equation (6-8) \( \mu = \mu_r \mu_0 \) with \( \mu_r \) changing from \( \mu_{r\text{unsat}} \) to a smaller \( \mu_{r\text{sat}} \) during saturation. There is also a difference in effective magnetic path lengths and areas before and after saturation. This is because the flux is almost totally contained in the magnetic core material in the unsaturated state, while in the saturated state flux leakage takes place resulting in lowering the value for \( l \) and increasing the value of \( A \).

If the core is composed of two distinct magnetic and geometrical regions \( l_1A_1 \) and \( l_2A_2 \) (figure 6-6), the inductance of the coil can be written as in equations (6-4), (6-5) and (6-6).

The effective magnetic path lengths and areas will now also be different in the saturated and unsaturated case as explained for equation (6-8). The question now arises of what value \( \frac{L_{\text{unsat}}}{L_{\text{sat}}} \) will assume for the case of a core with two distinct magnetic and geometrical areas?

By taking the ratio of \( \frac{L_{\text{unsat}}}{L_{\text{sat}}} \) from equations (6-5) and (6-6) and by substituting equation (6-8) for \( (\frac{L_{\text{unsat}}}{L_{\text{sat}}})_{\text{max}} \) where applicable, the following is obtained for the case where the core saturates in the volume \( l_1A_1 \):

\[
\frac{L_{\text{unsat}}}{L_{\text{sat}}} = \frac{1}{1 + \frac{A_1}{A_2} \left( \frac{L_{\text{unsat}}}{L_{\text{sat}}} \right)_{\text{max}}} \quad (6-9)
\]

It was assumed that \( A_{1,2} \approx A_{1,2\text{unsat}} \) and \( l_{1,2} \approx l_{1,2\text{unsat}} \), or in other words that the geometrical lengths and areas \( l_1, l_2, A_1 \) and \( A_2 \) roughly equal their unsaturated equivalent effective path lengths and areas. This is a good approximation since most of the flux is contained in the core during the unsaturated state.

The second important factor in the core design is the relative volumes of saturated to unsaturated material. As stated earlier, the smaller the volume of saturated magnetic material, the smaller the hysteresis losses of the whole core be. The ratio of the volumes of saturated to unsaturated parts is simply expressed as:

\[
\frac{(\text{Volume})_{\text{sat}}}{(\text{Volume})_{\text{unsat}}} = \frac{l_1A_1}{l_2A_2} \quad (6-10)
\]

The value of expression (6-10) will therefore have to be minimised to obtain the minimum amount of core losses.
Figures 6-7 and 6-8 graphically express the two design factors of volume and unsaturated to saturated inductance ratio to be used for optimal design purposes. In figure 6-7 the value of $L_{\text{unsat}}/L_{\text{sat}}$ is expressed as a fraction of the maximum attainable ratio of $(L_{\text{unsat}}/L_{\text{sat}})_{\text{max}}$. This is a function of $l_1/l_2$ and $A_1/A_2$ directly obtained from equation (6-9). Figure 6-8 shows ratios of the saturated to unsaturated volumes of core material (equation (6-10)) as function of $l_1/l_2$ and $A_1/A_2$.

![Diagram of Fractions of $(L_{\text{unsat}}/L_{\text{sat}})_{\text{max}}$](image)

**Figure 6-7:** Dependence of the ratios of magnetic path lengths and areas on the fraction of the maximum obtainable ratio of unsaturated to saturated inductance.

To be able to design the core, the two ratios of $l_1/l_2$ and $A_1/A_2$ will have to be chosen in such a way that the value of $l_1A_1/l_2A_2$ be minimised and the fraction of $L_{\text{unsat}}/L_{\text{sat}}$ to $(L_{\text{unsat}}/L_{\text{sat}})_{\text{max}}$ be maximised. This will typically occur at low values of $A_1/A_2$ and either high or low values of $l_1/l_2$, depending on the criterion of either volume (figure 6-8) or inductance ratio (figure 6-7). It is therefore a trade-off between the minimum volume saturating and the highest possible nonlinear inductance ratio $L_{\text{unsat}}/L_{\text{sat}}$. Exactly how this trade-off will be made, relies on some practical factors.
It will now be shown from practical results that core losses were reduced by only saturating a part of the core. Data were obtained for two different E-cores. One core was cut to obtain two magnetic regions $I_1A_1$ and $I_2A_2$, while the other core was used in standard form. Results are summarised in table 6-1. Details of the proportions in which the second E-core was cut, are given in table 6-2. Measurements were made at an ambient temperature of 21 °C with forced convection, while 3E1 core material was used. Different numbers of turns were used for the two cores so that the frequency of operation could be the same in both cases. Core losses were estimated using different values of core volumes multiplied with normalised figures for core losses (as obtained from the 3E1 data sheet).

Although it would have been better to use an area ratio ($A_1/A_2$) of around 1/10, it was practically difficult to achieve in practice and an area ratio of 2/10 was used instead. The ratio of $L_{\text{unsat}}/L_{\text{sat}}$ was reduced by almost two thirds for the cut core. If $A_1/A_2$ was smaller (say 1/10) then $L_{\text{unsat}}/L_{\text{sat}}$ would have been closer to $(L_{\text{unsat}}/L_{\text{sat}})_{\text{max}}$ (or the ratio of unsaturated to saturated inductances of the normal E-65 core).
The current through \( L_r \) in the nonlinear resonant pole would have the same effect on the operation of the inverter because the resonant period starts at the same current value. This is in accordance with the necessary factors for good inverter operation.

From table 6-1 it can be seen clearly that the power dissipated due to core losses are considerably less for the core with reduced saturating volume than for the normal shape. This has the effect that (under the same conditions) the final temperatures also differ considerably.

Table 6-1: Summary of measured results:

<table>
<thead>
<tr>
<th>Type of core</th>
<th>Number of turns</th>
<th>Supply Voltage ( V_s )</th>
<th>Freq. ( f_s )</th>
<th>Total dissipated power</th>
<th>Estimated Core losses</th>
<th>Estimated Winding losses</th>
<th>Core temp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal E-65</td>
<td>6</td>
<td>169 V</td>
<td>20 kHz</td>
<td>29 W</td>
<td>26 W</td>
<td>3 W</td>
<td>34°C</td>
</tr>
<tr>
<td>E-65 with reduced section</td>
<td>17</td>
<td>169 V</td>
<td>20 kHz</td>
<td>5 W</td>
<td>2 W</td>
<td>3 W</td>
<td>24°C</td>
</tr>
</tbody>
</table>

Table 6-2: Parameters for core with reduced section

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_1/I_2 )</td>
<td>0.35</td>
</tr>
<tr>
<td>( A_1/A_2 )</td>
<td>0.2</td>
</tr>
<tr>
<td>Fraction of ( (L_{unsat}/L_{sat})_{max} )</td>
<td>0.64</td>
</tr>
<tr>
<td>( I_1A_1/I_2A_2 )</td>
<td>0.07</td>
</tr>
</tbody>
</table>

6-2 Winding Losses

In this section it will be shown how winding losses can be reduced by optimising the inductor winding for the frequency contents of \( i_{Lr}(t) \).

There are two main electro-magnetic effects that influence the losses in a high frequency, high power winding. These are the skin and proximity effects.

With the skin effect, the AC resistance of the winding is increased with the increase in frequency. The tendency of the current to "crowd" on the outer "skin" of the conductor increases the current density on the outer layers so that the effective resistance increases. The skin depth \( \delta \) can be approximated by:

\[
\delta = \frac{2}{\sqrt{\omega \mu \sigma}}
\]  
(6-11)
where $\omega = 2\pi f$, $\mu$ is the permeability of the material and $\sigma$ the conductivity.

Using Litz wire (as opposed to a solid conductor) can reduce winding losses. This is especially true if the diameter of the wire (or thickness of a plate conductor) is larger than $\delta$. The diameter of each strand of Litz wire is made less than the penetration or skin depth $\delta$. The improvement in the AC resistance of Litz wire compared to solid wire of equal radius is shown in figure 6-9.

The electro-magnetic field in each strand of Litz wire influences the current density distribution in the other strands. The deviation in current density distribution due to either the magnetic fields of the nearby strands or even the field distribution in the window of a core, is called the proximity effect. When calculating the optimal diameter and number of strains in a Litz bundle, both the skin and proximity effects must be taken into account.

![Figure 6-9: Resistance of Litz wire compared to solid wire of equal radius, where M is the number of strands in the bundle, "p" the packing factor and "a" the bundle radius.](image)

In figure 6-9, a relation is given between the resistance of a solid wire and Litz bundle of equal radius "a" [12]. The x-axis is given as a normalised radius in terms of the skin depth $\delta$ and the packing factor $p$, where:

$$p = M \left( \frac{r_{st}}{a} \right)^2$$

(6-12)
with "a" the radius of the bundle, \( r_{st} \) the radius of a strand and \( M \) the number of strands.

With \( a \) and \( p \) constants, the x-axis can be viewed as a frequency scale. In optimising a winding design, one would ideally have the resistance of the Litz wire as small as possible over the range of frequencies that are prominent in the current \( i_{LR}(t) \). After the frequency contents of \( i_{LR}(t) \) have been established, it can be compared to a graph like that of figure 6-9 and the number of strands as well as radii for the total bundle and individual strands can be determined.

To make an optimal winding design, the frequency contents of \( i_{LR}(t) \) must first be known. The problem is that the shape of the current flowing through the resonant inductor varies with different modulation depths and load currents.

An estimation of the bandwidth for which the winding has to be designed, can be made from practical measurements.

In figure 6-10 the time function of \( i_{LR}(t) \) is given at zero modulation depth and with no load current flowing. The magnitude spectrum of the waveform in figure 6-10, with relative amplitudes in dB, is given in figure 6-11. In figure 6-12, an amplitude spectrum is shown for \( i_{LR}(t) \) where the converter is supplying a sinusoidal current to the load. In both cases the switching frequency at \( m=0 \) was 11.4 kHz.

When estimating the bandwidth for which the winding has to be designed, the relative amplitudes of the harmonics must be compared to the fundamental. At -20 dB, the \( i^2R \) losses of the harmonic is 1/100 of the fundamental and at -30 dB it is 1/1000 of the fundamental (assuming that \( R \) is constant). A bandwidth can therefore be established at the frequencies where the relative amplitudes are -20dB to -30dB smaller than the fundamental. At higher frequencies the losses would not
play a significant role any more. In the case of figures 6-11 and 6-12, this point would be around 300 kHz.

Figure 6-11: Amplitude spectrum of the waveform in figure 6-10, with $m=0$ and $f_s=11.4$ kHz

After a bandwidth has been selected from the practical data, it can be compared to a graph of winding resistance - such as figure 6-9. The number of strands for the Litz wire as well as bundle and strand diameters can now be chosen. The bandwidth of the measured data must be compared to the frequency scale of figure 6-9 and fitted to a curve that will result in the lowest AC resistance.

Figure 6-12: Measured amplitude spectrum of $i_{Lr}(t)$ where sinusoidal modulation takes place and with the load drawing a sinusoidal current. ($f_s=11.4$ kHz at $m=0$)
6-3 Component Layout

In this section it will be shown how the correct component layout can improve converter performance. To improve performance and counter negative effects, current loops have to be minimised. The specific current loops that have to reduced will be shown.

In any practical power electronics converter, the physical converter layout leads to parasitic inductances and capacitances [5]. When switching occurs, current is commutated from one current path to another. Commutation of current leads to current flanks which has an associated bandwidth of frequencies. The voltages and currents produced by the parasitics, manifest themselves as high frequency oscillations with voltage and current overshoots.

As a rule of thumb the bandwidth $B$ of a signal (in MHz) is associated with its rise time $t_r$ (in $\mu$sec) by the approximation:

$$B \approx \frac{350}{t_r}$$  \hspace{1cm} (6-13)

From (6-13) it can be seen that commutation of the current in a few micro-seconds can lead to harmonic frequencies in the MHz region. This is true even if the switching frequency is at a few kHz.

There are a number of reasons why parasitics effects in a converter must be reduced. These include:
- Breakdown of the device due to over-voltage or over-current.
- Increase in switching losses.
- Increased Electro-Magnetic Interference (EMI).

Depending on the electronic switches used, over-voltage or over-current may damage the device. Parasitic impedances tend to be inductive which result in large voltage overshoots during switch-off. In figure 6-13 oscillations during switch-off are shown. The voltage overshoot is clearly shown.

Switching losses are represented by the area underneath the voltage current product curve. It is calculated for the time that the device is either switching on or off. In figure 6-13 the voltage during the switch-off time is higher than the steady state voltage normally across the switch. Voltage overshoot can therefore increase the switching losses.

The oscillations seen in figure 6-13 can cause either radiated or conducted emissions. Conducted emissions are injected into the supply network through the rectifier circuit. Radiated emissions are emitted from the converter structure that acts as an antenna. Since the high frequency oscillations can typically be in the radio frequency (RF) range ([29] pp. 427-429), interference in the operation of sensitive
equipment may occur. EMI is therefore an important consideration in the design and reduction of parasitics.

In practical converters, parasitics may be due to transmission line effects [5] or the inclusion of flux by current loops. As current loops have the largest contribution to parasitic effects it will be discussed in this study.

Figure 6-13: Voltage across and current through a switch during switch-off and with parasitic inductance present.

Figure 6-14: Current loop that is supplying the load, which must be minimised to reduce stray inductance.
Current loops that link with the flux, produced by the current flowing through those loops, manifest themselves as stray inductances. Voltages produced by the stray inductances are responsible for the type of overshoot shown in figure 6-13. This voltage is related to the current flowing through the loop and stray inductance $L_{\text{stray}}$ by:

$$v(t) = L_{\text{stray}} \frac{di(t)}{dt} \quad (6-14)$$

The inductance $L_{\text{stray}}$ is a function of geometry so that $v(t)$ in (6-14) is largely dependent on the size of $di(t)/dt$. The most important current loops to minimise are therefore those which are subject to high $di(t)/dt$'s.

In the NLRP converter two types of current loops have to be minimised. The first are those associated with a normal hard switching inverter and the second are loops more specific to the NLRP.

![Diagram](university-of-johannesburg.com)

Figure 6-15: Current loops that have to be minimised if a DC-link capacitor is present.

