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FRACTURING AND DEFORMATION AT THE EDGES OF TABULAR GOLD MINING EXCAVATIONS AND THE DEVELOPMENT OF A NUMERICAL MODEL DESCRIBING SUCH PHENOMENA

RICHARD KENNETH BRUMMER
I declare that this thesis is my own unaided work and that it has not been submitted, in part or in full, for a degree at any other University.

This thesis describes an investigation into the nature of the fracture and deformation mechanisms which occur at the edges of tabular gold mining excavations. Published information on these phenomena is reviewed, and the necessary underground investigation required to consolidate the previous work is described. It is concluded that the rock near the reef plane at the edges of these mining excavations is subject to stresses sufficiently high to cause it to fracture through the formation of regular patterns of shear planes. These fractures can form in the solid rock some distance ahead of the mining excavation. Nearer the mining face, extension fractures form which result in slabbing or splitting of the exposed rock.

An idealization of the observed rock behaviour is proposed, which is then incorporated with conventional boundary element techniques into a numerical model (SEAMS) which is capable of analyzing two-dimensional tabular mining excavations where the rock near the reef plane at the edge of the mining excavation fractures, deforms and sheds load.

A sensitivity analysis of the numerical model is described which identifies those mining parameters capable of being used to advantage in controlling the size of the fracture zone. The model is then used to analyze two mining situations, namely a longwall face and a series of hypothetical pillars of various width to
height ratios. In both applications the model is shown to produce results consistent with behaviour observed in the field and laboratory.

It is concluded that the method of analysis described realistically and simply models the way a seam of rock fractures and deforms under high stress. This method can form the basis for the development of stress analysis packages which will permit rock mechanics engineers to design mine layouts taking into account the finite strength of most mined reefs. Although the numerical model described here is two-dimensional, the principle of the method can be extended to the analysis of three-dimensional tabular excavations.
Hierdie proefskrif beskryf 'n ondersoek na die aard van die breuk- en deformasie-mekanismes wat op die rand van tafelvormige goudmynbou-uitgravings voorkom. 'n Oorsig word gegee van die gepubliseerde inligting oor hierdie verskynsels, en die ondergrondse ondersoek wat nodig is om die voorafgaande werk te konsolideer, word beskryf. Daar word tot die gevolgtrekking gekom dat die rots naby die rifvlak op die rande van hierdie mynbou-uitgravings onderworpe is aan spanning wat hoog genoeg is om dit deur middel van die vorming van reelmatige skulpvlakke te laat verbrokkel. Hierdie breuke kan in die solide rots 'n afstand voor die mynbou-uitgrawing plaasvind. Nader aan die mynboufront vind verlengingsbreuke plaas wat tot bladskilfering of splitsing van die blootgelegde rots leid.

'n Idealisering van die waargeneemde rotsgedrag word voorgestel wat dan met konvensionele grenselementtegnieke geinkorporeer word in 'n numeriese model (SEAMS) wat tweedimensionale tafelvormige mynbou-uitgravings kan ontleed waar die rots naby die rifvlak op die rand van die mynbou-uitgravings verbrokkel, deformeer en belasting verloor.

'n Gevoeligheidsontleding van die numeriese model word beskryf wat daardie mynbouparameters identifiseer wat met vrug gebruik kan word om die grootte van die breukesone te beheer. Die model word dan gebruik om twee mynbusituasies te ontleed, naamlik 'n
langfront en 'n reeks hipotetiese pilare met verskillende breedte-tot-hoogteverhoudings. In albel toepassings lewer die model resultate wat ooreenstem met gedrag wat in die veld en die laboratorium waargeneem is.

Die gevolgtrekking word gemaak dat die ontleedingsmetode wat beskryf is, die manier waarop 'n rotslaag onder hoe spanning verbrokkel en deformeer, realisties en eenvoudig modelleer. Hierdie metode kan die grondslag vorm vir die ontwikkeling van spanningsontledingsprogramme wat rotsmeganika-ingenieurs in staat sal stel om mynultlegte met inagneming van die eindige sterkte van die meeste gemynde riwwe te ontwerp. Hoewel die numeriese model wat hier beskryf is, tweedimensionaal is, kan die beginsel van die metode tot die ontleiding van driedimensionale tafelvormige uitgrawings uitgebrei word.
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1. INTRODUCTION

1.1 Rock Pressure in the South African Gold Mining Industry

For the past 100 years, almost since mining began on the Witwatersrand, rock pressure has posed hazards for the mining industry. The problem first manifested itself in the form of earth tremors, and it was concluded as early as 1908 that these were due to the "shattering of support pillars" (Ophirton Earth Tremors Committee, reported by Cook et al, 1966). At that stage, the mining activities on the central Witwatersrand were being carried out at depths shallower than 300 m.

The rock pressure problems which began at mining depths of 300 m, have become more severe as the depth of mining has increased; some mines are now operating at depths greater than 3600 m and even deeper mines are being planned. Experience since the early years of mining on the Witwatersrand has identified the rock pressure problem as the greatest hazard, threat and technological challenge to deep level mining. During 1985, rockbursts and rockfalls (hazards directly attributable to rock pressure) accounted for 59% of all fatal accidents and 26% of all injuries which occurred in South African gold mines (Legge, 1987). More significantly, rock pressure related fatal accidents have formed an increasing proportion of all South African gold mine fatalities since 1926 (ibid.). Significant advances in the field of rock mechanics and its application in mines will have to be made before safe mining operations at depths greater than 4000 m can be planned.
with confidence.

It is assumed that the reader of this thesis is familiar with South African gold mining conditions, in particular the mining methods and geometries generally used. Terms such as stope, energy release rate (ERR), face etc. will not therefore be defined, as this would make the thesis laborious, and these have been defined elsewhere (see Budavari, 1983 for example).

1.2 The rock fracturing problem

The act of mining redistributes the virgin stress and induces increased stresses at the edges of tabular gold mining excavations. This results in extensive fracturing of the rock near the reef plane. The stope faces have to be advanced through this fractured region, and this causes several severe problems for the industry, as outlined below:

1. Support problems - The loose blocks of rock bounded by mining induced fractures and geological discontinuities need adequate support to prevent them falling in on mine workers.

2. Control of stoping width - This problem is closely related to the support problem, and occurs where extensive falls of ground result in the unavoidable removal of large quantities of barren rock along with the reef. This reduces the grade of the reef which has to be treated and can make the mining operations uneconomic.
3. Drilling difficulties - Conventional mining is carried out by drill-and-blast methods. Experience has shown that it is more difficult to drill into heavily fractured rock than into intact rock.

4. Problems regarding mechanization - Research is currently being carried out into the feasibility of large scale mechanization of gold mining. One of the techniques being examined is non-explosive mining of the reef by means of various mechanical rock breaking systems. It has been observed in an experimental mechanically mined stope that occasionally the regular pattern of fractures on the face does not form, and this makes it very difficult to mine mechanically. Current mechanical engineering technology is such that an unfractured face cannot be broken economically without explosives.

5. Transport of the broken rock - Rock handling difficulties are caused by the occasional production of large blocks of unfractured rock, even by conventional blasting. The rough footwall produced by the heavily fractured rock also makes it difficult to move the broken rock out of the stope.

1.3 Energy considerations

In South Africa two major advances in the field of rock mechanics have occurred in the last two decades. The first is the
recognition that outside the fracture zone surrounding mine excavations, the rockmass behaves elastically. The second is the development of the Energy Release Rate (ERR) concept (Cook, 1967 or summarised in detail by Salamon, 1984) and its use to determine how seismically hazardous a mine layout is likely to be (Hodgson and Joughin, 1967). These two advances have provided rock mechanics engineers with a rational method for designing the layout of a deep mine.

If the rockmass behaved entirely elastically, the ERR would correspond to the amount of strain energy actually stored in the rock removed from the face during mining (Salamon, 1984). In practice, the removed rock does not "contain" this amount of strain energy; the energy is released or dissipated in fracturing and deforming the rock near the stope face. Since stored strain energy can potentially be released catastrophically in the form of seismic events, a better understanding of the energy changes which occur in the fractured rock could thus hold the key to designing safer mining layouts.

1.4 Scope of work

The research described in this thesis was undertaken in order to improve the understanding of the behaviour of the fracturing rock at the edges of tabular gold mining excavations, and to develop a numerical model capable of describing the essential aspects of the observed behaviour. In order to achieve these objectives, the following investigations were necessary:
1. Firstly it was necessary to determine the nature and pattern of the fractures which form ahead of stope faces and at the edges of tabular mining excavations. This was done by reviewing the extensive published literature on the subject, and by carrying out underground fracture mapping in greater detail and over larger areas than had been done before. In this way, it was possible to combine the previously published work with the detailed fracture mapping and thereby to generate a representative fracture pattern.

2. In parallel with the investigation of the fracture pattern, it was necessary to determine the deformation mechanisms which occur in association with the formation of fractures. This was done by reviewing the published literature on the subject, and by making additional supplementary measurements and observations of the nature and amount of deformations which occur in the fractured rock near the reef plane.

3. After the fracture pattern and mode of failure of the reef rock had been identified, an idealized mechanistic model for the behaviour of the fracturing reef rock was proposed, and the behaviour of this failed rock model was fully investigated. It was demonstrated that the model can be viewed as defining the constitutive law for the fracturing rock and that this constitutive law is qualitatively similar to the behaviour of triaxial rock specimens tested in the
laboratory. With appropriate model parameters, the model also produces quantitatively realistic behaviour. This model for the falling rock was then incorporated into a boundary element scheme which enables one to model a two dimensional stope, and allow the rock near the reef plane to deform inelastically, while the rock remote from the mining excavation behaves according to the laws of elasticity.

4. Parameter studies were then performed, where the sensitivity of the model to the various model parameters was investigated. This was done partly to verify that the model was capable of modelling a relatively wide range of situations, and partly to investigate whether any of the parameters could be used in a real mining situation to control the behaviour of the fracture zone.

5. The results produced by the model were then shown to be consistent with both the observed depth of fracturing and amount of dilation measured in a well documented investigation. A further study was then carried out to demonstrate that the behaviour of brittle rock pillars can be realistically modelled by means of this technique.

The numerical model developed and described here requires several input parameters, some of which are very difficult or impossible to measure in the field. However, it is usually possible to back-calculate the required parameters from known situations.
underground, or to make reasonable assumptions. In this way, the numerical model can be tuned to match a particular set of circumstances, and then used to extrapolate beyond known conditions. Aspects of the model which are not well established, or where information is unavailable due to a lack of experimental data (stresses, for example are notoriously difficult to measure in highly stressed fractured rock) are critically reviewed near the end of the thesis.

1.5 Structure of thesis

This thesis is arranged broadly around the investigations described above, and consists of the following sections:

Section 2 contains a review of the relevant literature on mining-induced fracturing, deformation and numerical modelling of mining excavations.

Section 3 describes work done to investigate and explain the pattern of fracturing commonly observed to exist at the edges of tabular mining excavations at depth and concludes with a proposed typical fracture pattern.

Section 4 identifies and describes deformation mechanisms which are observed to occur and shows how these mechanisms operate together to permit the large scale overall deformations of the rock mass. A mechanism for the gross inelastic deformations observed is proposed.
In Section 5 the details of a numerical model (called SEAMS) based on the proposed mechanism and standard boundary element theory are then developed. The section concludes with various examples demonstrating the results produced by the numerical model and comparing these results with analytical solutions and other published data.

Section 6 contains a brief discussion of the parameters required by the model, suggesting and justifying suitable values for these. A sensitivity analysis is also presented which investigates the effect which each parameter has on the results produced for a typical case, i.e., the fracturing at a stope face.

Section 7 comprises a validation exercise, and demonstrates applications of the model for the analysis of the fracture zone which forms at the stope face and the behaviour of pillars of various width to height ratios.