The first are those associated with a normal hard switching inverter. These include the DC-link loop as well as the loop between the devices and the DC-link capacitor. In figure 6-14, the total loop supplying the load is shown. If $S1$ is conducting, current is carried via the positive rail to the load. When $S1$ is switched off, current is commutated to the bottom rail and $D2$ starts to conduct. During commutation, the $di(t)/dt$ in the loop is high as current in the positive rail falls to zero and current in the bottom rail climbs from zero to $I_{\text{load}}$. The same applies to a circuit where a DC-link capacitor is present. The two loops that have to be minimised are shown as ① and ② in figure 6-15.
Loops that are more specific to the NLRP are those associated with the resonant capacitors. These are shown as ① and ② in figure 6-16. From figure 6-16 it can also be seen that the current flowing through $C_r$ has a very high $di/dt$. This is because the two resonant capacitors must supply current during resonant switchover. During this time all the devices are turned off and current are supplied by the resonant capacitors.

Voltage overshoot due to the stray inductance in the resonant capacitor loop can be reduced by physically mounting the two $C_r$'s as close as possible to the devices. It was also shown by de Villiers [7] that the introduction of nonlinear resonant capacitors can reduce voltage overshoot across the devices.
In this chapter three aspects of converter design were discussed. The first two included the minimisation of core as well as winding losses in the nonlinear magnetic component. The third is the reduction of current loops that leads to reduced switching losses, as well as reduced EMI and overshoots.

It was shown that core losses can be reduced by only saturating a part of the core, but this leads to reduced nonlinearity. Winding losses can be minimised by specifying Litz wire that would have a low AC resistance for the most prominent frequencies in the inductor current. Because of the complexity of the $i_L(t)$ waveform it was suggested that the frequency content be determined by measurement.

In the last section it was shown that the reductions of current loops by proper physical lay-out reduces stray inductance. The reduction of this parasitic inductance leads to improved converter operation.
Chapter 7

Simulation and Experimental Results
Simulation and Experimental Results

7-1 Circuit Simulation

A simulation of the NLRP with feedback winding was made using PSPICE. The simulation is a small signal analysis of a few switching cycles around a specific working point of load current and modulation depth. The simulation was done for the following reasons:

- To gain insight into the behaviour of the resonant inductor current for different working points of modulation depth and load current.
- To determine the ripple on \( v_{CS}(t) \).
- To determine the overshoot and high frequency oscillations caused by parasitic inductances during switch over.

The circuit used for the simulations is shown in figure 7-1. A complete listing of the code is given in Appendix C.

![Circuit Diagram](image)

Figure 7-1: Circuit diagram for the PSPICE simulation.

The simulation circuit has two features that distinguishes it from the analytical description of chapter 3.

In the analytical description, two discrete inductance values were used to describe the saturating inductor. In the simulation, the resonant inductor consist of a model of
a physical E-core with the number of turns for the inductor and feedback windings specified. The inductor model also includes the magnetic parameters of the 3C8 magnetic core material used. The result of this is that the current $i_{L_r}(t)$ obtained in the simulation bears a closer resemblance to the true current than the analytical description.

![Figure 7-2: AND-circuit controlling top switch XIGBT1.](image)

The second advantage of the simulation is the ease with which parasitic elements can be modelled. This is in contrast with the analytical description where new equations have to be derived for each new element included in the circuit. It was found by De Villiers [8] that the simulation had an advantage over the analytical model in showing the effects of nonlinear resonant capacitors in the presence of parasitic oscillations and overshoots.

In addition to figure 7-1, two control circuits are used to supply the switching signals to XIGBT1 and XIGBT2. Each of the two circuits performs a logical AND function. The switches are turned on if two conditions are met. The first is that the current through $L_r$ (i.e. $L_1$) must have a minimum threshold value $I_{\text{min}}$ and secondly the voltage across the switch must be zero or below a few volts. In the AND circuit these two conditions are implemented by a current controlled switch (measuring current through $V_{\text{L_measure}}$) and a voltage controlled switch that monitors the voltage across XIGBT1 or XIGBT2. The controlling AND circuit for XIGBT1, is shown in figure 7-2. If both $Wg1$ and $Sg1$ are on, the $5V$ across $Rg1$ will turn XIGBT1 on.
Table 7-1: Parameters and component values used for results of figures 7-3 and 7-4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_S$</td>
<td>200 V</td>
</tr>
<tr>
<td>$C_S$</td>
<td>12 μF</td>
</tr>
<tr>
<td>$C_r$</td>
<td>55 nF</td>
</tr>
<tr>
<td>Core</td>
<td>E-65</td>
</tr>
<tr>
<td>$n_r$</td>
<td>13 turns</td>
</tr>
<tr>
<td>$m$</td>
<td>0.67</td>
</tr>
<tr>
<td>$I_{\text{thresh}1}$</td>
<td>35 A</td>
</tr>
<tr>
<td>$I_{\text{thresh}2}$</td>
<td>-15 A</td>
</tr>
<tr>
<td>$I_{\text{load}}$</td>
<td>0 A</td>
</tr>
</tbody>
</table>

Figures 7-3 and 7-4 show measured and simulated results for $i_{Lr}(t)$ and $v_{Cs}(t)$. The parameters and component values used are given in table 7-1.

From figures 7-3 and 7-4 it can be concluded that the simulated results generally resemble the measured waveforms. In figure 7-3, it is seen that the model used for the core does not truly represent the actual nonlinear inductor. This causes the saturated inductance to vary, giving rise to a discrepancy in waveforms once the core has saturated. The simulation is however a very useful tool in assessing the general shape of $i_{Lr}(t)$.

In figure 7-4 it is seen that the simulated waveform is a good indication of the shape and amplitude of $v_{Cs}(t)$. This is useful because the ripple on $v_{Cs}(t)$ must be established by simulation as it is difficult to obtain a precise $v_{Cs}(t)$ waveform from an analytical equation (see also chapter 3-6). In figure 7-4 the simulated value $v_{Cs}(t)$ increases between $t_1$ and $t_2$, since the model used for simulation does not include a
control system that keeps $v_{CS}(t)$ at a constant modulation depth. In the practical system represented by the measured waveform this control is already included.

![Graph showing measured and simulated voltage $v_{CS}(t)$ with time (sec) on the x-axis and voltage (in V) on the y-axis.]

**Figure 7-4**: Measured and simulated voltage $v_{CS}(t)$.

![Graph showing simulated waveforms and stray inductances effects during switch-off of S1 with time (sec) on the x-axis and current (A) and voltage (V) on the y-axis.]

**Figure 7-5**: Simulated waveforms showing the effects of stray inductances during switch-off of S1.
In the beginning of this section it was stated that the simulation can also be used to study oscillations and overshoots caused by parasitic inductances. An example of this is given in figure 7-5. The parameters and component values used are the same as in table 7-1. The only difference is the added stray inductance in the loops between the switches and source capacitors. The value used is 350 nH.

Figure 7-5 gives the waveforms of $i_{LR}(t)$, the voltage $v_S(t)$ across the top switch and current through the top resonant capacitor $i_{C_R}(t)$. The traces are at the time of resonant transition from $S_1$ to $D_2$. The voltage overshoot across $S_1$ and the oscillations in the resonant capacitor current can clearly be seen.

## 7-2 Practical System

### 7-2-1 Total System

The system used for an experimental set-up is shown in figure 7-6. It can be divided into two sub-systems. The first is the control system and the second the power circuit or physical NLRP.

![Figure 7-6: Half-bridge NLRP used in the practical set-up.](image)
The control system is a mixture of analog and digital electronics, with the reference current signal as input. Two switching signals S1 and S2 are used as commands to the gate drive circuits for the switches S1 and S2. These commands are relayed via optical links while measurements of the load and inductor currents are made with Hall-effect current transducers. The control system is therefore galvanically isolated from the power electronics section. Galvanic isolation reduces interference in the control electronics, which can be induced by the adjacent high current switching.

**7-2-2 Control**

The control circuit used in the experimental set-up is based on the integral threshold control described in chapter 4-3. This is a refinement of the current threshold control that was used in other studies ([4],[7],[8],[13],[14],[15],[17] and [33]).

With integral threshold control the error signal is compared to the integral of $i_{Lr}(t)$ and the switches are turned off accordingly. A block diagram of the circuit is given in figure 7-7.

![Block diagram of the integral threshold control used.](image)
The error signal is obtained by subtracting the load current from the reference input. The error signal is multiplied with a constant $K$ and compared to the output of the integrator. There are two comparators controlling switch-off of the two switches.

The integrator is reset by a signal which is controlled by the reference input and switchover of the two signals $S1$ and $S2$. If the reference current is positive, the load current is controlled by switching off $S1$ only. A positive reference would therefore cause the integrator to reset only when $S1$ is switched off. Likewise the load current is controlled by $S2$ only if the reference is negative. The integrator resets after $S2$ is switched off and a new switching cycle is started. For a zero reference input, the load current is controlled by both switches and the integrator resets if either $S1$ or $S2$ is switched off.

To ensure that resonant switchover will be complete, a minimum inductor current condition must be adhered to. The current through the resonant inductor is compared to a minimum threshold before switch-over can take place. If this condition is not met, switchover will be delayed until the inductor current has reached the threshold value.

Over-current protection is done by monitoring the switch current $i_s(t)$. The switch current is the sum of $i_{Lr}(t)$ and $i_{load}(t)$. A maximum value of switch current is set and compared to $i_s(t)$. If the current through a switch exceeds the pre-set maximum, that switch will be turned off by a logical high applied to the OR-gate that controls the input to the flip-flop.

![Diagram](image-url)

**Figure 7-8: Gate drives with zero voltage detection and optical link.**

Zero voltage switching is ensured by the gate drives. To keep the control system isolated from the power section, the switching signals are relayed via optical links. Detection of zero voltage conditions across the switches is therefore done by the
gate drives and not the control system. Typically a gate drive would receive a signal to switch on an IGBT. The gate drive measures the voltage $v_{\text{sense}}$ (figure 7-8) across the switch. If a zero voltage condition does not exist the voltage across the gates of the IGBT's is kept negative by $v_{\text{gate}}$. Only after an "on"-signal has been received and the voltage $v_{\text{sense}}$ is at least zero volt, the IGBT will be turned on by making $v_{\text{gate}}$ high.

7-2-3 Start-up

The control system described in 7-2-1 cannot automatically be used for start-up, since the gate drives in figure 7-8 will only turn on a switch under zero voltage conditions. When the DC power source is turned on, the voltage $v_{\text{c}}$ (figure 7-6) will be midway between $+V_{S}/2$ and $-V_{S}/2$. A zero voltage condition does not exist across S1 or S2 and neither of the gate drives will switch on an IGBT. Start-up can only be achieved by ensuring a zero voltage condition with the use of an external circuit.

Figure 7-9: External circuit for starting operation of the NLRP.

At power on, the control circuit applies a logical high to the gate drive of S2. For S2 to be switched on, a zero voltage condition must exist. This is achieved by temporarily short circuiting S2 with the circuit shown in figure 7-9. An extra IGBT is used to connect an inductor across S2 for a specified length of time. The inductor is used to prevent a large current from flowing through the external IGBT. Zero voltage across S2 is achieved the moment the external IGBT is turned on and the NLRP starts to operate.
7-3 Results

In this section different practical results are given. The measurements were taken from the experimental system described in Appendix A.

Load Voltage and Current

Figure 7-10 shows typical waveforms of reference and load currents. The average output voltage $v_{CS}(t)$ is also shown. The reference signal is a sine wave with a frequency of 50 Hz. The ripple on $i_{load}(t)$ as well as $v_{CS}(t)$ is clearly visible. The switching frequency is between 6.5 and 9.3 kHz.

![Figure 7-10: Typical waveforms of reference and load current as well as the average output voltage $v_{CS}(t)$.

Integral Threshold Controller

Section 7-2-2 describes the control logic used in the experimental converter. Fundamental to controller operation is the output of the integrator. Integrator output signals with the input inductor current $i_{Lr}(t)$ are shown in figures 7-11 and 7-12.

Figure 7-11 shows signals at zero modulation depth. With each switchover the integrator is reset.
Figure 7-12 shows signals at a positive modulation depth and with a positive load current. Resetting of the integrator takes place when S1 is turned off.

Figure 7-11: Measured resonant inductor current and output of integrator at zero modulation depth.

Figure 7-12: Measured resonant inductor current and output of integrator at non-zero modulation depth and with a positive load current flowing.
Switching Frequency

Measured values of the switching frequency at different modulation depths are shown in figure 7-13. System parameters are as described in Appendix A. Also shown in figure 7-13 is a linear approximation of switching frequency from the analysis in chapter 3-3. Details of the calculation can be found in Appendix B. Within the limits of modulation depth the measured values correspond well with those calculated from the estimation in chapter 3-3.

Figure 7-13: Measured and calculated switching frequency.

Large Signal Transfer Characteristics

Chapter 4-4-3 describes the estimation of the large signal transfer function by linearising the controller. The following linear relationships were used to estimate large signal response:

\[
\frac{I_{\text{load}}(s)}{I_{\text{ref}}(s)} = \frac{K \cdot R_s}{2 \cdot s^2 \cdot C_s \cdot R_s \cdot L_{\text{load}} + (2 \cdot C_s \cdot R_s \cdot R_{\text{load}} + 2 \cdot L_{\text{load}}) \cdot s + 2 \cdot R_{\text{load}} - K \cdot R_s} \tag{7-1}
\]

\[
\frac{V_{C_s}(s)}{I_{\text{ref}}(s)} = \frac{K \cdot R_{\text{load}} + s \cdot L_{\text{load}} \cdot R_s}{2 \cdot s^2 \cdot C_s \cdot R_s \cdot L_{\text{load}} + (2 \cdot C_s \cdot R_s \cdot R_{\text{load}} - 2 \cdot L_{\text{load}}) \cdot s + 2 \cdot R_{\text{load}} - K \cdot R_s} \tag{7-2}
\]

These estimations were made so that \( C_s \), a fundamental design parameter, can be chosen.
Results for the transfer function from input reference signal to output load current are given in figures 7-14 and 7-15. Figure 7-14 is a Bode plot of relative amplitude and figure 7-15 of relative phase shift. The solid lines are calculated from the linear estimation (7-1) for different gain factors $K$. The gain factor $K$ resembles the total open loop gain and includes the controller.

![Figure 7-14: Relative amplitude plot of input reference to output load current.](image)

In figures 7-14 and 7-15 two sets of measured results are shown. The parameters and practical system used are given in appendix A. The first set of measurements (indicated by the triangular stars) was taken with a combined gain (open loop gain and integral controller) of 0.004. The measurements indicated with dots were taken with a combined gain of 0.01.

Figures 7-14 and 7-15 show that the estimation is not an exact match. It does however, resemble the general trends of the measured points, concluding that the estimation is a good enough indication of large signal response.

In chapter 4 it was explained that the linear estimation include a parameter $R_S$ which has to be chosen from practical measurements. The transfer characteristics from reference to load current as well as reference to average output voltage $v_{CS}(t)$ can be used for this purpose.

Figures 7-16 and 7-17 show relative amplitude and phase plots for the transfer function of input reference to average output voltage $v_{CS}(t)$. The combined open loop and integral controller gain for the practical system was 0.01. The estimated plots for phase angle were independent of gain.
Figure 7-15: Relative phase angle for the transfer function from input reference to average output voltage.

Figure 7-16: Relative amplitude difference for transfer function of reference input to average output voltage $v_{CS}(t)$. 
From the type of results shown in figures 7-16 and 7-17, a value for $R_s$ in the estimations of (7-1) and (7-2) can be chosen. Once this has been done, the large signal frequency response of the whole system can be estimated and a value of $C_s$ be chosen to fit the frequency response to design criteria.

In this chapter the simulation of current and voltage waveforms with PSPICE was discussed. It was shown that these simulations compared favourably to measured waveforms. Some aspects of practical system operation were given. These include control and start-up schemes. Practical results of waveforms, control signals and low frequency transfer function behaviour were also shown.
Conclusion

In this chapter final comments about this study are made. The contributions of this study, the extent of the Nonlinear Resonant Pole technology and possible future work are discussed.

In this study an effort was made to establish the technology of the Nonlinear Resonant Pole. This was done by describing important aspects of converter operation and design.

In chapter 2 a systematic approach was used to describe the evolution of the NLRP and to distinguish it from other soft-switching topologies. Chapter 3 gave an analytical description of operation as well as constraints in operation. Chapter 4 dealt with the control of the NLRP. In chapter 5, different aspects in the overall design were addressed. Chapter 6 dealt with the design of the nonlinear magnetic component and converter lay-out. Circuit simulation and practical results were given in chapter 7.

Contributions of this study

Specific contributions were made, including:

- The systematic outlining of the evolution of the NLRP and its relation to other soft-switching topologies.
- The analytical description of currents and voltages during the resonant transition interval.
- Estimation of the switching frequency - taking into account the effect of load current.
- A description of the limiting factors of modulation depth and therefore successful operation. These include the range of voltage and load current values for a given set of component values. These limits are crucial in the design process.
- The development of an Integral Threshold Controller.
- Development of a linearised low frequency model that is used in overall dimensioning.
- Description of the steps and trade-offs in the design of an NLRP converter.
- The reduction of core losses by only saturating part of the core.
- The circuit simulation for operation at switching frequencies.

Taking all the above into account, the main contribution is an outline of aspects of the NLRP that will establish this specific technology.

Advantages and disadvantages

The Nonlinear Resonant Pole is a generic soft-switching topology that can be used for different reasons. These include:

- Reduction in switching losses and an increased over-all converter efficiency.
- Reduced $di/dt$ and $dv/dt$ stresses on the devices which lead to longer device life or the possibility of using smaller or cheaper devices.
- Reduced EMI.
- The use of diodes with slow reverse recovery times.

The aim in employing NLRP technology is therefore to construct cheaper, smaller and more reliable converters.

Using the NLRP as a converter structure has the following disadvantages:
- The topology requires extra components as part of the resonant circuit.
- The resonant components are subject to losses. This is especially true for the nonlinear inductor, which is driven into saturation. Relatively large losses occur in both the windings and core.
- The switching frequency is not constant and is a function of modulation depth.
- The amplitude of the average output voltage is limited by the modulation depth constraints. A lower output voltage as compared to equivalent hard-switching converters is obtained.
- Successful resonant transition has to occur during the switch-over process. If this does not happen, zero voltage switching may be lost or the converter may stop operating.
- Successful resonant transition is a function of the load phase angle. As the power factor of the load becomes smaller, it limits the maximum obtainable modulation depth.
- Because of the resonant transition process, the NLRP requires complex control circuitry.

Possible future work

Possible future work includes the following:
- A more accurate description of the commutation limits. This may include the re-modelling of the nonlinear inductor to include energy stored in the unsaturated state. It can also include losses that occur in different components of the converter.
- As discussed in chapter 2, the use of nonlinear capacitors for $C_S$ can improve the modulation depth constraint.
- Work can still be done on the control system. This includes high frequency modelling and the inclusion of sliding-mode control.
- The winding design of the nonlinear inductor can be optimised.

In comparison with hard switching converters, the Nonlinear Resonant Pole shows promising improvements in some aspects of operation. The main feature of this topology seems to be the reduction of stresses on switching devices. This technology can especially be useful where the cost of devices is large in comparison with other converter components.

Thus, usage of the Nonlinear Resonant Pole as a power electronics converter is a novel idea with numerous advantages and some disadvantages. Whether this topology will withstand the test of time and be incorporated into industrial technology, remains to be seen.
Chapter 9

References
References


Appendix A

Detail of Experimental Phase Arm
Detail of Experimental Phase Arm

The detail of the experimental phase arm used in some of the measurements of this study is given in this appendix. A diagram of the experimental phase arm is given in figure A-1.

The phase arm of figure A-1 is in a half bridge configuration. Instead of using two separate DC voltage sources, two large capacitors of 6 mF each are used.

Two resistors are connected across $C_{s1}$ and $C_{s2}$. These have two functions. The first is for added stability of voltage $v_{Cs}(t)$ (and to keep it from floating) and the second is to act as bleeding resistors.

Specific values and other detail:

$$C_{r1} = C_{r2} = C_{r} = 55nF$$
$$C_{s1} = C_{s2} = C_{s} = 12 \mu F$$

Nonlinear magnetic component:

Core: E-65 (3C8 Material) $n_{r}=13$  $n_{fb}=3$
Appendix B

MathCAD™ Listings
Appendix B-1

Analytical Description
Analytical description of the Nonlinear Resonant Pole with Feedback Winding

\[ V_s/2 \]
\[ C_{s1} \]
\[ - \]
\[ L_r \]
\[ i_{lr}(t) \]
\[ S_r/D_1 \]
\[ C_{r1} \]
\[ V_n(t) \]
\[ LOAD \]
\[ i_{load}(t) \]
\[ V_{out}(t) \]
\[ n \]
\[ n_{fb} \]
\[ C_{s2} \]
\[ V_s/2 \]

**Constants**

- \( V_s = 200 \)  
  Input DC voltage (V)
- \( C_s = 3.3 \cdot 10^{-6} \)  
  Value of single source side half bridge capacitor (F) (3.3 mF)
- \( C_r = 24 \cdot 10^{-9} \)  
  Value of single resonant capacitor (F)
- \( L_{unsat} = 1.35 \cdot 10^{-3} \)  
  Value of unsaturated inductance (H)
- \( k_{sat} = 0.65 \)  
  Fraction of saturated to unsaturated inductance
- \( I_{sat} = 0.8 \)  
  Value of current for which the resonant inductor saturates (A)
- \( n_r = 200 \)  
  Number of turns for the resonant inductor
- \( n_{fb} = 50 \)  
  Number of turns for the feedback winding
- \( I_{load} = 3.0 \)  
  Load current
- \( I_{thresh1} = 12 \)  
  Value of inductor current \( (i_{Lr}) \) for which switch \( S_1 \) is turned off (2.0 A)
- \( m = 0.8 \)  
  Modulation index \( (-1 < m < 1) \)

**Calculated parameters**

\[
L_{sat} = \frac{L_{unsat}}{k_{sat}} \quad \text{Value of saturated inductance}
\]

\[
k_{fb} = \frac{n_{fb}}{n_r} \quad \text{Feedback factor}
\]

Value for \( I_{thresh2} \) (the resonant inductor current value at which \( S_2 \) is switched off) is given for steady state by:

\[
I_{thresh2} = -\frac{1}{\sqrt{L_{sat}}} \cdot \sqrt{4 \cdot k_{fb} \cdot I_{load} \cdot I_{sat} \cdot L_{unsat} - L_{sat} \cdot I_{thresh1}}^2 \quad I_{thresh2} = -17.251
\]

**Calculation of time intervals**

**Time loop for all intervals:**

- \( t = 0.8 \cdot 10^{-7} \ldots 18 \cdot 10^{-5} \)
- \( v_{Cs,0} = m \cdot \frac{V_s}{2} \)
- \( v_{Lr,0} = 0 \)
- \( i_{Lr}(t) = I_{thresh2} - k_{fb} \cdot I_{load} \) (where \( i_{Lr}(t) = i_L(t) + k_{fb} \cdot I_{load} \))

**Functions for plotting**

- \( v_{Cs,plot}(t) = 0 \)
- \( i_{Lr,plot}(t) = 0 \)
\[\begin{align*}
V_L(t) &= i_L(t) \\
\text{Time interval } [t_0; t_1] & \\
& \quad v_{Cs}(t_0) = v_{Cs_0} \quad i_L(t_0) = -18.001 \quad L_r = L_{sat} \quad \omega = \frac{1}{\sqrt{2 C_s L_r}} \\
& \quad t_0 = 0 \quad v_{Cs}(t) := \left( v_{Cs}(t_0) - \frac{V_s}{2} \right) \cos[\omega(t - t_0)] + \frac{k_{fb} I_{load} + i_L(t_0)}{2\omega C_s} \sin[\omega(t - t_0)] + \frac{V_s}{2} \\
& \quad i_L(t) := \left( i_L(t_0) + k_{fb} I_{load} \right) \cos[\omega(t - t_0)] - \frac{v_{Cs}(t)}{\omega L_r} - \frac{V_s}{2} \sin[\omega(t - t_0)] - k_{fb} I_{load} \\
& \quad v_{Cs_plot}(t) := \text{if}(t_0 < t, v_{Cs}(t), v_{Cs_plot}(t)) \quad i_{Lr_plot}(t) := \text{if}(t \geq t_0, i_L(t) + k_{fb} I_{load}, i_{Lr_plot}(t)) \\
& \quad \text{Estimate : } x = 1 \times 10^{-6} \quad t_1 = \text{root}(i_L(x) + I_{sat}, x) \\
& \quad t_1 = 1.159 \times 10^{-5} \quad i_L(t_0) = -18.001 \quad i_L(t_1) = -0.8 \quad v_{Cs}(t_0) = 80 \quad v_{Cs}(t_1) = 63.442 \\
\text{Time interval } [t_1; t_2] & \\
& \quad v_{Cs}(t_1) = 63.442 \quad i_L(t_1) := I_{sat} \quad L_r := L_{unsat} \quad \omega = \frac{1}{\sqrt{2 C_s L_r}} \\
& \quad v_{Cs}(t) := \left( v_{Cs}(t_1) - \frac{V_s}{2} \right) \cos[\omega(t - t_1)] + \frac{k_{fb} I_{load} + i_L(t_1)}{2\omega C_s} \sin[\omega(t - t_1)] + \frac{V_s}{2} \\
& \quad i_L(t) := \left( i_L(t_1) + k_{fb} I_{load} \right) \cos[\omega(t - t_1)] - \frac{v_{Cs}(t_1) - \frac{V_s}{2}}{\omega L_r} \sin[\omega(t - t_1)] - k_{fb} I_{load} \\
& \quad v_{Cs_plot}(t) := \text{if}(t_1 \geq t, v_{Cs}(t), v_{Cs_plot}(t)) \quad i_{Lr_plot}(t) := \text{if}(t_1 \geq t_1, i_L(t) + k_{fb} I_{load}, i_{Lr_plot}(t)) \\
& \quad \text{Estimate : } x = t_1 \quad t_2 = \text{root}(i_L(x) - I_{sat}, x) \\
& \quad t_1 = 1.159 \times 10^{-5} \quad t_2 = 7.49 \times 10^{-5} \quad i_L(t_1) = -0.8 \quad i_L(t_2) = 0.8 \quad v_{Cs}(t_1) = 63.442
\end{align*}\]
Time interval $[t_2; t_3]$

$$v_{CS}(t_2) = 70.919 \quad i_L(t_2) = I_{sat} \quad L_r = L_{sat} \quad \omega = \frac{1}{\sqrt{2} C_s L_r}$$

$$v_{CS}(t) = \left( v_{CS}(t_2) - \frac{v_s}{2} \right) \cos[\omega \cdot (t - t_2)] + \frac{k_{fb} \cdot I_{load} + i_L(t_2)}{2 \omega C_s} \sin[\omega \cdot (t - t_2)] + \frac{v_s}{2}$$

$$i_L(t) = \left( i_L(t_2) + k_{fb} \cdot I_{load} \right) \cos[\omega \cdot (t - t_2)] - \frac{v_{CS}(t_2) - \frac{v_s}{2}}{\omega L_r} \sin[\omega \cdot (t - t_2)] - k_{fb} \cdot I_{load}$$

$$v_{CS \_plot}(t) = \text{if}(t \geq t_2, v_{CS}(t), v_{CS \_plot}(t)) \quad i_{Lr \_plot}(t) = \text{if}(t \geq t_2, i_L(t) + k_{fb} \cdot I_{load} + i_{Lr \_plot}(t))$$

Find the value for $t_3$ where $i_{Lr}(t_3) = i_{Lr}(t_3) + k_{fb} \cdot I_{load} = I_{thresh1}$

Estimate: $x = t_2 \quad t_3 = \text{root}(i_L(x) + k_{fb} \cdot I_{load} - I_{thresh1}, x)$

$$t_2 = 7.49 \times 10^{-5} \quad t_3 = 8.238 \times 10^{-5} \quad i_L(t_2) = 0.8 \quad i_L(t_3) = 11.25$$