Section 8 contains a critical appraisal of the method developed and discusses the major uncertainties and assumptions which have had to be made in developing the model. These uncertainties indicate areas where further research is necessary. The conclusions of the thesis are presented in Section 9.

A listing of the computer code developed in the course of the research is contained in the Appendix.
2. REVIEW OF PREVIOUS RESEARCH

Since the objectives of this research required a detailed study of the fracture and deformation mechanisms at the edges of tabular mining excavations, a review of published literature on these subjects was necessary. In addition, a review of the various attempts to model such phenomena was also completed. These reviews were necessary in order to provide a background to the work, to summarise what was known about these phenomena, and to highlight discrepancies and conflicting arguments. The results of these reviews are discussed below.

2.1 Mining Induced Fracturing

Leeman (1958) described fractures observed to form around tunnels in mines of the Central Witwatersrand at depths of up to 3 000 m below surface. He noted that "two types of failure may be distinguished, namely, the so-called 'ring stress' failure, ... and the so-called 'slabbing' failure".

Leeman attributed the "ring stress" failure to a shear mechanism, although he gave no description of the surface condition of or displacements on these fractures. He noted the similarity between the "slabbing" phenomenon and the tendency for axial splitting in uniaxial test specimens. On the basis of his observations, Leeman speculated that the fracture pattern around large span excavations would be as shown in Figure 2.1. He did not specifically describe the nature of the fractures.
Figure 2.1  Leeman's proposal for the fracture zone around a deep stope (Leeman, 1958)
Cook (1962) showed that the strength of the quartzite host rock would be exceeded by the stresses induced at the face. He also measured "considerable" stress concentrations between 3 m and 10 m ahead of an advancing longwall face. Cook showed that a Mohr-Coulomb type of failure criterion applied to the quartzite in the laboratory, and that the samples which he tested failed by the formation of shear planes which often formed wedges, subsequently splitting the samples in tension.

Joughin (1965) showed by means of a seismic network that most of the rock failure to occur as a result of mining was confined to a region relatively near to the reef plane and that only a very small proportion of the energy released by mining was radiated seismically. He further concluded that the "...large quantities of energy that are dissipated stably can only be dissipated by friction in the fracture zone...".

Hodgson (1967) investigated the behaviour of the failed zone of rock ahead of the face by means of seismic observations and convergence measurements. He made some interesting conjectural observations concerning the rate at which energy is dissipated in the failed zone ahead of a face, suggesting that the seismic hazard would be increased by mining the face at too high a rate. This was because he observed considerable time dependence in his convergence measurements, noting that movements continued some time after the face was blasted.
Kersten (1969) described the fractures which he observed in the hangingwall of an advancing stope at the Hartebeestfontein Gold Mine. He distinguished between three classes of fractures as follows:

Class I: Fractures exhibiting clean surfaces similar to those caused by pure tension.

Class II: Fractures having characteristics intermediate between Class I and Class III.

Class III: Fractures which exhibit definite evidence of shear displacement.

Kersten's extensive measurements of the dip of the fractures in the hangingwall reveal a consistent difference in dip between his Class I and Class III fractures. As an example, the data from his Figure 5 are reproduced here as Figure 2.2, together with least squares regression lines for the Class I and Class III fractures.

From this figure it is apparent that the Class I fractures dip away from the mined area while the Class III fractures dip towards the mined area. A relatively constant dip difference of a little over 30° is also apparent between the Class I and Class III fractures.

McGarr (1971) stated that there "... seem(ed) to be two types of fracture plane ...", one of which dips towards the stope and the other away from the stope in the hangingwall. McGarr classified
Figure 2.2 Shear and extension fractures based on Kersten's observations (Kersten, 1969)
his fractures as:

Type 1: Fractures which show little or no comminution
Type 2: Fractures which show considerable comminution

McGarr's diagram is reproduced as Fig 2.3.

McGarr assumed that the Type 1 fractures were "... planes of shear failure ...", an assumption which possibly contradicts his own description of their exhibiting "... little or no comminution ...

Gay (1976) described a series of laboratory tests carried out in order to simulate the mining-induced fracturing. Holes (which corresponded to mining excavations) were created in blocks of rock subjected to uniaxial and biaxial compression. Although the relatively uniform blocks were probably not a good model for the layered rockmass insitu, some interesting observations were made. Gay noted the occurrence of two types of fracture which he called type 1 and type 2, the disposition of which agree with that of McGarr (1971), pictured in Figure 2.3. Gay observed that his type 1 fractures formed first at lower stress and at smaller "spans". Type 2 fractures occurred later, i.e. at higher stress and larger span. Gay noted that the type 2 fractures initiated long before they became visible, and that shear displacement subsequently occurred on them, causing comminution.
Figure 2.3 McGarr's fracture classification (McGarr, 1971)
Van Proctor (1978) published findings on fractures ahead of a face which were observed by means of a petroscope in boreholes. Although he did not attempt to classify the fractures, he did note that there was some regularity in the formation of 'solid zones' and 'fracture zones' ahead of the face.

Roerling (1978) performed a systematic investigation of mining-induced fracturing ahead of an advancing face. He addressed the question of whether the fractures which he observed formed 'stably' or were 'burst fractures'. Very significantly, Roerling showed that most of the shear structures which he observed were "... simply related to a systematic set of inclined fractures ahead of the face which allows the rock to be shortened vertically and extended horizontally into the stope, " and that the "... entire process proceeds non-violently".

Later Roerling (1979) classified these fractures into the following types:

Type 1: Steep fractures parallel to the stope face without any displacement in the plane of the fracture

Type 2: Fractures parallel to the stope face but which dip towards, or away from the stope ... and which may reveal a pronounced component of displacement in the plane of the fracture

Type 3: Low angle fractures morphologically identical to Type 1 fractures but having shallow dip.
Because he could find no data to suggest that Type 1 and Type 2 fractures occurred at different times or stages, Roering concluded that they form "... essentially at the same time".

Adams & Jager (1980) showed a correlation between the depth of fracturing ahead of the face and the energy release rate. They also showed evidence for a "zoning" phenomenon in that "...fractures form(ed) in discrete zones separated by solid rock..."

Adams et al (1981) summarised previous research on fracture patterns and described the factors which influenced the development of mining induced fractures. They used Roering’s (1979) classification, illustrated diagrammatically in Figure 2.4. The existence of Type 1 fractures was inferred from observation of vertical splitting or slabbing on the face, and from their borehole observations.

They held the view that the first fractures to form were the Type 1 fractures, as their boreholes revealed zones of localized vertical splitting some distance ahead of the face.

Joughin and Jager (1984) described the same classification system, but also stated that certain fractures were observed to be confined to an individual stratum bounded by parting planes. They also noted that large horizontal slip on parting planes
Figure 2.4  Fracture classification according to Adams et al. (1981)
occasionally occurred.

Cook (1984) used arguments based on laboratory testing to suggest that two types of fracture could be expected to occur ahead of the face, namely shear fractures and cleavage fractures. Shear fractures are commonly observed under high confinement and cleavage fractures occur under low confinement, for example in uniaxial tests. He argued that the cleavage fractures can only form near the free surfaces, i.e. on the stope face.

The previous work as outlined above highlighted several areas of agreement between researchers, and some inconsistencies. These are discussed below.

Firstly, most workers agree that at least two types of fracture form. By their own conclusions, (Leeman, 1958, Kersten, 1969, Roering, 1979, Adams et al, 1981, and Cook, 1984) these two categories of fractures form due to a shear mechanism and due to an extension (or indirect tension) mechanism respectively. The same conclusion can be reached in the case of the work of McGarr (1971), since he suggested that all of the fractures which he observed were formed by a shear mechanism, although his Type 1 fractures exhibited "little or no comminution". McGarr's type 1 fractures were thus probably also extension fractures. Gay's (1976) work also indicated the existence of two types of fractures, and he noted that his Type 2 fractures formed at higher stresses, and exhibited shear displacements.
The existence of third categories of fractures was proposed by Kersten (1969) and by Roerling (1979) whose classification was later followed by Adams et al (1981). Kersten's third category (his Class II) was introduced in order to accommodate fractures which exhibited characteristics intermediate between shear and extension fractures and whose classification was thus uncertain. Roerling's third category (his Type 3) was said to be "morphologically identical" to his Type 1 (extension) fractures, and appears thus not to constitute (or warrant) a separate category.

There is less agreement in the literature on the position of formation of the fractures. Roerling (1979) suggested that the two types of fracture (his Type 1 and Type 2 fractures) formed contemporaneously, while Adams et al felt that the extension fractures (Roerling's Type 1) formed first, that is furthest ahead of an advancing face. This conclusion was based on observations in boreholes drilled into the face. Gay (1976) observed that the shear fractures formed under higher stress conditions than did the extension fractures, but did not comment on the position of formation of these fractures.

A very interesting observation, first reported by van Proctor (1978), and subsequently by Adams and Jager (1980), and by Adams et al (1981) is that the fracture pattern as observed on the reef plane ahead of the face exhibits a periodic nature.
Adams and Jager proposed that this periodicity was due to the formation of zones of extension (their Type 1) fractures, and showed that this periodicity was not due to the drill and blast cycle, since it was observed in a stope which was mechanically mined.

2.2 Deformation near the edges of tabular excavations

Relatively little quantitative data on the inelastic deformations which occur near the edges of gold mining excavations have been reported.

Thompson (1962) and Deacon (1962) observed that horizontal deformations on bedding planes occurred. In some cases these deformations were large enough to shear through roof bolts. Deacon used these observations to propose an explanation for the changes in the face rock brought about by "caving" (leaving the stope unsupported a few metres away from the face). He reported that the maximum vertical stress peak appeared to move further ahead of the face as the cave was initiated. The cave apparently reduced the confinement on the face rock, permitting it to shed vertical stress.

Kersten (1969) noted that the orientation of the fractures which formed ahead of the face could be altered by relaxing the confinement provided by the hangingwall strata, i.e. by caving.

It has been shown by several researchers that remote from the

It was noted by researchers who were able to record data continuously that most of the near stope movements were discrete and coincided with seismic events (Hodgson, 1967, McGarr & Green, 1975, Legge, 1984).

Researchers who measured larger stope convergence than elastic theory predicted, have usually ascribed the discrepancies to "bedding plane separation" or to "slip on cracks" (Walsh et al, 1977).

Gay (1976) in a series of model stope tests showed that at the stope faces shear fractures formed, and postulated that these fractures, together with movement on bedding planes permitted overall rockmass movement into the stope.

Legge (1984) carried out a large number of measurements of rock deformation at, ahead of and behind the stope face. Legge showed that as the rock ahead of the face fractures, it dilates towards the mined out area. In the horizontal direction, he proposed that the rock ahead of the face could be divided into 3 regions. The first region, furthest ahead of the face, dilated by a relatively
large amount as the fractures first formed. The second region, between about 1.5 m and 4.0 m ahead of the face dilated relatively little as the face advanced, and the third region, up to 1.5 m ahead of the face dilated by a large amount again as the face fractured. Legge's measurements all referred to relative dilation, since his reference points, which were installed in fractured rock, could not detect dilation which had already occurred. Although Legge comprehensively described the deformations measured, his work did not provide an explanation for the fracture and deformation processes which occur. For this reason, some of Legge's previously published data have been analysed in more detail later in this thesis (see Section 4.1).

2.3 Numerical modelling of rock deformation

The use of numerical models based on elasticity is now well established for the modelling of rock stress and deformation, remote from regions where the rock fractures and deforms non-linearly.

Salamon (1963) showed that provided one knew the convergence distribution in a mined region, the stresses at other points in the rock could be calculated. It was later found that these ideas could be used to find the convergence distribution itself (Salamon 1964, Plewman et al, 1969). Crouch (1970, 1976), Crouch and Starfield (1983) and others have extended and popularised these principles, which are now known broadly as the Boundary Element Method (BEM). Commercial numerical modelling programs
based on elastic Boundary Element theory are now routinely used in South African mines for the purpose of mine design.

Non-linear effects due to crushing of the stressed rock near the edges of excavations are inherently far more difficult to model. Researchers such as Deist (1966) and Crouch (1970) implemented numerical models which permitted non-linear behaviour near the mining excavations, but certain simplifying assumptions had to be made. Deist for example developed an analytical solution for the strain softening behaviour of rock around an axisymmetric tunnel. He also showed that excavations such as square tunnels could be modelled by finite difference approximations, but the method was cumbersome, and not amenable to solution by the computers available to him. If Deist had had access to modern computing facilities, it is likely that his work could have been carried much further. Crouch (1970) developed a method combining boundary element and finite element theory but the method was applicable only to a repetitive rib-pillar mining geometry, and was intended to demonstrate the influence of the fractured rock on the elastic region, and not to model in detail the fracturing of the reef rock.

Peirce (1983) and Peirce & Ryder (1983) described a non-linear boundary element formulation for modelling the fracturing processes which occurred near mining excavations. Dilatancy and strain softening were incorporated into the scheme in order to model rock behaviour observed in laboratory tests. Peirce used
his technique to model a hypothetical square two-way symmetric tunnel. He found that a localization of damage occurred which unfortunately was influenced by his choice of element size. At low stress levels the numerical technique was stable but beyond a certain value of virgin stress the model became numerically unstable. Peirce hypothesised that the numerical instability could correspond to a real physical instability such as "strain bursting", but this was not substantiated. The method was never extended to model more general excavations which are not two-way symmetric.

The Finite Element Method has reached a high level of sophistication in the modelling of non-linear phenomena but suffers certain major disadvantages when compared to Boundary Element Methods. Peirce (1983), for example, carried out a comparison between his non-linear boundary element model and a commercially available finite element programme (Owen & Hinton, 1980). He found that in cases where a relatively small region of the rockmass behaved inelastically, his boundary element technique was more economical.

Hybrid techniques based on boundary elements and discrete elements (Lorig, 1985), appear at present to show promise for modelling rock behaviour in South African gold mines, but to date, none have been used successfully to model the fracture zone ahead of a stope face.
Based on the above review, it can be concluded that no numerical model has yet been applied successfully to the study of the non-linear deformation phenomena which occur near the edges of the tabular gold mine excavations.
3. FRACTURING AT THE EDGES OF TABULAR MINING EXCAVATIONS

The literature review of fracture studies described in Section 2.1 identified several areas where additional research was necessary, mostly to clarify inconsistencies in the published work. Moreover, none of the publications reviewed was based on extensive mapping of large areas; in some cases only relatively small exposures of fractured rock were mapped, and only Roering (1978, 1979) actually published examples of the areas mapped. The work described by Adams and Jager (1980) and Adams et al (1981) was based largely on observations made in boreholes drilled into the face, although this work employed the fracture classification developed by Roering (1978, 1979). For this reason it was decided to carry out detailed mapping over large areas, to produce dip sections of the fracture patterns as exposed behind the mining face, as well as sufficient other exposures to build up a complete picture of the fracture pattern which exists ahead of a mining face.

3.1 Method of observation

The investigation of mining-induced fracturing as described here was based largely on mapping underground using a stereophotographic mapping technique developed specifically for this work and described elsewhere (Rorke and Brummer, 1985). This technique enabled large areas of exposed fracturing to be mapped with relative speed and accuracy. A major consequence of the use of this technique was that far larger mapped areas were available
to the author than have been available to previous workers. The pattern of fracturing described later is far more apparent when one views large exposures of mining-induced fracturing; in particular the regular pattern of shear fractures is not apparent on mapped areas smaller than about 10 m.

3.2 Description of sites mapped

In order to sample a representative cross section of typical mining faces, mapping was carried out at several mines. However, mining-induced fracturing is difficult to observe in conventionally blasted mining stopes, as the blast-induced fractures tend to conceal fractures induced solely by mining-induced stress. At the time that this investigation was carried out, the Chamber of Mines Research Organization was engaged in the development of mechanical rock-breaking mining techniques at a site at the Doornfontein Gold Mine. Stopes at this site were mined without the use of explosives, and this provided an ideal opportunity to study mining-induced fractures. Experimental work was also in progress at a site at the Hartebeestfontein Gold Mine, and this meant that mining there was under the control of the Research Organization. Mining at this site could therefore be temporarily stopped in order to enable exposed areas to be cleaned and mapped. For these reasons, the larger mapped areas were obtained from these two sites, and are presented here.

The geology of the Doornfontein site in the Carletonville mining area has been described in detail by Legge (1984) and
Rorke (1984), and is therefore briefly outlined below.

The stope lies at a depth of 2400m below surface and mining is carried out on the Carbon Leader reef, which dips at about 22° to the south at this locality. A face length of about 300 m was available for experimental purposes, and the ERR varied from about 10 MJ/m² to 40 MJ/m² along the face. The ERR of the mapped areas presented here was about 24 MJ/m², and is thus typical of mining faces in the Carletonville area. The mining layout was as shown in Figure 3.1. The region was geologically relatively simple, and because the faces were mined using experimental mechanical mining machinery, the fracture patterns observed were not complicated by fractures caused by blasting. Waste rock was packed some 4 m behind the face as backfill; this 'wastepacking' provided the regional support and contributed to the relatively low (in relation to the spans) ERR values which existed at the site. The average stoping width was 1.1 m, though local falls occasionally resulted in stope widths of up to 1.8 m.

The hangingwall quartzites are generally hard and glassy and contain about 90% SiO₂. The footwall quartzites are slightly weaker and contain about 80% SiO₂. A simplified stratigraphic column of the rock near the reef plane is shown in Figure 3.2, together with representative values of the elastic properties of each stratum.

The second site, at Hartebeestfontein Gold Mine, was also mined
Figure 3.1 Plan of the Doornfontein Experimental site
Figure 3.2  Simplified stratigraphy near the Carbon Leader Reef at Doornfontein Gold Mine
for experimental purposes. This site was situated at a depth of 2100 m and mining was carried out on the Vaal Reef. The quartzites were slightly weaker than those encountered at the Doornfontein site, and numerous bedding planes occurred both in the footwall and hangingwall. These bedding planes were on average 300 mm apart. The ERR at this site was 10 MJ/m$^2$, which is typical for mining faces in the Klerksdorp mining area.

3.3 Fracture patterns observed.

When the fracture mapping was carried out, it was decided that the fractures which were exposed would be classified according to only one criterion, namely whether or not the fracture plane exhibited signs of shear displacement, since this characteristic could be expected to indicate the mode of formation of the fracture. The system used here therefore groups fractures into the following two categories:

1. Extension fractures - which form in the direction of maximum principal stress when the minimum principal stress is low or tensile. They exhibit no signs of shear displacement.

2. Shear fractures - which are often complex macroscopic features comprising many closely spaced extension fractures but which have the overriding common characteristic of permitting relative lateral movement between the intact rock on either side of the overall shear zone.

In the brittle quartzites, even relatively small shear movements
cause sufficient comminution of the material along their surfaces for them to be easy to identify. In the mapping described here, any fracture which showed some evidence of shear movement was classified as a shear fracture. Any fracture which did not, was called an extension fracture. It is argued that in laboratory testing, fractures are regarded as either shear or extension, and it therefore is not necessary to establish any third category, as was done by Kersten (1969) and Roering (1978, 1979). In the present study, shear fractures were observed to range in width from 5 mm up to 200 mm.

Geological features such as joints, faults, quartz veins and parting planes, of which the latter are by far the most important in their occurrence and effect, were also observed and are included on the fracture maps which follow. The parting planes are often mere discontinuities between the material on either side but can also contain a layer of softish shaley material up to 100 mm thick.

Figure 3.3 is a hangingwall map (plan view) of a single panel between 11 Gully and 12 Gully (see Figure 3.1) at the Doornfontein site. Because this panel was mined without explosives and because there was generally a very good hangingwall parting plane, shear fractures were easily observed. Figure 3.3 shows that these shear fractures are relatively continuous and in some instances extend the full length of the 20 m long panel. (In a study by Roering (1982), similar fractures were observed to continue
Figure 3.3 Plan view of single panel of stope hangingwall at Doornfontein Gold Mine (after Brummer and Rorke, 1985)
parallel to the face for distances of up to 60 m down dip.) They are sub-parallel to the stope face and appear not to be affected by the shape of the immediate face. The extension fractures however tend to follow the shape of the immediate mining face, as can be seen from the fact that there is more variability in direction of the extension fractures, with orientations similar to the face shapes marked on the map.

Observations by the author at Blyvooruitzicht Gold Mine where the longwall examined was mined with five metre lags between panels showed that the shear fractures strike parallel to the general longwall direction, while the extension fractures observed at the same site were parallel to the individual panel faces. This suggests that the shear fractures formed some distance ahead of the face where the stress field orientation was influenced by the general longwall direction. The extension fractures must form later, as the face advances nearer to the site of fracture formation, since they follow more closely the shape of the immediate mining face. The shear fractures are probably analogous to the longwall parallel fractures described by Hagan (1980).

Figure 3.4 shows a section (looking updip) of a hangingwall strike slot which was excavated in 5 Gully (see Figure 3.1) some 30 m back from the face in order to expose the mining-induced fracturing. Because the rock was relatively destressed at this distance from the face, the excavation was performed relatively easily. The sidewalls of the slot were cleaned by means of pinch
Figure 3.4  North sidewall of hangingwall strike slot at Doornfontein Gold Mine
bars so that all blast induced fracturing was effectively removed and the fractures shown in this figure were virtually all induced by mining.

The extension fractures shown in this exposure dip toward the unmined area. Their distinguishing features are that the majority have a consistent dip of about 15° and they are relatively short in length. In the field, they exhibited no evidence of shear displacement. They are not intersected by parting planes or shear fractures, indicating that they formed after the shear fractures. The shallow dip of 15° observed here was unusually low; extension fractures observed in hangingwall exposures are usually steeper, and dip at about 60°.

The shear fractures observed in this exposure have the following characteristics; they show evidence of shear displacement in the form of whitish powdery gouge on their surfaces and they dip closer to the vertical than the extension fractures. They are spaced at intervals of about 1 m, and there are two complementary directions as can be seen in Figure 3.4.

Figure 3.5 shows the North sidewall of the 28 Level Reef Drive (see Figure 3.1). The two classes of fracture can be seen very clearly in this exposure. The shear fractures are regularly spaced, with a spacing of 1.0 to 1.5 m and do themselves vary in width up to about 500 mm. This regular pattern is the only characteristic of the exposure which exhibits a cyclic nature. In
Figure 3.5 North sidewall of Reef Drive at Doornfontein Gold Mine
examining boreholes drilled into the face on the same site, van Proctor (1978), Adams et al (1981) and Joughin and Jager (1984) all noted a cyclic pattern of fractured and unfractured zones with a spacing of roughly 1.0 to 1.5 m. It must thus be concluded that this cyclic pattern is due to the formation of these shear fractures at regular intervals some distance ahead of the face, as there is no other feature of the fracture pattern which is cyclic. It would thus appear that these shear fractures manifest themselves in boreholes as zones of intense apparently vertical fractures, and form at some distance ahead of the mining face.

Figure 3.6 is a view of a fracture pattern exposed in a face at the Hartebeesfontein site. A plan view of this site is also given in the figure. A lead of about 30 m existed between the panel below and the panel in which the fracture pattern was observed.