Time interval $[t_3; t_4]$

$$v_c(t_3) = \frac{V_s}{2} \quad i_L(t_3) = I_{thresh1} \cdot k_{fb} \cdot I_{load} \quad v_{CS}(t_3) = 72.763 \quad L_r = L_{sat} \quad \omega = \frac{C_r + C_s}{\sqrt{2} C_s C_r L_r}$$

$$i_L(t) = \left[ i_L(t_3) + I_{load} \left( \frac{C_s}{C_s + C_r} + k_{fb} \right) \right] \cos[\omega \cdot (t - t_3)] - \frac{v_{c}(t_3) - v_{CS}(t_3)}{\omega L_r} \sin[\omega \cdot (t - t_3)] - I_{load} \left( \frac{C_s}{C_s + C_r} + k_{fb} \right)$$

$$v_{CS}(t) = \left[ v_{CS}(t_3) - \frac{v_{c}(t_3)}{2 \omega C_s} \left( \frac{C_r}{C_s + C_r} \right) \right] \cos[\omega \cdot (t - t_3)] - \frac{I_{load}}{2 \omega} \left( i_L(t_3) + k_{fb} \cdot I_{load} \right) \sin[\omega \cdot (t - t_3)]$$

$$+ \frac{v_{c}(t_3) - v_{CS}(t_3)}{2 \omega} \left( \frac{C_s}{C_s + C_r} \right) \cos[\omega \cdot (t - t_3)]$$

$$v_c(t) = \left[ v_c(t_3) - \frac{v_c(t_3)}{2 \omega} \left( \frac{C_r + C_s}{C_r + C_s + C_s} \right) \right] \cos[\omega \cdot (t - t_3)]$$

$$+ \frac{I_{load}}{2 \omega} \left( \frac{1}{C_r + C_s} \right) \left[ (1 + k_{fb}) \cdot I_{load} + i_L(t_3) \right] \sin[\omega \cdot (t - t_3)]$$

$$+ \frac{v_{c}(t_3) - v_{CS}(t_3)}{2 \omega} \left( \frac{C_r + C_s}{C_r + C_s + C_s} \right) \cos[\omega \cdot (t - t_3)]$$

$$v_{CS \_plot}(t) = \text{if}(t \geq t_3, v_{CS}(t), v_{CS \_plot}(t)) \quad i_{Lr \_plot}(t) = \text{if}(t \geq t_3, i_L(t) + k_{fb} \cdot I_{load} + i_{Lr \_plot}(t))$$

Solve for $t$ where $v_c(t) = -\frac{V_s}{2}$

Estimate: $x = t_3 + 10^{-6} \quad t_4 = \text{root}\left(v_c(x) + \frac{V_s}{2}, x\right)$

$$t_3 = 8.238 \times 10^{-5} \quad t_4 = 8.305 \times 10^{-5} \quad i_L(t_3) = 11.25 \quad i_L(t_4) = 8.788$$
Time interval \([t4;t5]\)

\[v_{Cs}(t_4) = 73.913 \quad i_{L}(t_4) = 8.788 \quad L_r = L_{sat} \quad \omega = \frac{1}{\sqrt{2C_s L_r}}\]

\[v_{Cs}(t) = \left(v_{Cs}(t_4) + \frac{V_s}{2}\right) \cos(\omega(t-t_4)) + \frac{k_{fb} I_{load} + i_{L}(t_4)}{2 \omega C_s} \sin(\omega(t-t_4)) - \frac{V_s}{2}\]

\[i_{L}(t) = \left(i_{L}(t_4) + k_{fb} I_{load}\right) \cos(\omega(t-t_4)) - \frac{v_{Cs}(t_4) + \frac{V_s}{2}}{\omega L_r} \sin(\omega(t-t_4)) - k_{fb} I_{load}\]

\[v_{Cs}_{plot}(t) = \text{if} (t \geq t_4, v_{Cs}(t), v_{Cs}_{plot}(t)) \quad i_{Lr}_{plot}(t) = \text{if} (t \geq t_4, i_{L}(t) + k_{fb} I_{load}, i_{Lr}_{plot}(t))\]

Estimate: \(x = t_4\)

\(t_4 = 8.305 \times 10^{-5}\) \(t_5 = 8.410 \times 10^{-5}\) \(i_{L}(t_4) = 8.788\) \(i_{L}(t_5) = 0.8\)

Time interval \([t5;t6]\)

\[v_{Cs}(t_5) = 74.713 \quad i_{L}(t_5) = I_{sat} \quad L_r = L_{unsat} \quad \omega = \frac{1}{\sqrt{2C_s L_r}}\]

\[v_{Cs}(t) = \left(v_{Cs}(t_5) + \frac{V_s}{2}\right) \cos(\omega(t-t_5)) + \frac{k_{fb} I_{load} + i_{L}(t_5)}{2 \omega C_s} \sin(\omega(t-t_5)) - \frac{V_s}{2}\]

\[i_{L}(t) = \left(i_{L}(t_5) + k_{fb} I_{load}\right) \cos(\omega(t-t_5)) - \frac{v_{Cs}(t_5) + \frac{V_s}{2}}{\omega L_r} \sin(\omega(t-t_5)) - k_{fb} I_{load}\]

\[v_{Cs}_{plot}(t) = \text{if} (t \geq t_5, v_{Cs}(t), v_{Cs}_{plot}(t)) \quad i_{Lr}_{plot}(t) = \text{if} (t \geq t_5, i_{L}(t) + k_{fb} I_{load}, i_{Lr}_{plot}(t))\]

Estimate: \(x = t_5\)

\(t_5 = 8.492 \times 10^{-5}\) \(t_6 = 9.674 \times 10^{-5}\) \(i_{L}(t_5) = 0.8\) \(i_{L}(t_6) = -0.8\)

Time interval \([t6;t7]\)

\[v_{Cs}(t_6) = 83.212 \quad i_{L}(t_6) = I_{sat} \quad L_r = L_{sat} \quad \omega = \frac{1}{\sqrt{2C_s L_r}}\]

\[v_{Cs}(t) = \left(v_{Cs}(t_6) + \frac{V_s}{2}\right) \cos(\omega(t-t_6)) + \frac{k_{fb} I_{load} + i_{L}(t_6)}{2 \omega C_s} \sin(\omega(t-t_6)) - \frac{V_s}{2}\]

\[i_{L}(t) = \left(i_{L}(t_6) + k_{fb} I_{load}\right) \cos(\omega(t-t_6)) - \frac{v_{Cs}(t_6) + \frac{V_s}{2}}{\omega L_r} \sin(\omega(t-t_6)) - k_{fb} I_{load}\]

\[v_{Cs}_{plot}(t) = \text{if} (t \geq t_6, v_{Cs}(t), v_{Cs}_{plot}(t)) \quad i_{Lr}_{plot}(t) = \text{if} (t \geq t_6, i_{L}(t) + k_{fb} I_{load}, i_{Lr}_{plot}(t))\]
Estimate: \[ x = \frac{t_6}{t_7} = \frac{1}{\frac{t_6}{t_7}} \]

\[ t_6 = 9.674 \times 10^{-5} \quad t_7 = 9.87 \times 10^{-5} \quad i_L(t_6) = -0.8 \quad i_L(t_7) = -18 \quad i_{Lr\_plot}(t_7) = -17.25 \]

**Time interval \([t_7; t_8]\)**

\[ v_c(t_7) = \frac{v_s}{2} \quad i_L(t_7) = \text{thresh}_\text{2} - k_{fb} I_{load} \quad v_{Cs}(t_7) = 80.638 \quad L_r = L_{sat} \]

\[ i_{Lr\_plot}(t_7) = -17.25 \quad i_L(t_7) = -18.001 \]

\[ \omega = \sqrt{\frac{C_r + C_s}{\sqrt{2}C_s C_r L_r}} \]

\[ i_L(t) = \left[ i_L(t_7) + I_{load} \left( \frac{C_s}{C_s + C_r} \right) \right] \cos\left( \omega (t - t_7) \right) \]

\[ + v_c(t_7) - v_{Cs}(t_7) \left( \frac{v_c(t_7)}{C_s} \frac{v_{Cs}(t_7)}{C_r} \right) \sin\left( \omega (t - t_7) \right) \]

\[ + \frac{1}{2 \omega L_r} \left[ \frac{v_c(t_7)}{C_s} \frac{v_{Cs}(t_7)}{C_r} \right] \frac{v_c(t_7)}{C_s} \frac{v_{Cs}(t_7)}{C_r} \]

\[ + \frac{I_{load}}{2 \omega (C_s + C_r)} \left( \frac{v_c(t_7)}{C_s} + \frac{v_{Cs}(t_7)}{C_r} \right) \sin\left( \omega (t - t_7) \right) \]

\[ + \frac{I_{load}}{2 \omega L_r} \left( \frac{v_c(t_7)}{C_s} + \frac{v_{Cs}(t_7)}{C_r} \right) \cos\left( \omega (t - t_7) \right) \]

\[ + \frac{I_{load}}{2 \omega (C_s + C_r)} \left( t - t_7 \right) \]

\[ v_{Cs\_plot}(t) = \text{if}(t_7 \leq t, v_{Cs}(t), v_{Cs\_plot}(t_7)) \quad i_{Lr\_plot}(t) = \text{if}(t \geq t_7, i_L(t) + k_{fb} I_{load}, i_{Lr\_plot}(t_7)) \]

Solve for \( t \) where \( v_c(t) = +V_s/2 \)

Estimate: \[ x = \frac{t_7}{t_8} = \frac{1}{\frac{t_7}{t_8}} \]

\[ t_7 = 9.87 \times 10^{-5} \quad t_8 = 9.93 \times 10^{-5} \quad i_L(t_7) = -18.001 \quad i_L(t_8) = -20.388 \quad v_{Cs}(t_8) = 78.91 \]
Plot of inductor current

\[ i_{Lr_{\text{plot}}}(t) = \text{if}(t > t_{s}, i_{Lr_{\text{plot}}}(t - t_{s}), i_{Lr_{\text{plot}}}(t)) \]

Plot of source capacitor voltage

Calculate the average of \( v_{Cs}(t) \):

\[ v_{Cs_{\text{ave}}} = \frac{1}{t_{s} - t_{0}} \int_{t_{0}}^{t_{s}} v_{Cs_{\text{plot}}}(t) \, dt \]

\[ v_{Cs_{\text{ave}}} = 69.663 \]

\[ v_{Cs_{\text{plot}}}(t) = \text{if}(t <= t_{s}, v_{Cs_{\text{plot}}}(t), \left(v_{s} - v_{Cs_{\text{plot}}}(t) + \frac{V_{s}}{2} - v_{Cs_{\text{ave}}}, v_{Cs_{\text{plot}}}(t)\right) \]

\[ v_{Cs_{\text{plot}}}(t) = \text{if}(t <= t_{s}, v_{Cs_{\text{plot}}}(t - t_{s}), v_{Cs_{\text{plot}}}(t)) \]
Appendix B-2

Switching Frequency
Switching Frequency of the Nonlinear Resonant Pole with Feedback Winding

**Constants**

- \( V_s = 200 \) Input DC voltage (V)
- \( C_s = 3.3 \times 10^{-6} \) Value of single source side half bridge capacitor (F) (3.3 mF)
- \( C_r = 24 \times 10^{-9} \) Value of single resonant capacitor (F)
- \( L_{unsat} = 1.35 \times 10^{-3} \) Value of unsaturated inductance (H)
- \( k_{sat} = 65 \) Fraction of saturated to unsaturated inductance
- \( I_{load} = 3.0 \) Load current
- \( I_{thresh1} = 2 \) Value of inductor current (\( I_{Lr} \)) for which switch S1 is turned off (2.0 A)
- \( m = 0.25 \) Modulation index (-1 < m < 1)

**Calculated parameters:**

\[
L_{sat} = \frac{L_{unsat}}{k_{sat}}, \quad L_{sat} = 2.07 \times 10^{-5} \quad \text{Value of saturated inductance}
\]

\[
k_{fb} = \frac{n_{fb}}{n_r}, \quad k_{fb} = 0.25 \quad \text{Feedback factor}
\]

\[
I_{thresh2} = \frac{-1}{\sqrt{L_{sat}}} \left[ 4k_{fb}I_{load}L_{sat} + L_{sat}I_{thresh1} \right]^2
\]

\[
I_{thresh2} = -12.554
\]

**Switching frequency**

Total period \( T \):

\[
T = 2L_{sat} \left[ I_{thresh1} - k_{fb}I_{load} - I_{sat} \right] \frac{1}{(1-m)V_s} + 2L_{sat} \left[ I_{thresh1} - k_{fb}I_{load} - I_{sat} \right] \frac{1}{(1+m)V_s} + 2L_{sat} \left[ I_{thresh2} + k_{fb}I_{load} - I_{sat} \right] \frac{1}{(1-m)V_s} + 2L_{sat} \left[ I_{thresh2} + k_{fb}I_{load} - I_{sat} \right] \frac{1}{(1+m)V_s} + 4L_{unsat} \frac{I_{sat}}{(1-m)V_s} + 4L_{unsat} \frac{I_{sat}}{(1+m)V_s} + 2t_{res}
\]

\[
T = 2L_{sat} \left[ I_{thresh1} - 4L_{sat}I_{sat} - 2L_{sat}I_{thresh2} + 4L_{sat}I_{unsat} - t_{res}V_s m^2 + t_{res}V_s \right] \frac{1}{(-1+m)(1+m)V_s}
\]
Frequency:

\[ f_s = \frac{V_s \left( 1 - m^2 \right)}{V_s \left( 1 - m^2 \right) + 2 \left( L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \right) + 2 \cdot t_{\text{res}} \cdot V_s \left( 1 - m^2 \right)} \]

\[ t_{\text{res}} = 0 \]

Graph of switching frequency for different load current values

\[ f_s = \frac{V_s \left( 1 - m^2 \right)}{V_s \left( 1 - m^2 \right) + 2 \left( L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \right) + 2 \cdot t_{\text{res}} \cdot V_s \left( 1 - m^2 \right)} \]

\[ L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \]

\[ t_{\text{res}} = 0 \]

Graph of switching frequency for different load current values

Estimate \( t_{\text{res}} \) as half of the resonant period (i.e, the maximum resonant time)

\[ t_{\text{res}} = \frac{\pi}{2} \cdot \frac{L_{\text{sat}}}{C} \]

\[ f_s \left( m, I_{\text{load}} \right) = \frac{V_s \left( 1 - m^2 \right)}{V_s \left( 1 - m^2 \right) + 2 \left( L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \right) + 2 \cdot t_{\text{res}} \cdot V_s \left( 1 - m^2 \right)} \]

\[ t_{\text{res}} = 0 \]

\[ f_s \left( m, I_{\text{load}} \right) = \frac{V_s \left( 1 - m^2 \right)}{V_s \left( 1 - m^2 \right) + 2 \left( L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \right) + 2 \cdot t_{\text{res}} \cdot V_s \left( 1 - m^2 \right)} \]

\[ t_{\text{res}} = 0 \]

\[ f_s \left( m, I_{\text{load}} \right) = \frac{V_s \left( 1 - m^2 \right)}{V_s \left( 1 - m^2 \right) + 2 \left( L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \right) + 2 \cdot t_{\text{res}} \cdot V_s \left( 1 - m^2 \right)} \]