The fracture pattern thus formed in the solid rock up-dip of a ledged gully siding which remained static (as opposed to a face which advances). The fracture pattern was subsequently exposed in the face during mining of the lagging panel. The exposure thus gives a unique view of the fracture pattern as it exists ahead of a face. Exposures more commonly seen are records of all the fractures superimposed on one another and subsequently viewed in a gully sidewall after the face is advanced, making it difficult to interpret the chronology of the fracture formation.

In this exposure, the fractures which occur deepest into the solid
Figure 3.6 Breast mining face advancing adjacent to a 30m long interpanel lead at Hartebeestfontein Gold Mine
rock are shear fractures; the extension fractures are only observed within a few metres of the 'stope face'. This is very strong evidence which shows that the shear fractures form first, i.e. furthest ahead of the face, and the extension fractures form at a later stage, due to the proximity of the actual stope face (See also Cook, 1984 and Section 7.2). The orientation of the shear fractures is such that they form conjugate pairs while the orientations of the extension fractures are such as to coincide with the expected direction of maximum principal stress associated with the stope, as indicated in Figure 3.6.

3.4 Summary of fracture characteristics

The fracture mapping carried out revealed that although there were variations in the observed fracture patterns between sites, certain common features existed. These characteristics of the fracture patterns are summarised below:

1. Shear fractures appear to form first, i.e. furthest ahead of an advancing face.

2. The shear fractures were observed to be spaced at fairly regular intervals. They were spaced at intervals of 1.0 m to 1.5 m at most of the sites mapped, which had ERR values of between 10 MJ/m$^2$ and 40 MJ/m$^2$. No consistent difference in spacing due to ERR was found.

3. In the footwall, a set of shear fractures forms which dips away from the mined area. The fractures curve slightly, such that they become vertical about five metres below the
reef plane, as can best be seen in Figure 3.5.

4. For reasons of symmetry, it is speculated that a similar but opposite pattern of shear fractures must exist in the hangingwall. For safety reasons it was not possible to investigate fracturing further than 2 m into the hangingwall, and this was only done at one site.

5. Near the reef plane, the shear fractures were observed to form conjugate pairs and it is suggested that this is due to an 'overlap' of the hangingwall and footwall fracture patterns.

6. Because the extension fractures were found to closely follow the shape of the stope face both on plan and in section, and because they were found to truncate on the shear fractures, it must be inferred that they form nearer to the advancing stope face, and after the formation of the shear fractures. This is clearly shown in Figure 3.6. It is thus probable that they form due to a lowering of the minor principal stress to the extent that the rock fractures in an almost uniaxial (plane strain) stress field, and are therefore phenomenologically identical to the 'slabbing' observed on tunnel sidewalls and the vertical splitting observed during uniaxial testing of rock samples.

3.5 Typical pattern of fracturing

Since the object of mapping the mining-induced fractures was to explain and understand the failure and deformation mechanisms producing these fractures, the common characteristics of the
fracture patterns observed have been combined to produce a representative picture of the fracture pattern which forms ahead of a typical stope face in a deep gold mine under similar geological and stress conditions to those mapped during this study. This pattern is shown in idealized form in Figure 3.7. A number of zones are indicated in the figure, each of which will be described in turn.

Zone 1
This represents the boundary between fractured rock and intact rock and is thus the position where the shear fractures first form. These fractures appear to separate the rock into distinct blocks of relatively intact material since the microfractures localize in a narrow plane of weakness, leaving the rest of the rock unfractured, as has been described in laboratory specimens by Hallbauer et al, 1973. From arguments presented earlier, it appears that in a borehole, the initial formation of these shear fractures appears as a zone of intense vertical fracturing. (See Adams, et al, 1981).

These shear fractures are parallel to the overall longwall direction and are spaced fairly regularly (see Figures 3.3 to 3.6). They curve slightly as can best be seen in Figure 3.5.

The formation of these shear fractures results in an initial dilation of the rock toward the stope, as was measured by Legge (1984).
Zone 2

The stresses in this zone can be described by points on a residual Mohr envelope, that is for rock in a state of limiting frictional equilibrium. Since the shear fractures have formed, the rock will have negligible cohesion along any fracture.

Two processes appear to occur in Zone 2:

1. On a macroscopic scale, movement is observed on the shear fractures formed in Zone 1. (This movement will be described in Section 4.)

2. On an intrablock scale, local high stresses at asperities and point contacts (Chappell, 1975) can cause individual blocks to split in a direction more or less parallel to the maximum principal stress direction, thus generating a number of extension or tensile fractures, with little or no relative displacement. (Where two shear fractures intersect, the "wedge tips" thus formed result in such a local high stress area, where severe crushing is commonly observed.)

Zone 3

This is a zone of local longitudinal splitting or extension fracturing right on the face, parallel to the direction of maximum principal stress, and either brought about or promoted by the presence of the free surface of the face. This process produces the slabbing on the immediate face which is usually observed in
stopes, and on the sidewalls of most tunnels at depth.

It is probable that in shallow or small-span stopes (having ERR values less than about 2 MJ/m²), this type of fracturing will be the only type of fracturing observed, as the stresses ahead of these faces will not be high enough to cause shear failure of the rock, as described in Zone 1. Hence the depth of fracturing ahead of these stopes will be only 0,5 m to 1,0 m of vertically split rock on the face. The sides of tunnels are probably the most common manifestation of this phenomenon.

Zone 4
This zone consists of the destressed strata in the hangingwall of the stope, where stresses are too low to result in the formation of new fractures.

Zone 5
A similar layer of fractured rock must exist in the footwall of the stope. This layer is often relieved of horizontal stress because of the development of dip gullies as shown in Figure 3.7.

This fracture pattern forms the basis for the numerical model developed in Section 5.
4. DEFORMATIONS AT THE EDGES OF TABULAR EXCAVATIONS

The underground mapping of the previous section resulted in the identification of the typical fracture pattern which forms at the edge of a tabular mining excavation. As the next logical step in the development of a useful numerical model, it is necessary to identify the deformation mechanisms which occur, so that these can be incorporated into the model in a realistic way. Those deformations which provide a mechanism for the observed gross inelastic deformation will be identified, and a model for the overall inelastic deformation will be proposed at the end of this section. This mechanistic model will provide the basis for the numerical model to be developed in Section 5.

Observations were made at several mines, and at various depths and excavation shapes. The displacements are discussed in a qualitative way, though quantitative data are quoted. ERR values are given in order for comparisons to be made between sites, since it has been found that as a rule, the observed phenomena all increase in extent at higher ERR values.

4.1 Displacement towards stope and dilation

Legge (1984) has described a series of extensometer installations designed to measure mainly horizontal dilations of the rock near the stope face. Legge's results show that extensometers installed directly into the face near the reef plane reveal a region of dilating rock, in some cases as deep as 10 m. Legge found a close
Figure 4.1
Horizontal dilation measured by means of extensometers (after Legge, 1984)

Dilation measured in three boreholes drilled horizontally into the face

Depth into borehole

Face position

Anchor points

End of hole

Deepest anchor

6 - 7 metres

Displacement of anchor point with respect to end of borehole (deepest anchor)
dilation profile which represents the total amount by which a particular point has dilated from its initial position before the rock first fractured. It is necessary to determine the absolute dilation of a point in the rock in this way so that comparisons can be made with the results of numerical models, which naturally give results in absolute terms.

It should be noted that even the absolute dilation profile depends on the assumption that the deepest anchor is stable. In the examples shown here, this is nearly true as can be seen by the 'flattening off' of the three extensometer curves shown in Figure 4.1.

The absolute dilation profile (derived by numerically integrating Legge's data as shown in Figure 4.1) is presented in Figure 4.2. To further demonstrate the significance of these profiles, two identical curves are shown displaced by 1.1 m (the mining step which gave rise to the dilations measured by Legge). The difference between the two curves may be compared with the dilation shown in Figure 4.1.

4.2 Sliding on parting planes

A phenomenon which has been observed by other workers (e.g. Deacon, 1962 and Thompson, 1962) and during the course of this study, is that slip displacements commonly occur near the stope face on the near-horizontal parting planes in the host rock.
Figure 4.2 Absolute dilation profile derived from Figure 4.1
An example which illustrates the magnitude of the possible movements was found at the Doornfontein site. A plan of the area is shown in Figure 4.3. The packs on both sides of the strike gully were photographed from the strike gully.

Looking down dip, all of the packs (initially installed vertically) were severely tilted, as shown in Figure 4.4. The figure shows that the footwall was displaced by 600 mm with respect to the hangingwall, and had apparently "relaxed" into a dip gully which is not shown in the photograph.

However, at the same position, looking up-dip, the packs were only slightly tilted, as shown in Figure 4.5. On this side of the gully, the dip gully was not present, and the footwall was thus unable to "relax" horizontally. This example clearly illustrates that the dip gullies play a major role in facilitating horizontal dilation of the footwall layers of rock.

In situations where no dip gully exists to relieve the induced horizontal stresses, the footwall strata have been observed to buckle upward. One example of this is illustrated in Figure 4.6. This photograph, taken at the Blyvooruitzicht Gold Mine suggests that the footwall actually buckled upward in response to a large horizontal stress. Elastic theory predicts a tensile stress in these strata. The crushing, and consequent dilation of the rock ahead of the face must be the mechanism responsible for generating such horizontal stresses in the footwall and hangingwall strata.
Figure 4.3  Plan showing location of Figures 4.4 and 4.5
Figure 4.4  View downdip showing how the footwall has displaced away from the face and relaxed into a footwall dip gully (out of view to the left in the figure). Packs were initially vertical.
Figure 4.6 Upward buckling of the footwall strata indicated by tilt of vertically installed pipesticks (photo by N. B. Legge)
A similar though more extreme case is shown in Figure 4.7 (Gay & Jager, 1980), this time in a shale footwall at the Geduld Gold Mine.

Because of the apparent significance of these slip movements, it was decided to attempt to measure bedding plane slip underground, in order to discover how far away from the reef plane this type of movement occurred. At the time, suitable sites were available at the Hartebeestfontein site and at Western Deep Levels Gold Mine. Vertical boreholes were diamond-drilled into the hangingwall at Hartebeestfontein, and both into the hangingwall and footwall at Western Deep Levels. In both cases, it was immediately found that bedding plane slip occurred well into the footwall and hangingwall.

Figure 4.8 illustrates step displacements near the bottom of a pillar at the Western Deep Levels site (in response to a face advance of 10 m in the direction shown) and Figure 4.9 shows bedding plane slip in the hangingwall at the Hartebeestfontein site observed after a face advance of 1 m, and the initiation of "caving" at the site. These stepwise displacements occurred within a few days and it was consequently only possible to make one observation in the boreholes at Western Deep Levels, and three at Hartebeestfontein, before it was no longer possible to insert instruments into the holes.
Figure 4.7  Upward buckling of footwall strata (Gay and Jager, 1980)
Figure 4.8 Lateral borehole displacements measured in the bottom of a pillar at Western Deep Levels Gold Mine due to a face advance of approximately 15 m.
Bedding plane slip which occurred after the start of caving.

Figure 4.9 Horizontal slip on bedding planes observed at Hartebeestfontein Gold Mine.
The fact that sliding on these parting planes occurs some distance above and below the stope leads one to the significant observation that there exists an "effective stope width", considerably wider than the actual width of rock stoped. The effective stope comprises the actual stope, together with the relatively passive loose strata in the hanging and footwalls and bounded by the parting planes on which significant horizontal slip occurs. The effective stope width must thus correspond to the overall height of the fracture zone which forms ahead of the stope face.

In the context of the pillar shown in Figure 4.8, this means that the pillar is "squeezing out" horizontally over a height of 5 to 7 metres, and not the 1 m pillar height commonly assumed in pillar design. This observation obviously significantly affects the width to height ratio of pillars. The true state of the pillar shown in Figure 4.8 probably lies between being a 20 m wide by 1 m high unconfined pillar to being a 20 m wide by 7 m high fractured pillar which is confined horizontally by the rock above and below the stope.

4.3 Sliding on Shear Planes

Early investigators often regarded sliding on shear planes (or mining induced shear fractures) as being synonymous with rockbursts. However, displacements on shear fractures were observed at all sites investigated (where clearly no rockbursts had occurred). The mere presence of a large shear fracture is therefore not an indication that a rockburst occurred at the site.
in question (see also Rorke and Roering, 1982). Certainly seismic events which result in rockbursts often manifest themselves as major shear movements. There is reason to believe that all of the major seismic events on certain goldfields are the result of slip on major faults, but shear failure of the stressed rock ahead of a face is not always the cause of rockbursts. Based on this study, it must be concluded that in almost all cases, stable shear failure occurs and benignly dissipates energy ahead of advancing stope faces, since all of the sites investigated exhibited regularly spaced shear fractures, and none had recently experienced rockbursts.

Shear displacements are difficult to measure in boreholes drilled into the face. No suitable instrument is, at time of writing, commercially available, and a special purpose instrument had to be developed for this task. This very simple device consists of a small wheeled trolley with a weighted cantilever arm projecting ahead of it. As the trolley is pushed down a hole, the steps in the floor of the hole are converted to rotation of the cantilever which is measured by means of a potentiometer in the trolley. The stepwise shear displacements were measured by means of this device.

Two examples of large shear displacements which occurred at the Doornfontein site are shown in Figures 4.10 and 4.11. These fractures were observed within a few metres of each other, and were photographed on the same day. In both cases the net
Figure 4.10 Shear displacement of 200mm observed ahead of a face at the Doornfontein Gold Mine - movement down to left
Figure 4.11 Shear displacement of 200mm observed ahead of face at Doornfontein Gold Mine - movement down to right
displacement between opposite surfaces was of the order of 200 mm, though the internal structure of the fracture zone is clearly far more complex in Figure 4.10. A seismic network covering the experimental site showed that although seismic events did occur in the area, these were all of magnitude less than 1.0, and resulted in no damage to the mining excavations. The observed fractures thus cannot be regarded as "burst fractures".

Figure 4.12 is a fracture exposed in the face at Hartebeesfontein, and is one of the photographs on which the mapping presented in Section 3 is based. As can be seen from this photograph, a shear displacement of 150 mm has occurred (near the "6" in the Figure). Reference to Figure 3.6 will show that this shear displacement was only one of a family of fractures which extended about 6 m into the solid rock at this site.

Shear displacements of this nature can be used to infer horizontal and vertical displacements. For example, the fractures exposed in the face at Hartebeesfontein all have displacement components in the horizontal direction. This has been done in Figure 4.13. This exercise illustrates the large movements which can occur. Converted into strain, the data reveal that the rock has strained in a horizontal direction by about 36 m/6, on average, ignoring elastic effects. Put into perspective, this amount of strain is more than three times the radial strain quoted by Hojem et al (1975) during their landmark triaxial testing of brittle quartzite (see Figure 5.5). This emphasizes the fact that the
Figure 4.12 Shear displacement near the edge of tabular excavation at the Hartebeesfontein Gold Mine
Total horizontal dilation = ± 250 mm  (= 36 millistrains)

Figure 4.13  Horizontal displacements calculated from shear displacements such as that shown in Figure 4.12, using detailed mapping data shown earlier in Figure 3.6
Inelastic strains occurring ahead of the face are extremely large. This is one of the main reasons why post failure rock behaviour must be synthesised from a consideration of fault and fracture behaviour, as has been done in this thesis.

Examples of shear displacement from a number of locations have been shown in this section. They exhibited varying internal structure, some were complex (Figure 4.10) while others were relatively simple (Figure 4.11). One characteristic has, however, been noted for all of the shear planes observed. That is that they are all similar to (geologically) normal faults. This means that they all form in response to a high vertical or near vertical stress - the rockmass falls in shear. This permits the rock to compress vertically, and results in an expansion horizontally, in this case towards the mined out area.

4.4 Stope Closure

Stope closure (or convergence) has been measured by a number of researchers. These measurements reveal that closure always occurs more rapidly than predicted by elastic models, and this is usually explained by the (unknown) amount of inelastic deformation which occurs. Some researchers have however reported measured convergence only a little larger than the elastically predicted convergence (Hodgson 1967). Inelastic deformations which occur ahead of the stope face can logically be expected to influence closure rates. For this reason, during the course of this and other research projects, routine closure measurements were made by
the author at Doornfontein Gold Mine, Blyvooruitzicht Gold Mine, Hartebeestfontein Gold Mine and at Western Deep Levels, where sites were available. Some of these measurements will be discussed below, as their interpretation led to some of the conclusions reached.

The author's convergence measurements at the Doornfontein site (previously described) indicated that the convergence was slightly larger than elastically predicted. This can be seen in Figures 4.14 and 4.15. The elastic curve shown for comparison is that for a slit with the same ERR as the average for the Doornfontein site. Figure 4.14 shows a fitted curve based on convergence measurements at the Doornfontein site. The single line is averaged from approximately 20 convergence profiles measured at this site, one of which is shown in Figure 4.15.

Convergence monitoring at Western Deep Levels and Hartebeestfontein revealed behaviour in which the measured convergence was far larger than that predicted by elastic theory. Convergence profiles measured at Western Deep Levels are shown in Figure 4.16. The large discrepancy between observed and calculated convergence can probably be best explained by the net effect of bedding plane separation, (the support at WDL was calculated to be supporting the weight of only about 3 m of hangingwall rock) and the slip on the horizontal parting planes caused by the dilation of the fracturing reef rock, as previously discussed. (Slip on horizontal planes is commonly regarded as being one cause of the
Figure 4.14 Composite closure profile of 20 measuring stations at the Doornfontein site.
Figure 4.15 Typical closure profile at the Chamber of Mines Experimental Stope at Doornfontein Gold Mine
Convergence profiles measured at Western Deep Levels Gold Mine in the middle of a stope between barrier pillars.

Figure 4.16
very severe subsidence profiles measured above coal longwalls.)
It is also probable that the horizontal layers of rock which form
the hangingwall and footwall are forced to buckle into the stope
by these dilation forces, in cases where there is no support to
prevent this from occurring. The differences between the two sets
of data presented must thus lie in the different internal support
systems used, as discussed below.

The Doornfontein stope was packed with waste rock (described in
detail by Joughin and Jager, 1978). Although this "waste pack"
does not develop significant support near the face, it does
provide a very good form of internal support (when significantly
compressed in the back areas) and this must prevent bedding plane
separation, and increase the frictional resistance between the
parting planes. Measurements taken by the author with a water
level closure instrument (described by Atkins and Keen, 1984) show
clearly how the vertical stress, which builds up in the waste
packing, forces the loose footwall strata back down after these
strata initially rise upward, as indicated in Figure 4.17.

The Western Deep Levels stope, in contrast, relied on pipesticks
for internal support. These support elements support virtually no
load when significantly compressed underground as has recently
been convincingly demonstrated by Roberts (1985).

The closure measurements described above confirm that the type of
internal support used in a stope can have a profound effect on the
Figure 4.17 Absolute movement of points on hangingwall and footwall at Doornfontein Gold Mine
closure, and therefore possibly also on the stope conditions, and the fracturing and deformation which occurs ahead of the stope face.

4.5 Deformation mechanism for gross inelastic movements

Based on the fracture patterns as described in Section 3 and the deformations described here, the following mechanism for the gross inelastic deformation is proposed:

High vertical stresses, induced in the solid rock ahead of the face by the mining operations, cause the rockmass to fall in shear, forming macroscopic inclined shear fractures, on which normal fault type displacements occur. This permits the rock to yield vertically, and shed vertical load, but results in a horizontal dilation, accompanied by an overall volumetric increase. The horizontal dilation is facilitated by the presence of the parting planes which exist in the host rock, since slip movements occur on these planes. These deformation processes occur over a height greater than the actual mined stope width, a height referred to here as the "effective stope width". The dilation of the fracturing rock ahead of the face appears to force the layers of rock in the hangingwall and footwall horizontally into the mined area. If insufficient support is installed in the stope, these layers can buckle into the stope. The layers of rock in the hangingwall and footwall must therefore provide confining stresses (the magnitudes of
which are not known at this stage) to the deforming rock ahead of the stope.

This proposed mechanism, illustrated in Figure 4.18, forms the basis for the numerical model developed in Section 5.
Figure 4.18 Mechanism for inelastic deformation near typical stope face

(Extension fractures omitted for clarity)
5. DEVELOPMENT OF THE FRACTURED ROCK MODEL

The investigation of the fracture and deformation processes resulted in the identification of the deformation mechanism described at the end of Section 4. This mechanism is qualitative, since it was idealized from many underground observations. In order for such a conceptual model to be useful, it is necessary that the model be shown to be quantitatively realistic. It was thus necessary to develop and expand the conceptual model within a numerical framework.

Since the primary objective of this research was to improve the understanding of the deformation phenomena, it was not necessary to develop a numerical model capable of being used as a mine design tool. Rather it was desired to have a model which could firstly be demonstrated to be a realistic model for the true processes which occur underground, and secondly which could be used to explore the sensitivity of the deformation mechanisms to various parameters which can possibly be used to influence the fracture zone.

5.1 Criteria for a suitable model

From the arguments presented above, it was possible to decide on the characteristics required of the proposed model. These requirements are outlined below:

1. The model should capture the essential elements of observed
rock behaviour at the edges of tabular mining excavations.

2. The model should be able to predict the horizontal extent (ahead of the face) of the fracture zone.

3. The model should be able to predict the amount by which the rock ahead of the face is crushed (vertically).

4. The model should be able to predict the amount by which the rock ahead of the face dilates.

5. The model should be able to account for the energy changes which occur in the fractured rock.

6. The model should explain or predict the stress distribution within the fracture zone ahead of the face.

In order to meet these criteria, the observed fracture pattern (Figure 5.1(a)) and inelastic deformation mechanism (Figure 5.1(b)) were idealized for the purpose of numerical modelling as shown in Figure 5.2.

The illustrated model embodies several simplifications to the observed rock behaviour. The most important of these is that the elastic rockmass is separated from the fracturing rock by distinct parting planes in the hangingwall and footwall. In practice, horizontal slip is usually observed on several parting planes both above and below the stope. However, as a first approximation, the movement is regarded as occurring on two planes only, in the interests of simplicity. This is felt to be reasonable, and refinements to this approximation can be implemented later should they be found to be necessary. In addition, since the model is
Figure 5.1 Fracture pattern (a) and inelastic deformations (b) near typical stope face
Elastic rockmass

Prominent parting plane on which slip occurs.

Passive hangingwall rock provides horizontal confinement to fracture zone

Prominent parting plane on which slip occurs.

Passive footwall rock provides horizontal confinement to fracture zone

Zone of fractured rock in a residual strength state. (Traversed by shear fractures at ±80 deg to vertical)

Zone of rock where shear fractures first form. Strength described by Hoek & Brown failure criterion.

Figure 5.2 Idealization of fractured rock near stope face
based on shear deformation mechanisms, it must be anticipated that the model will not be capable of describing the severe dilation which occurs when extension fractures form on the mining face.

The following sections describe the development of the numerical model SEAMS.

5.2 Behaviour of the Fracture Zone

Considerable insight into and simplification of the behaviour of the fracture zone can be gained by means of concepts used in the field of soil mechanics - that of limit equilibrium, and the notion of the Rankine active and passive states (see for example Smith, 1973). Knowing that a plane in a rockmass is sliding (in limit equilibrium) enables one to determine the direction of the resultant force on that plane, and also to determine the ratio between horizontal and vertical average stresses.