Graph of switching frequency for different load current values

\[ m \in [-1, 0.95, 1] \]

\[ I_{\text{load}} \in [0, 1.4] \]

\[ f_s \left( m, I_{\text{load}} \right) = \frac{V_s \left( 1 - m^2 \right)}{V_s \left( 1 - m^2 \right) + 2 \left( L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \right) + 2 \cdot t_{\text{res}} \cdot V_s \left( 1 - m^2 \right)} \]

Graph of switching frequency for different load current values

\[ m \in [-1, 0.95, 1] \]

\[ I_{\text{load}} \in [0, 1.4] \]

\[ f_s \left( m, I_{\text{load}} \right) = \frac{V_s \left( 1 - m^2 \right)}{V_s \left( 1 - m^2 \right) + 2 \left( L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \right) + 2 \cdot t_{\text{res}} \cdot V_s \left( 1 - m^2 \right)} \]

Graph of switching frequency for different load current values

\[ m \in [-1, 0.95, 1] \]

\[ I_{\text{load}} \in [0, 1.4] \]

\[ f_s \left( m, I_{\text{load}} \right) = \frac{V_s \left( 1 - m^2 \right)}{V_s \left( 1 - m^2 \right) + 2 \left( L_{\text{unsat}} \left( \frac{I_{\text{thresh1}} - I_{\text{thresh2}}}{2} \right) + \frac{L_{\text{sat}}}{k_{\text{sat}}} \right) + 2 \cdot t_{\text{res}} \cdot V_s \left( 1 - m^2 \right)} \]
Without the resonant period

\[ t_{\text{res}} = 0 \quad f_s(m, I_{\text{load}}) = \frac{V_s (1 - m^2)}{s L_{\text{unsat}} \left[ \left( \frac{I_{\text{thresh}1}}{2} - \frac{I_{\text{thresh}2}(I_{\text{load}})}{2} - I_{\text{sat}} \right) \right] + I_{\text{sat}} + 2 t_{\text{res}} V_s (1 - m^2)} \]
Appendix B-3

Commutation Limits
Commuation Limits

Constants:

\[
\begin{align*}
V_s &= 500 & \text{Input DC voltage (V)} \\
C_r &= 10 \cdot 10^{-9} & \text{Value of single resonant capacitor (F)} \\
L_{\text{unsat}} &= 1.0 \cdot 10^{-3} & \text{Value of unsaturated inductance (H)} \\
k_{\text{sat}} &= 100 & \text{Fraction of saturated to unsaturated inductance} \\
I_{\text{load}} &= 20 & \text{Load current (rms value)} \\
I_{\text{thresh}} &= -35 & \text{Value of inductor current (i}_{Lr}) \text{ for which switch S1 is turned off}
\end{align*}
\]

Calculated parameters:

\[
L_{\text{sat}} = \frac{L_{\text{unsat}}}{k_{\text{sat}}} = 1 \cdot 10^{-5}
\]

At 50 Hz:

\[
f_{\text{load}} = 50 \quad \omega_{\text{load}} = 2\pi f_{\text{load}} = 314.159
\]

Time range:

\[
t := 0, 0.5 \cdot 10^{-3}, \ldots, 1
\]

Derivation of modulation depth constraint:

Simplified 2nd order equation describing resonant switchover:

\[
v_c(t) = (v_c(t_0) - v_{CS}(t_0)) \cos(\omega_r(t-t_0)) - \left(\frac{i_{\text{load}}(t_0) + i_{Lr}(t_0)}{2 \omega_r C_r}\right) \sin(\omega_r(t-t_0)) + v_{CS}(t_0)
\]

where:

\[
\omega_r = \frac{1}{\sqrt{2 L_{\text{sat}} C_r}}
\]

For switchover from S1 to D2:

\[
-\frac{V_s}{2} \geq \left[\frac{v_c(t_0) - v_{CS}(t_0)}{2}\right]^2 + \left[\frac{i_{\text{load}}(t_0) + i_{Lr}(t_0)}{2 \omega_r C_r}\right]^2 \cos(\omega_r(t-t_0) + \phi) + v_{CS}(t_0)
\]

\[
-\frac{V_s}{2} \geq \left(\frac{V_s}{2} - m \frac{V_s}{2}\right) \cos(\omega_r t) - \left(\frac{i_{\text{load}}(t_0) + i_{Lr}(t_0)}{2 \omega_r C_r}\right) \sin(\omega_r t) + m \frac{V_s}{2}
\]

\[
\frac{V_s}{2} = \sqrt{\left(\frac{V_s}{2} - m \frac{V_s}{2}\right)^2 + \left(\frac{i_{\text{load}}(t_0) + i_{Lr}(t_0)}{2 \omega_r C_r}\right)^2} \left(1 + m \frac{V_s}{2}\right)
\]

For commutation from S1 to D2 the following apply:

\[
m_{\text{peak}} \cos(\omega_{\text{load}} t) \leq \frac{1}{\sqrt{2 \omega_r C_r}} \left(\sqrt{2 I_{\text{load}} \cos(\omega_{\text{load}} t - \phi_{\text{load}}) + I_{\text{thresh}}}\right)^2
\]

where:

\[
v_{\text{load}}(t) = m_{\text{peak}} \frac{V_s}{2} \cos(\omega_{\text{load}} t) \quad \text{and} \quad i_{\text{load}}(t) = \sqrt{2 I_{\text{load}} \cos(\omega_{\text{load}} t - \phi_{\text{load}})}
\]
For switchover from S2 to D1: \[ V_c(t) = \frac{V_s}{2}, \quad V_{Cs}(t_0) = m \frac{V_s}{2}, \quad V_c(t_0) = \frac{V_s}{2} \]

\[ V_s = \sqrt{\left(0 - \frac{V_s}{2} - m \frac{V_s}{2}\right)^2 + \left(\frac{i_{load t_0} + i_{Lr t_0}}{2 \omega C_r}\right)^2 \cdot \cos^2(\theta_{r t} - \phi) + m \frac{V_s}{2}} \]

Right hand side a max when cosine term = +1

\[ m = \frac{1}{2} \left(\frac{i_{load t_0} + i_{Lr t_0}}{V_s^2 C_r}\right)^2 \]

For commutation from S2 to D1 the following apply:

\[ m_{peak} \cdot \cos(\theta_{load t}) \geq \frac{1}{V_s^2 2 \omega C_r} \left(\sqrt{2} i_{load} \cdot \cos(\theta_{load t}) - \phi_{load} - 1 \text{ thresh}\right) \]

where: \[ v_{load}(t) = m_{peak} \frac{V_s}{2} \cos(\theta_{load t}) \quad \text{and} \quad i_{load}(t) = \sqrt{2} i_{load} \cdot \cos(\theta_{load t} - \phi_{load}) \]

Value of minimum threshold current in terms of \( v_{load}(t) \) and \( i_{load}(t) \)

\[ S1 \text{ to } D2: \quad m(t) \leq \frac{1}{V_s^2 2 \omega C_r} \left(\frac{i_{load}(t) + i_{Lr}(t)}{2 \omega C_r}\right)^2 \cdot v_{load}(t) = m(t) \frac{V_s}{2} \]

\[ v_{load}(t) \leq \frac{1}{2} \left(\frac{i_{load}(t) + i_{Lr}(t)}{V_s^2 2 \omega C_r}\right)^2 \]

For: \[ m = \frac{1}{V_s^2 2 \omega C_r} \left(i_{load} + 1 L_r\right)^2 \quad i_{Lr} = \]

\[ \frac{V_s^2 C_r}{L_r} \left[\frac{1}{V_s^2 C_r} i_{load} + \sqrt{2} \sqrt{L_r} \cdot \sqrt{m} \right] \]

\[ \frac{V_s^2 C_r}{L_r} \left[\frac{1}{V_s^2 C_r} i_{load} - \sqrt{2} \sqrt{L_r} \cdot \sqrt{m} \right] \]

\[ i_{Lr} = i_{load} + V_s \left[\frac{2 \cdot m \cdot C_r}{L_r}\right] \quad \text{or} \quad i_{Lr} = i_{load} - V_s \left[\frac{2 \cdot m \cdot C_r}{L_r}\right] \]
The minimum value of threshold current for successful commutation from S1 to D2 is on the Z-axis. Load current and load voltage are on the X and Y-axis respectively.
Phase plane representation of load voltage and current and constant current threshold curves

\[
S1 \text{ to } D2: \quad m(t) = \frac{L_r}{V_s^2 2 C_r} \left( i_{\text{load}}(t) + i_{\text{Lr}}(t) \right)^2 \quad v_{\text{load}}(t) = m(t) \frac{V_s}{2} \quad v_{\text{load}}(t) \leq \frac{1}{2} \frac{L_r}{V_s^2 2 C_r} \left( i_{\text{load}}(t) + i_{\text{Lr}}(t) \right)^2
\]

The maximum values of load current and voltage are now plotted for a constant threshold current that will ensure successful commutation.

\[
i_{\text{load}}(t, \phi_{\text{load}}) = \sqrt{2} I_{\text{load}} \cos(\omega_{\text{load}} t - \phi_{\text{load}}) \quad v_{\text{load}}(t, m_{\text{peak}}) = m_{\text{peak}} \frac{V_s}{2} \cos(\omega_{\text{load}} t)
\]

\[
v_{\text{lim}}(t) = \frac{V_s}{2} \left( t - t_{\text{load}} - \frac{1}{2} \right) \quad i_{\text{lim1}}(t, I_{\text{Lr}}) = \begin{cases} v_{\text{lim}}(t) \geq 0, 2 \sqrt{\frac{C_r}{L_r}} \sqrt{v_{\text{lim}}(t) - l_{\text{lim1}} I_{\text{Lr}}} \\ v_{\text{lim}}(t) \leq 0, 2 \sqrt{\frac{C_r}{L_r}} \sqrt{v_{\text{lim}}(t) + l_{\text{lim2}} I_{\text{Lr}}}
\end{cases}
\]

Phase plane plot of sinusoidal load current and voltage and the limits of successful commutation for a constant inductor current threshold value.
Appendix B-4

Nonlinear Control Functions
Determination of the nonlinear characteristics for the Direct Threshold Controller and Integral Threshold Controller

Constants

\[ V_s = 200 \]  
Input DC voltage (V)

\[ C_s = 3.3 \times 10^{-6} \]  
Value of single source side half bridge capacitor (F) (3.3 μF)

\[ C_r = 24 \times 10^{-9} \]  
Value of single resonant capacitor (F)

\[ L_{\text{unsat}} = 1.35 \times 10^{-3} \]  
Value of unsaturated inductance (H)

\[ k_{\text{sat}} = 65 \]  
Fraction of saturated to unsaturated inductance

\[ I_{\text{sat}} = 0.8 \]  
Value of current for which the resonant inductor saturates (A)

\[ n_r = 200 \]  
Number of turns for the resonant inductor

\[ n_{\text{fb}} = 50 \]  
Number of turns for the feedback winding

\[ I_{\text{load}} = 3.0 \]  
Load current

\[ I_{\text{min}} = 2.0 \]  
Minimum value of resonant inductor current for successful commutation

\[ m = 0.0 \]  
Modulation index (-1 < m < 1)

Calculated parameters:

\[ L_{\text{sat}} = \frac{L_{\text{unsat}}}{k_{\text{sat}}} \]  
Value of saturated inductance

\[ k_{\text{fb}} = \frac{n_{\text{fb}}}{n_r} \]  
Feedback factor

Direct Threshold Control

Under steady state conditions and for a positive error signal it is assumed that \( I_{\text{thresh2}} = I_{\text{min}} \). It is furthermore assumed that the amplified error signal changes the \( I_{\text{thresh1}} \) value from \( I_{\text{min}} \) to a new value that will change the average inductor current.

Derivation:

\[ I_{L_{\text{average}}} = \frac{8k_{\text{fb}}I_{\text{load}}I_{\text{sat}}(L_{\text{sat}} - L_{\text{unsat}}) - t_{\text{res}}I_{\text{thresh1}}V_s(m^2 + 1)}{V_s(m^2 - 1)} \]

\[ + \frac{-2L_{\text{sat}}(I_{\text{thresh1}}^2 - I_{\text{thresh2}}^2) - t_{\text{res}}I_{\text{thresh2}}V_s(m^2 - 1)}{V_s(m^2 - 1)} \]

Substitute:

\[ I_{\text{thresh2}} = I_{\text{min}} \]  
\[ I_{\text{thresh1}} = K_1i_{\text{err}} + I_{\text{min}} \]

\[ I_{L_{\text{average}}} = \frac{L_{\text{sat}}[(K_1i_{\text{err}} + I_{\text{min}})^2 - I_{\text{min}}^2] - 4k_{\text{fb}}I_{\text{load}}I_{\text{sat}}(L_{\text{sat}} - L_{\text{unsat}})}{2L_{\text{sat}}(K_1i_{\text{err}} + 2I_{\text{min}}) + 4I_{\text{sat}}(L_{\text{unsat}} - L_{\text{sat}})} \]
Under steady state, $I_{Lr\_average}=0$:

$$0 = 8 \cdot k_{fb} \cdot I_{load} \cdot I_{sat} \cdot (L_{sat} - L_{unsat}) - I_{res} \cdot I_{thresh1} \cdot V_s \cdot (m^2 + 1) + 2 \cdot L_{sat} \cdot (I_{thresh1}^2 - I_{thresh2}^2) - I_{res} \cdot I_{thresh2} \cdot V_s \cdot (m^2 - 1)$$