In Figure 5.3, what is required is a relationship between the vertical and horizontal stresses which act in the fracture zone. Since the stress distribution in a highly fractured rockmass is extremely discontinuous and variable, (see Chappell (1975) as an example), what is referred to here is merely the average vertical and horizontal stresses.

By considering the force components in the vertical and horizontal directions acting on the rock wedge, where the shear surfaces are assumed to be just sliding, one can show that:
Average vertical stress $\sigma_v$

Arbitrary element of fractured rock

Average horizontal stress $\sigma_h$

Shear fractures

Direction of stress resultant on shear fracture

Average vertical stress $\sigma_v$

Figure 5.3 Average stresses acting on an isolated wedge of rock
A comparison can be made between this result and the equation relating vertical and horizontal stresses in a soilmass which is falling in a Rankine active state (see for example Lambe & Whitman, 1969), where:

\[
\sigma_v = \sigma \frac{\tan(\theta + \lambda)}{\tan \theta} = \frac{\sigma \cdot \beta}{h} \quad \ldots 5.1
\]

Where:

\( \theta \) = inclination (from vertical) of shear fractures
\( \lambda \) = friction angle of shear fractures

Equation 5.1 represents the residual strength of a rockmass traversed by many shear fracture planes with orientations of \( \pm \theta \) to the vertical. Equation 5.1 can be shown to degenerate into Equation 5.2 should the angle \( \theta \) be set at \( 45^\circ - \phi/2 \), with \( \phi = \lambda \). Equation 5.1 is thus in agreement with conventional soil strength theory.

The ratio of vertical to horizontal stress in the fractured rockmass thus depends on the fracture inclination \( \theta \) and on the friction angles of the various shear planes. Values of this ratio are given for various \( \theta \) and \( \lambda \) values in Table 5.1.
Table 5.1  VALUES OF $\beta$ FOR VARIOUS $\lambda$ & $\theta$ VALUES

<table>
<thead>
<tr>
<th>SHEAR FRACTURE FRICTION ANGLE $\lambda$</th>
<th>SHEAR FRACTURE ANGLE $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$5^\circ$</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>5.33</td>
</tr>
<tr>
<td>$25^\circ$</td>
<td>6.60</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>8.00</td>
</tr>
<tr>
<td>$35^\circ$</td>
<td>9.59</td>
</tr>
<tr>
<td>$40^\circ$</td>
<td>11.43</td>
</tr>
</tbody>
</table>
In practice, most of the shear fractures observed during the course of this study were relatively steep with $\theta$ in the range of 10° to 20°, in hard glassy quartzites, and $\theta$ in the range of 20° to 30° in the weaker more argillaceous quartzites.

5.3 Local Wedge Tip crushing

An examination of the schematic fracture distribution and the idealized model presented in Section 4 reveals that for large amounts of overall crushing a geometric problem arises in that the tips of the wedges formed are forced to inter-penetrate. Underground observations of the rock in the region of two crossing shear fractures always reveals a local zone of intense crushing which corresponds to this "wedge tip" concept.

In order to simulate this effect in the model one can allow an extra "tip crushing" term which comes into effect after the rock has fractured and the deformations become large. This tip crushing offers increased resistance to vertical deformation. The basis for including an appropriate term into the analysis is outlined below.

Should a rock wedge be forced to impinge on a flat surface it would offer resistance which would depend on the local crushing strength and on the contact area, as shown in Figure 5.4.

A reasonable approximation to the average stress caused by this
tip crushing may be found at the centre of the wedge by:

\[ \sigma_{\text{vTC}} = \frac{F}{a} = \delta \cdot \Delta \cdot 2 \tan \theta / (d \cdot \tan \theta) \]

i.e. \[ \sigma_{\text{vTC}} = 2 \delta \varepsilon \]

Where:
- \( \sigma_{\text{c}} \) = Uniaxial strength of small samples
- \( \varepsilon_{\text{v}} \) = Average vertical strain.

The stress is determined at the centre, since it varies from a maximum near the tip to a minimum at the top of the wedge.

In practice the local tip crushing must result in a certain amount of confinement being generated at the wedge tips, which will cause the stress state near the crushed tip to become more triaxial than uniaxial. Tests carried out by Purrer (1983) where coal wedge tips were crushed in a similar manner to the mechanism discussed here, revealed that under average horizontal stresses between 2.5 MPa and 12.5 MPa, the tip crushing force could be approximated by 1.4 times the uniaxial strength multiplied by the theoretical area of interpenetration.

Thus: \[ \sigma_{\text{vTC}} = \delta \cdot \varepsilon \]

Where:
- \( \delta = 2.8 \)

The effect of the tip crushing term is relatively small, and only becomes significant at high vertical strains.
Based on the above arguments, the average vertical stress thus depends on the horizontal stress and on the vertical deformation, as follows:

\[ \sigma_v = \beta \sigma_h + \delta \varepsilon \]

...5.5

Where

\[ \beta = \tan(\theta + \lambda)/\tan\theta \]

and \[ \delta = 2.8 \]

It is perhaps worth noting that the equation has the same form as that describing stresses in a soil mass, but with an additional term to account for local crushing at asperities and point contacts.

5.4 Constitutive Law of Fractured Rock

Since the wedge theory relates horizontal stresses acting in the fractured rock to vertical stresses acting on the parting/wedge interface and vertical strains between partings, it can be thought of as defining the constitutive law of the rock mass for large strains. The simple model given by Equation 5.5 can thus provide information about the constitutive law for large strains, data for which are notoriously difficult to obtain in the laboratory.

In order to predict when the rock mass fails, a failure criterion must be used. The Mohr-Coulomb failure criterion is often used but most rock strength envelopes are distinctly curved at low
confining stresses, making the Mohr-Coulomb criterion unsuitable. For this and other reasons, Hoek & Brown (1980) have proposed an empirical failure criterion of the form:

\[ \sigma = \sigma_1 + (m \cdot \sigma_3 \cdot \sigma + s \cdot \sigma_c^2)^{\frac{1}{2}} \]

Where:

- \( \sigma_1 \) = maximum principal stress at failure
- \( \sigma_3 \) = minor principal stress (= \( \sigma_2 \) in triaxial tests)
- \( \sigma_c \) = uniaxial compressive strength of the intact rock in the specimen

This failure criterion fits the experimental data on quartzite strength well (with appropriate parameters \( m \) and \( s \)), and was thus used to predict the failure stress.

The constitutive behaviour of the rockmass can be divided into the following regions:

1. Elastic behaviour - defined by the elastic properties of the rockmass.
2. Failure point - defined by the failure criterion
3. Post failure behaviour - a negative slope is assumed equal to the positive elastic slope (for reasons outlined below).
4. Residual strength - this behaviour is defined by the parameters of the wedge model.
The assumption of a negative slope equal to that of the positive slope may at first sight seem a little arbitrary. In practice this slope is extremely difficult to measure, (a reliable value cannot at this stage be determined) and this necessitates an assumption. The slope is qualitatively similar to that shown by Hojem et al (1975) and reproduced in Figure 5.5. In practice, both by the strains discussed in Section 4, and by the results of the numerical model which will be shown later, most of the fractured zone is in the residual strength state and only a small volume of rock exists on the negative slope part of the constitutive law. It would be a relatively simple refinement to change the negative slope should additional data become available.

It is interesting to note that the relatively straightforward assumptions made so far permit one to define the complex stress strain curves for large strains. These curves (derived in Section 5.6) and shown in Figure 5.6, agree fairly well (in a qualitative sense) with the experimentally measured data of Stavropoulou (1982) shown in Figure 5.7 and Hojem et al (1975) shown in Figure 5.5.

Note in particular the transition from brittle to ductile behaviour, which results directly from the curvature of the failure envelope.

5.5 Details of Constitutive Law

For the numerical analysis developed later, it is necessary to
Figure 5.5  Stress/strain curves for quartzite (after Hojem et al., 1975)
Figure 5.6 Complete constitutive law based on fractured rock analysis
Figure 5.7 Stress/strain data for quartzite (after Stavropoulou 1984)
derive certain relationships between stress and strain. Note that
the equations given below are valid for plane strain conditions,
and that all stresses and strains represent sensible averages.
This is necessary, as discussed previously, because of the highly
discontinuous nature of the stresses and deformations within the
fractured rock. The quantities described are averages over the
full effective stope width (ESW), since this represents the
vertical extent of the fractured and deforming rock. The
horizontal region of interest is an element, one grid unit wide.
This is discussed in more detail later.

For a rectangular yielding element as shown in Figure 5.8, the
following analysis holds:

**Vertical Strain**
The vertical strain $\varepsilon_{v}$ of element 1 is defined as the convergence
on the element divided by the Effective Stope Width.

$$
\varepsilon_{v} = \frac{D}{SW} \quad ...5.7
$$

**Vertical Failure Strain**
By the Hoek & Brown failure criterion:

$$
\sigma_{v1} = \sigma + (m \cdot \sigma \cdot \sigma + s \cdot \sigma) \quad ...5.8
$$

By considering the intersection of lines 3 and 4, one can show
Figure 5.8 Rectangular yielding element and regions of constitutive law.
that:

\[ \varepsilon_{v2} = \frac{(2-\nu)\sigma_h + 2(m\sigma_c + s\sigma^2_c)^{1/2}}{\sigma_c + E/(1-\nu)} \]

\[ \ldots 5.9 \]

The constitutive law is thus as follows:

**Elastic Region**

\[ \varepsilon_v < \varepsilon_{v1} \]

\[ \sigma_v = \frac{E}{D} \frac{1}{v} \]

\[ \ldots 5.10 \]

**Post Failure Region**

\[ \varepsilon_{v1} < \varepsilon < \varepsilon_{v2} \]

\[ \sigma_v = \frac{2}{D} \frac{\sigma_{v1}}{v} - \frac{E}{D} \frac{1}{v} \]

\[ \ldots 5.11 \]

**Residual Strength Region**

\[ \varepsilon_{v2} < \varepsilon_v \]

\[ \sigma_v = \beta \cdot \sigma_h + \delta \cdot \sigma_c \cdot D \frac{1}{ESW} \]

\[ \ldots 5.12 \]
Strain Relationships

The relationship between vertical and horizontal strain is required in order to determine the dilation of the fractured rock. The equations which follow are in "rate" form, because of the changes in slope which occur. The rock behaviour is once again divided into the following regions:

Elastic Region

Since the rock is acting in plane strain with $\varepsilon_2 = 0$, it can be shown that, for a given horizontal stress:

$$\dot{\varepsilon}_3 = \varepsilon_h = -\frac{J}{1-J} \dot{\varepsilon}_1 \quad \ldots 5.13$$

Post Failure Region

In this region rock is observed in laboratory tests to be strongly dilatant, with an overall volumetric increase. This implies that:

$$\dot{\varepsilon}_3 = -\alpha \dot{\varepsilon}_1 \quad \ldots 5.14$$

where $\alpha$ is some constant greater than 1.0. In the analyses which follow, $\alpha$ is usually taken to be 1.50 (since this value results in dilatations in good agreement with those observed in triaxial tests), but it is anticipated that this parameter can be back-calculated from field data, provided the vertical strain in the fracturing rock can be measured. Data of this nature are not
available at present.

**Residual Strength Region**

Most of the post failure deformation occurs in this region and because large deformations occur (both underground and in the model), the rock is assumed to deform at constant volume. This is consistent with the laboratory testing of Hojem et al (1975), where it was found that most of the volumetric dilatation occurs during fracture formation; thereafter the total volume of the sample remains essentially constant. Thus:

$$\varepsilon_3 = -\varepsilon_1 \quad \ldots 5.15$$

As an example of the constitutive law, the following rockmass parameters were used to derive the complete constitutive law shown in Figure 5.6:

- $E = 70$ GPa
- $\nu = 0.20$
- $\theta = 20^\circ$
- $\beta = 3.39$
- $\lambda = 0.31$
- $m = 16.8$
- $s = 1.0$
- $\sigma_c = 200$ MPa

The "s" value is usually varied to account for the
discontinuities present in the rockmass, but in the work presented here, the rock first reaches failure when in a completely intact state and the "s" value has been maintained at 1.0.

5.6 Behaviour of Rockmass outside the fracture zone

By far the simplest procedure for representing the behaviour of the elastic rockmass away from the fracture zone is by means of boundary element methods.

Mined elements and fractured elements on the reef plane are represented by displacement discontinuities (DDs). The layout of elements is shown in Figure 5.9.

Convergences are taken to be positive for convenience. The vertical stress at the midpoint of the ith element (Q) due to a displacement discontinuity D at the jth element is then given by:

\[
Q_{ij} = \frac{-a \cdot G \cdot D_{j}}{1 + \frac{2}{\pi(1-\nu)} (x_{i} - x_{j}) - a_{j}}
\]

Where:

\( G \) = Shear Modulus of the elastic material

\( a_{j} \) = Halfwidth of element \( j \)

(See Crouch and Starfield (1983)).

The \( Q \) is related to all the DDs at all N elements as follows:
Figure 5.9 Displacement discontinuities for boundary element formulation
For any given boundary pressure distribution the DDs (convergences) are found by solving the system of equations:

\[ Q_i = \sum_{j=1}^{n} k_{ij} D_j \]

Where:

\[ k_{ij} = \frac{-G}{J} \frac{a_{ij}}{\pi(1-\nu)(x_i-x_j)^2 - a^2} \]

The above analysis assumes a horizontal reef plane; shear stresses are thus ignored.

For any given boundary pressure distribution the DDs (convergences) are found by solving the system of equations:

\[
\begin{bmatrix}
  k_{11} & k_{12} & \cdots & k_{1n} \\
  k_{21} & k_{22} & \cdots & k_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  k_{n1} & k_{n2} & \cdots & k_{nn}
\end{bmatrix}
\begin{bmatrix}
  D_1 \\
  D_2 \\
  \vdots \\
  D_n
\end{bmatrix}
= 
\begin{bmatrix}
  Q_1 \\
  Q_2 \\
  \vdots \\
  Q_n
\end{bmatrix}
\]

The system of linear equations may be solved by any convenient method.

However, the intention is to give certain elements yield characteristics, where their resistance to convergence depends on the convergence and on the confinement provided by adjacent elements. An iterative solution scheme will thus be far more convenient as the iteration for yield stresses can be done.
simultaneously with the iteration of the overall elastic problem, saving considerable time. The scheme adopted is as follows:

The equation for \( Q \) can be divided by \( k \) and transposed to give:

\[
D = \frac{1}{k} (Q \ -k \ D \ -k \ D \ \ldots \ -k \ D) \quad \ldots 5.19
\]

If the \( D \) vector on the RHS of Equation 5.19 is an estimate of the convergence distribution then an improved estimate of \( D \) is obtained by performing the multiplication in Equation 5.19. In addition, if each new estimate of \( D \) obtained on the LHS is immediately replaced in the RHS \( D \) vector (Gauss-Seidel iteration), then convergence is usually improved (see for example Crouch (1976) or Shoup (1979)).

A further technique which was found beneficial in the numerical model was that of over or under relaxation.

Each new estimate \( D_{\text{new}} \), before being updated in the RHS \( D \) vector is modified as follows:

\[
D_{\text{new}} = D_{\text{old}} + w (D_{\text{new}} - D_{\text{old}}) \quad \ldots 5.20
\]

where \( 0 < w < 2 \) for the scheme to be stable, and the value of \( D \) is obtained from Equation 5.19.

In the analyses presented later, \( w \) values of 1.4 to 1.6 generally
speeded up the solution considerably. Values of $w$ lower than 1,0 were not found to be necessary.

As an example, Figure 5.10 shows the convergence distribution within a 300 m span slot at a depth of 2,5 km calculated using the above procedure, compared to the analytical solution (Salamon, 1963, 1964). Symmetry was not invoked, 40 elements were used and convergence (to 0,1 mm difference between successive estimates) was obtained in 13 iterations (from an initial zero estimate) using $w = 1,5$.

5.7 Elastic Seam Elements

As an initial step towards the incorporation of fully deformable seam elements, and to test the soundness of the numerical technique described so far, elastic horizontally decoupled (Winkler type, see Bowles, 1974) spring elements were implemented.

When these seam "springs" are compressed by the convergence, they exert a resisting force on the boundary which tends to decrease the convergence. A further requirement is that the "springs" should be "prestressed" by the virgin vertical stress so that zero convergence is necessary to support the overlying strata where no mining takes place.

The spring stiffness is defined as follows:
Figure 5.10  Comparison between numerical and analytical convergence profiles for a 300 m span stope
Where:

\[ K = \text{Stiffness of element } i \]
\[ F = \text{Resisting stress exerted by seam element } i \]
\[ E_s = \text{Seam modulus} \]
\[ SW = \text{Height of seam (usually the ESW)} \]
\[ D_i = \text{Convergence on seam element } i \]

In the case of the elastic seam elements, the boundary stress can be adjusted every iteration step, as soon as the new convergence on a particular element is found.

Since the convergence affects the boundary stress, Equation 5.19 becomes:

\[
D_i = \frac{(Q - K D_i)}{k} - \frac{k}{k} D_i \quad \ldots 5.22
\]

or, solving for \( D_i \):

\[
D_i = \frac{[Q - k D_i - \ldots - k D_i]}{[k + K]} \quad \ldots 5.23
\]

To demonstrate the effectiveness of the algorithm, Figure 5.11 shows convergences and seam stresses for a single stope with span of 300m.
Stresses in elastic seam

Deformation of elastic seam

Figure 5.11 Behaviour of elastic seam elements
5.8 Yielding Seam Elements

As was demonstrated during the analysis of the fractured rock in Section 5.5, the vertical resistance of the fractured rock depends on the confinement provided by the adjacent rock and on the vertical strain.

The analysis led to the derivation of a rock mass constitutive law. A convenient method for interfacing between the elastic rock mass and the yielding rock mass is by means of a "degradation of seam stiffness" model. (See Owen & Hinton, 1980, or Crouch, 1976). This is indicated schematically in Figure 5.12.

An algorithm is thus required which, given an estimate of the vertical strain and the horizontal confinement, will return the degraded stiffness of the seam element.

Referring to Figure 5.12, it can be seen that although the actual stress strain curve is sharp and rapidly changing with discontinuous slope, the stiffness degrades relatively gradually with no severe changes in slope. Use of the seam stiffness in this way is thus likely to result in a numerically stable algorithm, amenable to solution by means of iteration.

5.9 Horizontal Stresses

Any yielding element will support a vertical load which is dictated by the minimum horizontal confinement provided by its neighbours. Because the yielding elements have an "effective
Peak stress

\[ \text{Residual stress} \]

\[ K = \text{Initial Stiffness} \]
\[ K' = \text{Degraded Stiffness} \]

\[ E' = E \left(1 - \lambda \right) \]

*Figure 5.12 Degradation of seam stiffness*
stope width" greater than the actual stope width, the first
yielding element is confined by the fractured hangingwall and
footwall rock, as indicated in Figure 5.2. Because of the
frictional restraint of the parting planes, each element will
provide an enhanced horizontal confinement to the adjacent element
(In a direction defined by the "yield direction vector", described later), as shown in Figure 5.13. A necessary assumption (which is
almost certainly true in practice) is that the parting planes are
slipping in the fracture zone. This enables one to determine the
shear stresses on the parting planes. The fact that the initial
formation of the shear fractures results in dilatation and
therefore dilatation of the fractured rock towards the mined area,
supports this notion.

Thus for any element 1:

\[ \sigma = \sigma_{h1} + 4a \tan\lambda \frac{\sigma_{h1-1}}{\tan\alpha} \]  \hspace{1cm} \ldots 5.24

In the numerical model, use is made of a yield direction vector
which is defined as the macroscopic direction of inelastic
displacement (usually towards the mined area).

In order to decide the yield direction, the confinement provided
by the adjacent elements is investigated as follows:
Figure 5.13  Stresses acting on yielding beam element
Rule 1:
If \( \sigma_{h-1} > \sigma_{h+1} \), then the rock will be forced to dilate preferentially in the \( +x \) direction and the element's yield code is \( +1 \).

Rule 2:
If \( \sigma_{h-1} < \sigma_{h+1} \), then the yield code of the element is \( -1 \), as it will be forced to dilate in the \( -x \) direction.

A further rule was found to be necessary to correctly describe the situation near the centre of an isolated pillar, where the unfractured core of the pillar could consist of three or less elements:

Rule 3:
If the element which provides the least confinement is itself elastic, then the yield direction vector of the element is zero (since it cannot be yielding) and the yield boundary is thus established.

5.10 Horizontal dilation

Each yielding seam element deforms both vertically and horizontally. When an element deforms horizontally (dilates), large-scale displacements of the adjacent elements must occur.

The yielding rockmass is compelled to dilate away from the unfractured (elastic) elements, and towards the more fractured
elements.

By the continuity equation, one can show that, for an element dilating in the \(-x\) direction, the displacement \(u\) is given by:

\[
\begin{align*}
\Delta u &= u - [a \varepsilon_{ij} + a \varepsilon_{ij+1}] \\
&= u - [a \varepsilon_{i1} + a \varepsilon_{i+1}] \\
&= \ldots \text{5.25}
\end{align*}
\]

For an element dilating in the \(+x\) direction, (established by examining the sign of the yield direction vector):

\[
\begin{align*}
\Delta u &= u + [a \varepsilon_{ij} + a \varepsilon_{ij-1}] \\
&= u + [a \varepsilon_{i1} + a \varepsilon_{i-1}] \\
&= \ldots \text{5.26}
\end{align*}
\]

The horizontal strain \(\varepsilon\) for each element is returned by the \(h\) constitutive law sub-routine, and the centre displacements \(u\) of each element are updated each iteration step.

Seam elements which are still elastic are therefore not dilating inelastically. These elements are however displacing elastically due to the movement of the surrounding rockmass.

The elastic displacements of the rockmass measured at the midpoints of the elements (in the horizontal sense) can be found as follows (Crouch, 1976):

\[
\begin{align*}
\Delta u &= \sum_{j=1}^{n} \left\{ \begin{array}{l}
\frac{(1-2v)}{(1-v)} \left\{ (x-x_j-a) - a \right\} \\
\frac{1-v}{(1-v)} \left\{ (x-x_j+a) + a \right\}
\end{array} \right\} \\
&= \ldots \text{5.27}
\end{align*}
\]
Each yielding element will thus displace inelastically with respect to its neighbours which will all displace with the surrounding elastic rockmass.

Underground measurements are usually made with respect to a "stable" local datum assumed to be in the elastic rockmass. Measurements such as those made by Legge (1984) are incremental, being differences between selected points anchored in the deforming rockmass, often taken before and after some mining steps have occurred. In the computer programme developed, absolute elastic displacements of seam elements may be calculated as an option.

5.11 Mining Steps

Because of the iterative solution scheme adopted, mining steps are relatively easy to implement. After a solution has been obtained, the element to be mined is simply assigned a "mined element" code, the horizontal and vertical stresses which that element was exerting are set automatically to zero, and execution is returned to the iterative solving sub-routine.