Let:

$$8 \cdot k_{fb} \cdot I_{load} \cdot I_{sat} = A$$
$$t_{res} \cdot V_s \cdot (m^2 + 1) = B$$

$$0 = A \cdot (L_{sat} - L_{unsat}) - I_{thresh1} \cdot B + 2 \cdot L_{sat} \cdot (I_{thresh1}^2 - I_{thresh2}^2) - I_{thresh2} \cdot C$$

$$I_{thresh1} = \frac{1}{(4 \cdot L_{sat})} \left[ B + \sqrt{B^2 - 8 \cdot L_{sat} \cdot (A - 2 \cdot L_{sat} \cdot I_{thresh2}^2 + A \cdot L_{unsat} + C \cdot I_{thresh2})} \right]$$

$$I_{thresh2} = \frac{1}{(4 \cdot L_{sat})} \left[ t_{res} \cdot V_s \cdot (m^2 + 1) + 8 \cdot L_{sat} \cdot \left( 8 \cdot k_{fb} \cdot I_{load} \cdot I_{sat} \cdot (L_{sat} - L_{unsat}) \right) + (2 \cdot L_{sat}) \cdot I_{thresh2}^2 + t_{res} \cdot V_s \cdot (m^2 - 1) \cdot I_{thresh2} \right]$$

Input to controller:

For $t_{res} = 0$:

$$I_{Lr\_ave}(i_{err}, I_{load}, K_1) = L_{sat} \cdot \left( K_1 \cdot i_{err} + I_{min} \right) + 4 \cdot k_{fb} \cdot I_{load} \cdot I_{sat} \cdot (L_{sat} - L_{unsat})$$

$$I_{Lr\_ave}(i_{err}, I_{load}, K_1) = \text{if}(i_{err} > 0.0, I_{Lr\_ave}(i_{err}, I_{load}, K_1), -I_{Lr\_ave}(i_{err}, I_{load}, K_1))$$
Integral Threshold Control

Derivation of describing function for integral threshold control

Assume that under steady state condition $I_{\text{LR\_average}}=0$. This is true if the threshold value is set at $I_{\text{thresh1}}$. To obtain a nonzero average value, the threshold is set at $I_{\text{thresh\_set}}$. The average value now is:

$$I_{\text{LR\_average}} = \frac{I_{\text{thresh\_set}} + I_{\text{thresh1}}}{2}$$

where:

$$I_{\text{thresh1}} = \frac{1}{\sqrt{L_{\text{sat}}}} \sqrt{4k_{\text{fb}}\text{load}\text{1}\text{sat}\left(L_{\text{sat}} - L_{\text{unsat}}\right) + L_{\text{sat}}\text{1}\text{thresh1}^2}$$

with: $I_{\text{thresh1}} = I_{\text{min}}$

The integral function calculates the area underneath function:

$$\text{Area} = I_{\text{thresh1}} \Delta t + \frac{1}{2} \left(I_{\text{thresh\_set}} - I_{\text{thresh1}}\right) \Delta t$$

where:

$$\Delta t = \frac{L_{\text{sat}}}{(1-m)V_s} \left(I_{\text{thresh\_set}} - I_{\text{thresh1}}\right)$$

$$\text{Area} = I_{\text{sat}} \left[I_{\text{thresh1}}^2 - I_{\text{thresh\_set}}^2\right] \left[(1-m)V_s\right]$$

With $K_2$ the gain of the integrator and $K_1$ the open loop gain:

$$K_1 \text{1}\text{i}\text{err} = K_2 L_{\text{sat}} \left[I_{\text{thresh1}}^2 - I_{\text{thresh\_set}}^2\right] \left[(1-m)V_s\right]$$

$$I_{\text{thresh\_set}} = \frac{1}{\sqrt{\left(K_2 L_{\text{sat}}\right)}} \frac{1}{\sqrt{1 + m}} \text{1}\text{i}\text{err} \frac{V_s}{\sqrt{1 + m}} - K_2 L_{\text{sat}} I_{\text{thresh1}}^2$$

where:

$$I_{\text{thresh1}} = \frac{1}{\sqrt{L_{\text{sat}}}} \sqrt{4k_{\text{fb}}\text{load}\text{1}\text{sat}\left(L_{\text{sat}} - L_{\text{unsat}}\right) + L_{\text{sat}}\text{1}\text{thresh1}^2}$$

$$I_{\text{thresh\_set}} = \frac{1}{\sqrt{\left(K_2 L_{\text{sat}}\right)}} \frac{1}{\sqrt{1 + m}} \frac{4k_{\text{fb}}\text{load}\text{1}\text{sat}\left(L_{\text{sat}} - L_{\text{unsat}}\right) + L_{\text{sat}}\text{1}\text{thresh1}^2}{2\sqrt{1 + m}} - K_2 L_{\text{sat}} I_{\text{thresh1}}^2$$

Substitute in eq. for average current:

$$I_{\text{LR\_average}} = \frac{1}{2} \left[I_{\text{thresh\_set}} + I_{\text{thresh1}}\right]$$

$$I_{\text{LR\_average}} = \frac{1}{2} I_{\text{thresh\_set}} + \frac{1}{2} \sqrt{4k_{\text{fb}}\text{load}\text{1}\text{sat}\left(L_{\text{sat}} - L_{\text{unsat}}\right) + L_{\text{sat}}\text{1}\text{thresh1}^2}$$

$$I_{\text{LR\_average}} = \sqrt{1-m} \frac{4k_{\text{fb}}\text{load}\text{1}\text{sat}\left(L_{\text{sat}} - L_{\text{unsat}}\right) - K_2 L_{\text{sat}} I_{\text{thresh1}}^2}{2\sqrt{K_2 L_{\text{sat}}}} + \frac{1}{2} \sqrt{4k_{\text{fb}}\text{load}\text{1}\text{sat}\left(L_{\text{sat}} - L_{\text{unsat}}\right) + L_{\text{sat}} I_{\text{thresh1}}^2}$$
\[
I_{\text{LR ave}}(i_{\text{err}}, m, K_1, K_2) = \frac{\sqrt{1 - m} \left[ K_1 |i_{\text{err}}| V_s (m - 1) \ldots \right] - K_2 L_{\text{sat}} L_{\text{min}}^2}{2 \cdot \sqrt{K_2 L_{\text{sat}}} \sqrt{m - 1}}
+ \frac{1}{2 \cdot \sqrt{L_{\text{sat}}}} \left[ 4 k_{fb} I_{\text{load}} K_2 L_{\text{sat}} (L_{\text{unsat}} - L_{\text{sat}}) \right] L_{\text{sat}} L_{\text{min}}^2
\]

\[
I_{\text{LR ave}}(i_{\text{err}}, m, K_1, K_2) = \begin{cases} 
\text{if } |i_{\text{err}}| \geq 0, & I_{\text{LR ave}}(i_{\text{err}}, m, K_1, K_2), 
\end{cases}
\]

\[
I_{\text{LR ave}}(i_{\text{err}}, 0.9, 1, 1)
I_{\text{LR ave}}(i_{\text{err}}, 0.9, 1, 10)
I_{\text{LR ave}}(i_{\text{err}}, 0.1, 1, 1)
I_{\text{LR ave}}(i_{\text{err}}, 0.1, 10, 1)
\]
Appendix B-5

Large Signal Analysis
Large Signal Transfer Function of total NLRP system

Constants:

- \( L_r = 0 \)  
  Low frequency equivalent inductance for \( L_r \)
- \( R_r = 0 \)  
  Low frequency equivalent series resistance of \( L_r \)
- \( C_s = 12.10^{-6} \)  
  Value of single source side half bridge capacitor (F)
- \( R_s = 70 \)  
  Parallel source resistance
- \( L_{\text{load}} = 5.10^{-3} \)  
  Series load inductance
- \( R_{\text{load}} = 1.5 \)  
  Series load resistance

Reference Current to Load Current Transfer Function

Transfer function of controller (integral threshold): \( F_{\text{ctrl}}(s) = 1 \)
Transfer function of feedback measurement of load current: \( F_{\text{ctrl}}(s) = 1 \)
Transfer function of series impedance of resonant inductor (i.e. \( L_r \) and \( R_r \)): \( Z_r(s) = R_r + sL_r \)
Transfer function of average inductor current to load current (i.e. NLRP phase arm components):

\[ V_{Cs}(s) + Z_r(s) I_L(s) = I_{\text{load}}(s) \left( R_{\text{load}} + sL_{\text{load}} \right) \quad \ldots \ldots \ldots \quad (1) \]

\[ V_{Cs}(s) = \frac{I_L(s)}{2 + 2sC_s} \quad \ldots \ldots \ldots \quad (2) \]

Substitute (2) in (1):

\[ \frac{I_L(s)}{I_{\text{load}}(s)} \left[ \frac{2}{R_s} + 2sC_s \right] + Z_r(s) I_L(s) = I_{\text{load}}(s) \left( R_{\text{load}} + sL_{\text{load}} \right) \]

\[ I_{\text{load}}(s) = \frac{1}{I_L(s)} \left[ \frac{2}{R_s} + 2sC_s \right] + Z_r(s) I_L(s) \]

\[ I_{\text{load}}(s) = \frac{1}{\left( R_{\text{load}} + L_{\text{load}} \right)} \left( \frac{R_s + 2Z_r(s) + 2Z_r(s)sC_sR_s}{2 \left( 1 + sC_sR_s \right) \left( R_{\text{load}} + sL_{\text{load}} \right)} \right) \]
Total loop transfer function:

\[
\frac{I_{\text{load}}(s)}{I_{\text{ref}}(s)} = \frac{K \cdot F_{\text{ctrl}}(s)}{1 + K \cdot F_{\text{ctrl}}(s) - \frac{I_{\text{load}}(s)}{I_L(s)}}
\]

\[
I_{\text{load}}(s) = \frac{K \cdot F_{\text{ctrl}}(s)}{1 + K \cdot F_{\text{ctrl}}(s)} \cdot \frac{I_{\text{ref}}(s)}{I_L(s)}
\]

\[
I_{\text{ref}}(s) = \frac{1}{1 + K \cdot F_{\text{ctrl}}(s) - \frac{I_{\text{load}}(s)}{I_L(s)}}
\]

If the following are substituted in:

\[
\frac{I_{\text{load}}(s)}{I_{\text{ref}}(s)} = 1, \quad F_{\text{ctrl}}(s) = 1, \quad F_{\text{cmf}}(s) = 0
\]

\[
I_{\text{ref}}(s) = K \cdot R_s
\]

Reference Current to Load Voltage Transfer Function

\[
\frac{V_{\text{CS}}(s)}{I_L(s)} = \frac{1}{2 \cdot s \cdot C_s + \frac{2}{R_s}}
\]

\[
I_L(s) = K \cdot F_{\text{ctrl}}(s) \cdot \left( I_{\text{ref}}(s) - I_{\text{load}}(s) \right)
\]

To obtain \(I_{\text{load}}(s)\) in terms of \(V_{\text{CS}}(s)\):

\[
V_{\text{CS}}(s) + Z_{r}(s) \cdot I_L(s) = I_{\text{load}}(s) \cdot \left( R_{\text{load}} + s \cdot L_{\text{load}} \right) \quad \ldots \ldots (1)
\]

\[
V_{\text{CS}}(s) = \frac{2}{R_s} + 2 \cdot s \cdot C_s \quad \ldots \ldots (2)
\]

Substitute (2) in (1):

\[
V_{\text{CS}}(s) + Z_{r}(s) \cdot \left[ V_{\text{CS}}(s) \cdot \left( \frac{2}{R_s} + 2 \cdot s \cdot C_s \right) \right] = I_{\text{load}}(s) \cdot \left( R_{\text{load}} + s \cdot L_{\text{load}} \right)
\]

\[
I_{\text{load}}(s) = \frac{V_{\text{CS}}(s) \cdot \left( \frac{R_s + 2 \cdot Z_{r}(s) + 2 \cdot Z_{r}(s) \cdot s \cdot C_s \cdot R_s}{R_s} \right)}{R_{\text{load}} + s \cdot L_{\text{load}}}
\]

Substitute \(I_{\text{load}}(s)\) in:

\[
V_{\text{CS}}(s) = \frac{1}{2 \cdot s \cdot C_s + \frac{2}{R_s}}
\]

\[
K \cdot F_{\text{ctrl}}(s) \cdot \left( I_{\text{ref}}(s) - I_{\text{load}}(s) \right)
\]
\[ V_{Cs}(s) = \frac{1}{2sC_s + \frac{R_s}{s} + K_F} \cdot \frac{I_{ref}(s)}{s} \cdot \frac{V_{Cs}^2}{R_s} \cdot \left( \frac{R_s + 2Z_r(s) + 2Z_r(s) \cdot s \cdot C_s \cdot R_s}{(R_{load} + s \cdot L_{load})} \right) \]

\[ V_{Cs}(s) = \frac{1}{I_{ref}(s)} \cdot \frac{(R_{load} + s \cdot L_{load}) \cdot R_s \cdot K_F}{s} \cdot \left( 2s^2C_s + R_s \cdot L_{load} \right) \cdot \left( R_s + \left( 2sC_s \cdot s + R_s \cdot L_{load} \right) \cdot s + 2 \cdot R_{load} + s \cdot R_s \cdot K_F \right) \]

If the following are substituted in:

\[ V_{Cs}(s) = \frac{I_{ref}(s)}{I_{ref}(s)} \cdot \frac{K_F}{(R_{load} + s \cdot L_{load}) \cdot R_s} \cdot \left( 2s^2C_s + R_s \cdot L_{load} \right) \cdot \left( R_s + \left( 2sC_s \cdot s + R_s \cdot L_{load} \right) \cdot s + 2 \cdot R_{load} + s \cdot R_s \cdot K_F \right) \]

Bode plot of total transfer function (reference current to load current):

\[ F_{total}(s,K) = 1 + F_{ctrl}(s) \cdot K \cdot F_{ctrl}(s) \cdot \left( \frac{R_s + 2Z_r(s) + 2Z_r(s) \cdot s \cdot C_s \cdot R_s}{(R_{load} + s \cdot L_{load})} \right) \]

Magnitude Plot

[Graph and figure showing the Bode plot with frequency and magnitude axes.]
Phase Angle Plot

\[ \phi(\theta) = \text{angle}(\text{Re}(\theta), \text{Im}(\theta)) \cdot \frac{180}{\pi} + 360 \]

Transfer function of input reference to \( V_{cs}(t) \)

\[
F_{vCs}(s, K) = \frac{R_{\text{load}} + sL_{\text{load}}}{s^2C_sR_s + sL_{\text{load}} + Z_{\text{cs}}(s)Z_{\text{load}}(s)R_{\text{load}} + 2sZ_{\text{load}}(s) + R_s + KF_{\text{ctrl}}(s)}
\]
Phase Angle Plot

$$\Phi(\theta) = \text{angle}(\text{Re}(\theta), \text{Im}(\theta)) \cdot \frac{180}{\pi}$$
Appendix C

PSPICE Listing
Program for the simulation of a nonlinear resonant pole inverter

A.S. de Beer

Industrial Electronics Research Group
Energy Laboratory
Rand Afrikaans University

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NLRP2.CIR
Last changes:
10 Dec. 1993

******************************************************************************
****** Variables ( Parametres ) **************

*** Inverter ***
.PARAM DC_bus_val = 200V ; DC-bus voltage if rectifier is not used .PARAM Snubber_Co = 55nF ; Value of resonant capacitor .PARAM Source_C = 12uF ; Source capacitor for half bridge configuration .PARAM Iload_val = 0A ; Steady state load current value

*** Magnetics ***
.PARAM Ni = 15 ; Number of turns for resonating inductor .PARAM N2 = 3 ; Number of turns on feedback winding .PARAM Path_length = 14.7cm ; Effective magnetic path length of core (cm) .PARAM Mag_Area = 5.32cm ; Effective magnetic core area in cm*cm

*** Parasitics ***
.PARAM Parasitic_Rsource = 0.01ohm ; Parasitic resistance between source and converter .PARAM Parasitic_R_bus = 0.2ohm ; Parasitic resistance between half-bridge capacitor and switch .PARAM Parasitic_R_SnubC = 0.2ohm ; Parasitic resistance in series with snubber capacitor .PARAM Parasitic_L_source = 50nH ; Parasitic inductance between source and converter .PARAM Parasitic_L_bus = 350nH ; Parasitic inductance between half-bridge capacitor and switch .PARAM Parasitic_L_SnubC = 0.1pH ; Parasitic inductance in series with snubber capacitor

*** Control ***
.PARAM I_min_pos = 35A ; Minimum pos inductor current to open switch .PARAM I_min_neg = 15A ; Minimum neg inductor current to open switch .PARAM m = 0.67 ; Modulation index for which simulation is done

******************************************************************************
****** Main circuit **************

*** Inverter ***

* Note : 1 usually means a top component and 2 a bottom one
Vs1 7 0 {DC_bus_val/2} ; Top Half bridge source of Vs/2
Vs2 0 8 {DC_bus_val/2} ; Bottom Half bridge source of Vs2
Cs1 12 1 {Source_C} IC = {(1-m)\*DC_bus_val/2} ; Startup in terms of m
Cs2 11 11 {Source_C} IC = {(1 + m)\*DC_bus_val/2} ; Startup in terms of m
XIGBT1 5 2 g1 IGBT_mod PARAMS: Iload_val_sub = {Iload_val}
* Upper switching device
XIGBT2 2 6 g2 IGBT_mod PARAMS: Iload_val_sub = 0
* Lower switching device
XLreson 4 2 3 Reson_Inductor
Iload 3 0 {Iload_val} ; Constant current source as load V_L_measure 4 10V ; Measures current through resonating inductor XC_snub1 5 9 Snub_Capacitor PARAMS: V_startup = 0 ; S1 is on
* Upper snubber capacitor
XC_snub2 10 6 Snub_Capacitor PARAMS: V_startup = {DC_bus_val}
* Lower snubber capacitor ; S2 is off
XParasitic_busl 12 5 Bus_parasitic PARAMS: Iload_val_sub = {0} XParasitic_bus2 11 6 Bus_parasitic PARAMS: Iload_val_sub = {0}
XParasitic_source1 7 12 Source_parasitic PARAMS: Iload_val_sub = {0} XParasitic_SnubCl 9 2 SnubC_parasitic
XParasitic_SnubC2 10 2 SnubC_parasitic

**************************************** Control circuits ****************************************

* Control of S1 and S2. Switch-on is only done with zero voltage across
* the switch, while a switch can only be turned off if a certain
* value of current I_min is present in the resonant inductor.

* AND-gate for S1
V_S1 g5 2 6V
Rg1 g1 2 1ohm
Cg1 g1 2 1nF IC = 6V ; S1 must immediately be on at start-up Sg1 g3 g1 5 2
V_AND_Switch
Wg1 g5 g3 V_L_measure I_AND_Switch1

* AND-gate for S2
V_S2 g6 6 6V
Rg2 g2 6 1ohm
Cg2 g2 6 1nF IC = 0V ; S2 must immediately be off at start-up
Sg2 g4 g2 26 V_AND_Switch
Wg2 g6 g4 V_L_measure I_AND_Switch2

.MODEL V_AND_Switch VSWITCH[ RON = 1mohm VON = 4.0V VOFF = 5V ] ; If voltage
* across S1 or S2 is smaller than 3V, the switch may be turned on.
.MODEL I_AND_Switch1 ISWITCH[ RON = 1mohm ION = {0.95*I_min_pos}
+ IOFF = {I_min_pos} ]
.MODEL I_AND_Switch2 ISWITCH[ RON = 1mohm ION = {-0.95*I_min_neg}
+ IOFF = {-I_min_neg} ]
* If current is larger than I_min_pos or smaller than I_min_neg,
* the switch may be turned off.

****************************** Subcircuits ******************************

*** Switching devices ***

.SUBCKT IGBT_mod Collector Emitter Gate PARAMS: Iload_val_sub = 0A
* L_switch Collector 2 1nH IC = {Iload_val_sub} ; Switch inductance
Switch 1 Emitter Gate Emitter SwitchMOD
Dswitch Collector 1 Ideal_D
Diode Emitter Collector DiodeMOD
.MODEL SwitchMOD VSWITCH( Ron = 3e-3ohm Roff = 1e6ohm Von = 5V Voff = 4V )
.MODEL DiodeMOD D ( Rs = 10e-3ohm CJO = 0pF )
.MODEL Ideal_D D
*** Resonating Inductor with Feedback Winding ***

.SUBCKT Reson_Inductor 1 2 3
L1 2 1 {N1} IC = {0} ; {N2/N1*Iload_val}
* Resonating Inductor with so many turns (last figure) L2 3 2 {N2} IC = {0} ; {-Iload_val} ; Feedback winding with so many turns K1 L1 L2 0.9999 E-42 3C8 ; Same core for L1 and L2 .MODEL E-42_3C8 CORE( MS = 250E3 ALPHA = 2E-5 A = 26 K = 18 C = 1.05 AREA = {Mag_Area} + GAP = 0.00cm PATH = {Path_length}); From PSPICE library * Except for PATH and AREA (which correspond with a given core size), * the other parameters are for 3C8 material .ENDS

*** Snubber Capacitors ***

.SUBCKT Snub_Capacitor 1 2 PARAMS: V_startup = 0V Cbase 1 2 {Snubber_Co} IC = {V_startup} ; Linear snubber capacitor .ENDS

*** Parasitic Subcircuits ***

.SUBCKT Bus_parasitic 1 2 PARAMS: Iload_val_sub = 0A Rparasitic 1 3 {Parasitic_R_bus} Lparasitic 3 2 {Parasitic_L_bus} IC = {Iload_val_sub} .ENDS

.SUBCKT Source_parasitic 1 2 PARAMS: Iload_val_sub = 0A Rparasitic 1 3 {Parasitic_R_source} Lparasitic 3 2 {Parasitic_L_source} IC = {Iload_val_sub} .ENDS

.SUBCKT SnubC_parasitic 1 2 Rparasitic 1 3 {Parasitic_R_SnubC} Lparasitic 3 2 {Parasitic_L_SnubC} IC = 0A .ENDS

*************************** General settings ****************************

* Iteration limit = inf Iterations per point .OPTIONS ITL5 = 0 ITL4 = 100

* Toleration settings .OPTIONS RELTOL = 0.02 VNTOL = 40mV ABSTOL = 20uA

* PRINT STEP FINAL TIME NO-PRINT STEP CEILING INIT. COND. .TRAN 0.1usec 90usec 70usec 0.03usec UIC .PRINT TRAN I(V_L_measure) V(5,2) I(XC_snub1.Cbase)

.PROBE V(1) I(V_L_measure) V(5,2) I(XC_snub1.Cbase)
+ I(XIGBT1.Switch) I(XIGBT1.Diode)

.END
Appendix D

Mathematical Manipulations
Mathematical Manipulations

D-1 Equations during Resonant Transition

In this section the equations for the inductor current $i_L(t)$, output voltage $v_c(t)$ and source capacitor voltage $v_{Cs}(t)$ are derived.

From figure D-1 the capacitor currents can be written in terms of the output voltage $v_c(t)$ and source capacitor voltage $v_{Cs}(t)$ as:

\[
\begin{align*}
    i_{Cs1}(t) &= -C_{s1} \frac{dv_{Cs}(t)}{dt} \\
    i_{Cs2}(t) &= C_{s2} \frac{dv_{Cs}(t)}{dt} \\
    i_{Cr1}(t) &= -C_{r1} \frac{dv_c(t)}{dt}
\end{align*}
\]
Looking at the nodes A and B as well as the voltage drop across the resonant inductor, the following differential equations can be written according to Kirchoff's laws:

\[ i_{Cr2}(t) = C_{r2} \frac{dv_{c}(t)}{dt} \]  

(D-1)

Substituting the different capacitor currents from (D-1) into (D-2) the following are obtained:

\[ v_{c}(t) - v_{Cs}(t) = L_{r} \frac{di_{L}(t)}{dt} \]

\[ i_{L}(t) = i_{Cs2}(t) - i_{Cs1}(t) - k_{fb} I_{load} \]

\[ i_{L}(t) + i_{Cr2}(t) - i_{Cr1}(t) + k_{fb} I_{load} + I_{load} = 0 \]

(D-2)

where:

\[ C_{r} = \frac{C_{r1} + C_{r2}}{2} \]

\[ C_{s} = \frac{C_{s1} + C_{s2}}{2} \]

(D-4)

Performing a Laplace transform on (D-3) and rearranging:

\[ V_{c}(s) - V_{cs}(s) - sL_{r}I_{L}(s) + L_{r}i_{L}(t_{0}) = 0 \]

\[ I_{L}(s) + k_{fb} \frac{I_{load}}{s} - 2sC_{s}V_{c}(s) + 2C_{s}v_{cs}(t_{0}) = 0 \]

\[ I_{L}(s) + (1 + k_{fb}) \frac{I_{load}}{s} + 2sC_{s}V_{c}(s) - 2C_{s}v_{c}(t_{0}) = 0 \]

(D-5)
Derivation of $i_L(t)$

To obtain the current $i_L(t)$, the Laplace equation $I_L(s)$ must first be obtained from (D-5). The Laplace form of the resonant inductor current is:

$$I_L(s) = \frac{2s^2L_Cr_Ci_L(t_0) + 2C_rC_s[v_c(t_0) - v_{C_s}(t_0)] - I_{load}\left[\left(1 + k_{ib}\right)C_s + k_{ib}C_r\right]}{s(2s^2L_rC_sC_r + C_r + C_s)}$$  \hspace{1cm} (D-6)

Substituting for $\omega^2 = \frac{C_r + C_s}{2L_rC_sC_r}$ and rearranging:

$$I_L(s) = \frac{s_i_L(t_0) + \frac{1}{L_r}\left[v_c(t_0) - v_{C_s}(t_0)\right] - \frac{I_{load}}{2sL_rC_sC_r}\left[C_s + k_{ib}(C_s + C_r)\right]}{s^2 + \omega^2}$$  \hspace{1cm} (D-7)

The inverse Laplace transforms for the different terms of (D-7) are given by:

$$\mathcal{L}^{-1}\left\{\frac{i_L(t_0)s}{s^2 + \omega^2}\right\} = I_L(t_0)\cos[\omega(t-t_0)]$$

$$\mathcal{L}^{-1}\left\{\frac{1}{L_r}\left[v_c(t_0) - v_{C_s}(t_0)\right]\right\} = \frac{1}{\omega L_r}\left[v_c(t_0) - v_{C_s}(t_0)\right]\sin[\omega(t-t_0)]$$

$$\mathcal{L}^{-1}\left\{-\frac{I_{load}}{s}\left[\frac{1}{2sL_rC_s} + \omega^2k_{ib}\right]\right\} = -I_{load}\left[\frac{C_s}{C_s + C_r} + k_{ib}\right]\left[1 - \cos[\omega(t-t_0)]\right]$$  \hspace{1cm} (D-8)

This means that:

$$i_L(t) = \left\{i_L(t_0) + I_{load}\left[\frac{C_s}{C_s + C_r} + k_{ib}\right]\right\}\cos[\omega(t-t_0)]$$

$$+ \frac{1}{\omega L_r}\left[v_c(t_0) - v_{C_s}(t_0)\right]\sin[\omega(t-t_0)] - I_{load}\left[\frac{C_s}{C_s + C_r} + k_{ib}\right]$$  \hspace{1cm} (D-9)
**Derivation of \( v_{CS}(t) \)**

From the Laplace equations (D-5), the equation for the source capacitor voltage is solved as:

\[
V_{Cs}(s) = \frac{V_{Cs}(t_0)}{s^2 + \omega^2} + \frac{1}{2C_s} \left( i_L(t_0) + k_f b I_{load} \right) + \frac{1}{2L_r} \left[ \frac{v_e(t_0)}{C_s} + v_{Cs}(t_0) \right] \frac{1}{s} - \frac{I_{load}}{4L_rC_sC_s} \frac{1}{s^2 + \omega^2}
\]  

(D-10)

where:

\[
\omega^2 = \frac{C_s + C_L}{2L_rC_sC_s}
\]

The inverse Laplace transform of equation (D-10) can be found by adding the different inverse transforms for each term, where:

\[
L^{-1}\left[ \frac{V_{Cs}(t_0)}{s^2 + \omega^2} \right] = v_{Cs}(t_0) \cos(\omega(t - t_0))
\]

\[
L^{-1}\left[ \frac{1}{2C_s} \left( i_L(t_0) + k_f b I_{load} \right) \right] = \frac{1}{2\omega C_s} \left( i_L(t_0) + k_f b I_{load} \right) \sin(\omega(t - t_0))
\]

\[
L^{-1}\left[ \frac{1}{2L_r} \left[ \frac{v_e(t_0)}{C_s} + v_{Cs}(t_0) \right] \frac{1}{s} \right] = \frac{1}{2\omega L_r} \left[ \frac{v_e(t_0)}{C_s} + v_{Cs}(t_0) \right] \{1 - \cos(\omega(t - t_0))\}
\]

\[
L^{-1}\left[ \frac{-I_{load}}{4L_rC_sC_s} \frac{1}{s^2 + \omega^2} \right] = L^{-1}\left[ \frac{-I_{load}}{2s^2(C_r + C_s)} + \frac{I_{load}}{2(s^2 + \omega^2)(C_r + C_s)} \right]
\]

\[
= \frac{-I_{load}}{2(C_r + C_s)} (t - t_0) + \frac{I_{load}}{2\omega(C_r + C_s)} \sin(\omega(t - t_0))
\]  

(D-11)

Which gives the time equation for \( v_{CS}(t) \) as:
\[ v_{C_s}(t) = \left( v_{C_s}(t_0) - \frac{1}{2\omega^2 L_r} \left[ \frac{v_{c}(t_0)}{C_s} + \frac{v_{C_s}(t_0)}{C_r} \right] \right) \cos(\omega(t-t_0)) \]

\[ + \left\{ \frac{I_{\text{load}}}{2\omega(C_r + C_s)} + \frac{1}{2\omega C_r} \left( i_{L}(t_0) + k_{fb} I_{\text{load}} \right) \right\} \sin(\omega(t-t_0)) \]

\[ - \frac{I_{\text{load}}}{2(C_r + C_s)(t-t_0)} + \frac{1}{2\omega^2 L_r} \left[ \frac{v_{c}(t_0)}{C_s} + \frac{v_{C_s}(t_0)}{C_r} \right] \]

(D-12)

**Derivation of the output voltage \( v_c(t) \)**

The expression of the output voltage \( v_c(t) \) can be derived from equations (D-5). In the Laplace form:

\[ V_c(s) = \frac{v_c(t_0)s - \frac{1}{2C_r} \left[ i_L(t_0) + (1 + k_{fb}) I_{\text{load}} \right] + \frac{v_c(t_0)C_r + v_{C_s}(t_0)C_s}{2L_r C_r C_s} \frac{1}{s} - \frac{I_{\text{load}}}{s^2 + \omega^2}} \]

(D-13)

The inverse transform is calculated from the terms of \( V_c(s) \) (D-13) where:

\[ L^{-1} \left[ \frac{v_c(t_0)s}{s^2 + \omega^2} \right] = v_c(t_0) \cos(\omega(t-t_0)) \]

\[ L^{-1} \left[ \frac{1}{2C_r} \left[ i_L(t_0) + (1 + k_{fb}) I_{\text{load}} \right] \right] = \frac{1}{2\omega C_r} \left[ i_L(t_0) + (1 + k_{fb}) I_{\text{load}} \right] \sin(\omega(t-t_0)) \]

\[ L^{-1} \left[ \frac{v_c(t_0)C_r + v_{C_s}(t_0)C_s}{2L_r C_r C_s} \frac{1}{s} \right] = \frac{v_c(t_0)C_r + v_{C_s}(t_0)C_s}{C_r + C_s} \left[ 1 - \cos(\omega(t-t_0)) \right] \]

(D-14)
The output voltage can now be expressed as:

\[
v_c(t) = \left\{ v_c(t_0) - \frac{v_c(t_0)C_r + v_{cs}(t_0)C_s}{C_r + C_s} \right\} \cos[\omega(t - t_0)]
+ \left\{ \frac{I_{load}}{2\omega(C_s + C_r)} - \frac{1}{2\omega C_r} \left[ i_c(t_0) + (1 + k_f)I_{load} \right] \right\} \sin[\omega(t - t_0)]
- \frac{I_{load}}{2(C_r + C_s)} (t - t_0) + \frac{v_c(t_0)C_r + v_{cs}(t_0)C_s}{2(C_r + C_s)}
\]

(D-15)
In this section the average inductor current $I_{Lr\_average}$ are calculated or:

$$I_{Lr\_average} = \frac{1}{T_s} \int_{t_0}^{t_f} i_{Lr}(t) \, dt$$

(D-16)

where $T_s$ is the period over one switching cycle ($t_0 - t_f$ in figure D-2). During steady state operation the average inductor current is zero. From this condition the threshold levels $I_{\text{thresh1}}$ and $I_{\text{thresh2}}$ can be calculated.

![Diagram](image)

**Figure D-2:** Linear approximation of the resonant inductor current $i_{Lr}(t)$ for the calculation of the average over one switching cycle.

In figure D-2, a linear approximation of $i_{Lr}(t)$ is given. This approximation is used to simplify the derivation of the average inductor current. It is used as opposed to the more complex trigonometric forms of equations 3-6, 3-9 and 3-15.

In the linear approximation it is assumed that the capacitors $C_s$ are infinitely large and that they act as perfect voltage sources. This leads to a linear function of time as opposed to the trigonometric forms of section 3-2.

The average value can be calculated by first obtaining the time current area under $i_{Lr}(t)$ in figure D-2. This is done by calculating the different triangular or rectangular areas for each time interval. The different time intervals are:
\[ (t_1 - t_0) = 2L_{\text{sat}} \frac{I_{\text{thresh}2} + k_{fb}I_{\text{load}} - I_{\text{sat}}}{(1-m)V_s} \]

\[ (t_2 - t_1) = 4L_{\text{unsat}} \frac{I_{\text{sat}}}{(1-m)V_s} \]

\[ (t_3 - t_2) = 2L_{\text{sat}} \frac{I_{\text{thresh}2} - k_{fb}I_{\text{load}} - I_{\text{sat}}}{(1-m)V_s} \]

\[ (t_4 - t_3) = t_{\text{res}} \]

\[ (t_5 - t_4) = 2L_{\text{sat}} \frac{I_{\text{thresh}2} - k_{fb}I_{\text{load}} - I_{\text{sat}}}{(1+m)V_s} \]

\[ (t_6 - t_5) = 4L_{\text{unsat}} \frac{I_{\text{sat}}}{(1+m)V_s} \]

\[ (t_7 - t_6) = 2L_{\text{sat}} \frac{I_{\text{thresh}2} + k_{fb}I_{\text{load}} - I_{\text{sat}}}{(1+m)V_s} \]

\[ (t_8 - t_7) = t_{\text{res}} \]

The area above the zero line in figure D-2 can be written as:

\[
\frac{1}{2} \left[ \frac{L_{\text{unsat}}}{(1-m)V_s} + \frac{L_{\text{unsat}}}{(1+m)V_s} \right] \left( I_{\text{sat}} + k_{fb}I_{\text{load}} \right)^2 + t_{\text{res}}I_{\text{thresh}1} \ldots
\]

\[
\frac{1}{2} \left[ \frac{L_{\text{sat}}}{(1-m)V_s} + \frac{L_{\text{sat}}}{(1+m)V_s} \right] \left( I_{\text{thresh}1} - k_{fb}I_{\text{load}} - I_{\text{sat}} \right) \left( I_{\text{sat}} + k_{fb}I_{\text{load}} \right) \ldots
\]

\[
\frac{1}{2} \left[ \frac{L_{\text{sat}}}{(1-m)V_s} + \frac{L_{\text{sat}}}{(1+m)V_s} \right] \left( I_{\text{thresh}1} - k_{fb}I_{\text{load}} - I_{\text{sat}} \right)^2 \]

The area underneath the zero line in figure D-2 can be written as:

\[
\frac{1}{2} \left[ \frac{L_{\text{unsat}}}{(1-m)V_s} + \frac{L_{\text{unsat}}}{(1+m)V_s} \right] \left( I_{\text{sat}} - k_{fb}I_{\text{load}} \right)^2 + t_{\text{res}}I_{\text{thresh}2} \ldots
\]

\[
\frac{1}{2} \left[ \frac{L_{\text{sat}}}{(1-m)V_s} + \frac{L_{\text{sat}}}{(1+m)V_s} \right] \left( I_{\text{thresh}2} + k_{fb}I_{\text{load}} - I_{\text{sat}} \right) \left( I_{\text{sat}} - k_{fb}I_{\text{load}} \right) \ldots
\]

\[
\frac{1}{2} \left[ \frac{L_{\text{sat}}}{(1-m)V_s} + \frac{L_{\text{sat}}}{(1+m)V_s} \right] \left( I_{\text{thresh}2} + k_{fb}I_{\text{load}} - I_{\text{sat}} \right)^2 \]

The average current per switching cycle equals the area above the zero line minus the area underneath, divided by the time over one cycle or \( t_8 - t_0 \). The total area can be calculated by subtracting (D-19) from (D-18). The time over one
switching cycle is taken from the calculation of switching frequency in chapter 3-3. This leads to the following expression for the average resonant inductor current:

\[
I_{Lr_{average}} = \frac{8k_{fb_{load}}I_{sat}(L_{sat} - L_{unsat}) - t_{res}I_{\text{thresh1}}V_s(m^2 + 1)}{V_s(m^2 - 1)} + \frac{-2L_{sat}(I_{\text{thresh1}}^2 - I_{\text{thresh2}}^2) - t_{res}I_{\text{thresh2}}V_s(m^2 - 1)}{V_s(m^2 - 1)}
\]

where:

\[
f_s = \frac{(1 - m^2)V_s}{2}\left[2L_{sat}(I_{\text{thresh1}} - I_{\text{thresh2}}) + 4I_{sat}(L_{unsat} - L_{sat})\right] + t_{res}V_s(1 - m^2)
\]

Therefore:

\[
I_{Lr_{average}} = -\frac{8k_{fb_{load}}I_{sat}(L_{sat} - L_{unsat}) + t_{res}I_{\text{thresh1}}V_s(m^2 + 1)}{4L_{sat}(I_{\text{thresh1}} - I_{\text{thresh2}}) + 8I_{sat}(L_{unsat} - L_{sat}) + 2t_{res}V_s(1 - m^2)} + \frac{-2L_{sat}(I_{\text{thresh1}}^2 - I_{\text{thresh2}}^2) + t_{res}I_{\text{thresh2}}V_s(m^2 - 1)}{4L_{sat}(I_{\text{thresh1}} - I_{\text{thresh2}}) + 8I_{sat}(L_{unsat} - L_{sat}) + 2t_{res}V_s(1 - m^2)}
\]

Substituting \(t_{res} = 0\) in the above equation yields:

\[
I_{Lr_{average}} = \frac{L_{sat}(I_{\text{thresh1}}^2 - I_{\text{thresh2}}^2) - 4k_{fb_{load}}I_{sat}(L_{sat} - L_{unsat})}{2L_{sat}(I_{\text{thresh1}} - I_{\text{thresh2}}) + 4I_{sat}(L_{unsat} - L_{sat})} + \frac{2}{t_{res}V_s(1 - m^2)}
\]

In a steady state condition the average inductor resonant current is zero. Setting (D-22) equal to zero and solving for either \(I_{\text{thresh1}}\) or \(I_{\text{thresh2}}\) yields:

\[
I_{\text{thresh1}} = \frac{1}{\sqrt{L_{sat}}}\sqrt{4k_{fb_{load}}I_{sat}(L_{sat} - L_{unsat}) + L_{sat}I_{\text{thresh2}}^2}
\]

\[
I_{\text{thresh2}} = \frac{-1}{\sqrt{L_{sat}}}\sqrt{4k_{fb_{load}}I_{sat}(L_{unsat} - L_{sat}) + L_{sat}I_{\text{thresh1}}^2}
\]
D-3 Resonant Transition Time

The resonant transition time is the time it takes for switchover or commutation from S1 to D2 or S2 to D1.

If it is assumed that the capacitors $C_s$ are infinitely large the output voltage during resonant transition is:

$$v_c(t) = (v_c(t_0) - v_{Cs}(t_0)) \cos(\omega_{res} t) - \left[ \frac{(1 + k_{fb})I_{load} + i_L(t_0)}{2\omega_{res} C_r} \right] \sin(\omega_{res} t) + v_{Cs}(t_0)$$

where:

$$\omega_{res} = \frac{1}{\sqrt{2C_rL_{sat}}}$$

For transition from S1 to D2:

$$v_c(t_0) = \frac{V_s}{2} \quad v_{Cs}(t_0) = \frac{V_s}{2} \quad t = t_{res} \quad v_c(t_{res}) = \frac{V_s}{2}$$

$$i_L(t) = i_{Lr}(t) - k_{fb}I_{load} \quad \text{and} \quad i_{Lr}(t_0) = I_{thresh} \quad \therefore \quad i_L(t_0) = I_{thresh} - k_{fb}I_{load}$$

$$\frac{V_s}{2} = \left( \frac{V_s}{2} - \frac{V_s}{2} \right) \cos(\omega_{res} t_{res}) \cdot \left[ \frac{I_{load} + I_{thresh}}{2\omega_{res} C_r} \right] \cdot \sin(\omega_{res} t_{res}) + \frac{V_s}{2}$$

and:

Solve for $t_{res}$:

$$t_{res} = \frac{\text{atan} \left[ \frac{1}{2mV_s^2\omega_{res} C_r} \frac{I_{load} + I_{thresh}}{\sqrt{(I_{load} + I_{thresh})^2 - 4V_s^2\omega_{res}^2 C_r^2m}} \right]}{\omega_{res}}$$

For transition from S1 to D2:

$$v_c(t_0) = \frac{V_s}{2} \quad v_{Cs}(t_0) = \frac{V_s}{2} \quad t = t_{res} \quad v_c(t_{res}) = \frac{V_s}{2}$$

$$i_L(t) = i_{Lr}(t) - k_{fb}I_{load} \quad \text{and} \quad i_{Lr}(t_0) = I_{thresh} \quad \therefore \quad i_L(t_0) = I_{thresh} - k_{fb}I_{load}$$
\[
\frac{V_s}{2} = \left( \frac{V_s}{2} - \frac{m}{2} \right) \cos (\omega \ res \ t \ res) - \left( \frac{I_{load} + I_{thresh2}}{2 \omega \ res \ C_r} \right) \sin (\omega \ res \ t \ res) + m \frac{V_s}{2}
\]

and:

Solve for \( t_{res} \):

\[
t_{res} = 2 \ \frac{\text{atan} \left[ \frac{1}{2 \cdot m \ V_s \ \omega \ res \ C_r} \left( I_{load} + I_{thresh2} + \sqrt{\left( I_{load} + I_{thresh2} \right)^2 + 4 \cdot m \ V_s^2 \ \omega \ res \ C_r^2} \right) \right]}{\omega \ res}
\]

(D-27)