The solution method thus follows, in a semi-realistic way, the processes which actually occur in practice when rock on the mining face is removed. The removed rock no longer provides confinement to the rock ahead of it, which yields slightly, providing less confinement to the rock ahead of it, until all rock zones or
elements are in equilibrium with the surrounding rockmass and satisfy their constitutive laws. Figures 5.14 to 5.16 illustrate the operation of the mining steps. Figure 5.14 represents a 240 m span stope, the "face" of which was mined in two steps, to produce Figures 5.15 and 5.16, which are essentially identical to the initial state, with stress profiles advanced by the size of the mining step.

5.12 Work done on the fracture zone

When the mining face is advanced, or an element is "mined", the equilibrium between the remaining seam elements and the surrounding elastic rockmass is disturbed. In order for equilibrium to be re-established, the remaining seam elements deform non-linearly and the fracture zone advances ahead of the face by approximately the same distance that the face was advanced.

If it is assumed that the readjustment process occurs quasistatically, then it is possible to calculate the amount of work which the elastic rockmass does on the seam elements. Previous workers have merely stated that part of the "released" energy is expended in doing work on the fracture zone. The numerical technique described in this thesis makes it possible to quantify the amount of energy expended in this way.

Since the elastic boundary (neglecting second order effects) only crushes the rock in the vertical direction, with no net ride
Figure 5.14  Results of analysis demonstrating the operation of the mining steps routine - 1
Figure 5.15 Results of analysis demonstrating the operation of the mining steps routine - 2
Figure 5.16 Results of analysis demonstrating the operation of the mining steps routine - 3
component, no shear work is done on the yield zone. It is thus sufficient to account for the work done by the elastic boundary in the normal direction only, as follows:

Let the vertical stress and convergence on any seam element before the mining step occurs be $Q$ and $D$ respectively. After the mining step occurs, let the values be $Q'$ and $D'$. This is illustrated in Figure 5.17.

The work done ($U_i$) on the seam element $i$ is thus:

$$U_i = (Q + Q').(D - D').a$$

...5.28

This work represents stored elastic energy in the seam material as well as irrecoverable energy lost mainly through shear deformation processes within the seam. The total work done on the seam is obtained by summing the contribution of each seam element.

The element on the face which is "mined" also contains stored elastic strain energy which may be estimated in exactly the same way.
Before mining

Quasistatic process

After mining

Figure 5.17 Boundary stress and convergence changes during mining
6. DETERMINATION OF PARAMETERS AND INPUT VARIABLES

The numerical model described in the previous section embodies several parameters and input variables. In order to make meaningful use of the model, appropriate values for these must be selected. In some cases, it would be extremely difficult to actually measure the parameter concerned, and suitable values must thus be chosen on a rational basis (for example by means of back-analyses).

Since the real purpose of the model at this stage of development is to improve the understanding of the behaviour of the fractured rock, it is also vitally important to know how the numerical model's results are influenced by variations in the actual values of the parameters used. For this reason, the sensitivity of the model's results to variations in the input parameters was assessed.

While it is not the objective of this thesis to investigate, measure and publish suitable values of all the required parameters, it is appropriate that some attention be given to the selection of the parameters necessary to carry out the example analyses described in Section 7.

In this section, suitable values for the necessary parameters are suggested and justified. These values are used in the example analyses which follow later in the thesis. In addition, the
results of the sensitivity analysis are presented and discussed.

6.1 Elastic moduli

The elastic modulus (E) and Poisson's Ratio (\( \nu \)) as derived from laboratory triaxial tests on small intact rock samples (carried out in accordance with the ISRM's (1978) guidelines) have been used in all analyses, since this is the common practice in South Africa. The use of these moduli was originally justified on the basis of the experimental work described by Ryder and Officer (1964) and Ortlepp and Cook (1964). The insitu moduli are probably lower than these values, but large scale field experiments to measure insitu moduli were not undertaken during the course of this research, and data are therefore not available.

6.2 Strength properties

The uniaxial compressive strength (\( \sigma_c \)) and the Hoek/Brown failure parameter (m) as derived from triaxial tests carried out on small samples tested in the routine manner have been used. The Hoek/Brown "s" parameter has been assigned a value of 1.0 in all analyses. The reason for not using a reduced s parameter, as is customary when describing rockmass strength, is that in the model (and in the underground situation) the intact rock falls some distance ahead of the face, where it is well confined by the surrounding rock. The initial fractures thus form in rock which is almost completely intact, in a high stress environment; the discontinuities do not influence the initial formation of the shear fractures. The constitutive law described in Section 5 is
Intended to account for the post failure behaviour of the rockmass, once the discontinuities have formed; it would therefore be inappropriate to degrade the peak strength of the rock by means of a reduced "s" value.

6.3 Friction properties

The friction angles $\lambda$ (shear fractures) and $\lambda_p$ (parting planes) should be measured on relatively large surfaces at stresses which match those experienced by these planes in the real situation. Such large scale tests were not possible during the course of this research, and friction angles had to be determined from relatively small-scale tests (triaxial and torsional).

It was found from the limited number of tests done that friction coefficients varied fairly widely from test to test and even during a single test. Rotary friction tests carried out on 50 mm diameter samples having quartzite/rock flour/quartzite and quartzite/parting plane material/quartzite surfaces are shown in Figures 6.1. & 6.2.

Data published by Byerlee (1977) suggest that a friction coefficient for shear fracture surfaces of about 0.6 is the limiting value for large surface areas, high stresses and large displacements. However, an average value for the parting plane friction coefficient of about 0.5 was obtained for these surfaces during testing, and this is considered to be reasonable, in view
Figure 6.1  Coefficients of sliding friction of quartzite/rock flour/quartzite surfaces
Figure 6.2 Coefficients of sliding friction for quartzite/parting plane material/quartzite surfaces
of their flat, lubricated nature (see Figure 6.2).

During the tests done to establish the sliding coefficients of friction, the effect of water on the slip plane was also investigated. As can be seen from Figure 6.1, it was found that the presence of water reduces the value of \( \mu \) from about 0.60 to about 0.45, depending on the amount of displacement. This water was not introduced under pressure; the surfaces were merely wet prior to testing. The significance of this finding will be discussed later, in the context of possible modifications to the fracture zone size.

Based on the limited number of tests carried out, the following values for the friction coefficients have been used in the analyses presented later in the thesis:

- Shear plane friction angles \( 31.0^\circ (\mu = 0.6) \)
- Parting plane friction angles \( 26.6^\circ (\mu = 0.5) \)

6.4 Geometric properties

The actual angle at which the macroscopic shear fractures form on the reef plane ahead of an advancing face varies within certain limits. In the very hard, glassy quartzites the shear fractures are relatively steep, being \( 10^\circ \) to \( 20^\circ \) off vertical (i.e. \( 10^\circ < \theta < 20^\circ \)). In the slightly weaker, more argillaceous quartzites, the shear fractures form at between \( 20^\circ \) and \( 30^\circ \) off vertical (i.e. \( 20^\circ < \theta < 30^\circ \)). This is probably symptomatic of the fact that the
apparent angle of internal friction based on peak strength is higher for the stronger quartzites, and the classical Mohr envelope type analysis has it that the shear fractures should form at angles of \( 45° + \theta/2 \) to the direction of minimum principal stress (horizontal in this case).

In the analyses which follow, a value for \( \theta \) of 20° has been used.

6.5 Effective Stope Width and Initial horizontal confinement

The effective stope width and the initial horizontal confinement provided by the hangingwall and footwall are very difficult parameters to quantify.

Observations have shown that in ledging stopes (Span < 10m), the effective stope width is equal to the actual stope width, since the hangingwall and footwall rock immediately above and below the stope is relatively unfractured and is not subject to large deformations. In large span or very deep stopes, observations of the fracture zone size reveal that the rock is fractured for a considerable height above and below the stope. In addition, slip along bedding planes can occur several metres above and below the stope, as can be clearly seen in Figures 4.8 and 4.9. The occurrence of parting planes must therefore play a significant role in determining to what vertical extent the rock is disturbed by the fracture processes.

The initial horizontal confinement provided to the fracturing rock
ahead of the face by the hangingwall and footwall is virtually impossible to measure, in view of the highly discontinuous nature of the fractured rock.

For these reasons, it would be inappropriate to suggest values of ESW for use in analyses; the choice of ESW is best made by direct investigation at a site, or by means of back analyses, while the initial horizontal confinement can probably only be determined by means of back-analyses.

6.6 Effect of Grid Size and Iteration Sequence

It was found that provided the grid size is small enough to ensure that more than 2 elements are yielding, the grid size has no appreciable effect on the results (except as discussed below). This is demonstrated by the results shown in Figures 6.3, 6.4 and 6.5 where the yielding seam elements were reduced in width from 2.0 m to 0.5 m. The extent of the yield zone never differed by more than 1 grid from the finest element arrangement used, and the convergence and stress distributions are essentially identical, to the limit of resolution of the grid size.

In order to test the effect of iteration sequence, the order of iteration was reversed for certain analyses. No differences in final result or solution time were noted.

6.7 Sensitivity analysis of model results

The data used in any numerical scheme which attempts to describe a
Figure 6.3 Analysis using 2m seam elements
Figure 6.4 Analysis using 1m seam elements
geotechnical phenomenon are by nature extremely variable and imprecise. For this reason, no single analysis of a given mining geometry is acceptable without an exploration of the effect which each input variable has on the final answer.

The numerical model described previously was thus subjected to a series of test runs, the object of which was to evaluate the effect which each input variable had on the horizontal extent of the yield zone (loosely known as the "fracture depth"). The reason for choosing the size of the fracture zone as the yardstick for evaluating the model's performance is simply that the fracture depth is one of the parameters most readily measured insitu, and a large amount of data is available for comparison purposes. It is obviously not possible to use stress distribution as a test since no data is available, due to there being no way to actually measure the stresses in the fracture zone.

The following input variables all have an effect on the model output and can potentially influence the size of the fracture zone:

1. Depth of stope below surface.
2. Stope span.
3. Effective stope width.
4. Uniaxial strength of rock.
5. Friction angle of parting planes.
6. Friction angle of shear planes.
7. Young's Modulus of rockmass.
8. Poisson's ratio of rockmass.
9. Shear fracture angle.
10. Horizontal confinement provided by the hangingwall and footwall strata.

Since each variable can take on a wide range of values it is impractical to try all combinations of all variables. Thus a base analysis was chosen using standard values of the input variables. Each variable was then varied while all other variables were assigned their base values, as follows:

- Depth of stope: 2.5 km
- Stope span: 240 m
- Effective stope width: 7 m
- Uniaxial strength of intact rock: 200 MPa
- Friction angle - parting planes: 26.6°
- Friction angle - shear planes: 31.0°
- Young's Modulus - intact rock: 70 GPa
- Poisson's Ratio - intact rock: 0.20
- Shear fracture angle: 20°
- Horizontal confinement provided by hanging and footwall rock at face: 1.0 MPa

These parameters correspond to conditions at the Doornfontein Experimental site and describe a stope face with an ERR of 23.5 MJ/m². A large number of observations of fracture zone size
and extent of fracturing are available for comparison purposes at this site. The value of horizontal confinement provided to the fractured rock was estimated by using the support pattern installed at Doornfontein, together with the ESW and the positions of dip gullies at the site. A limit equilibrium analysis was then done in order to determine how much "push" from the fractured rock ahead of the face was necessary to develop slip on the parting planes; this defines the maximum possible confinement which can be provided by the fractured hangingwall and footwall rock.

The element arrangement chosen was as follows:

- 0 - 200 m - 10 mined elements
- 200 - 230 m - 6 mined elements
- 230 - 240 m - 5 mined elements
- 240 - 250 m - 20 yielding seam elements

As can be seen, the elements were graded gradually from coarse, to fine in the region of interest, and a total of only 41 elements was required. Within the yielding region, all elements were kept the same size, since very large variations in the size of adjacent elements can lead to numerical problems. This grading made the many runs required fairly quick to carry out, but still gave an acceptable accuracy for comparison purposes. The element arrangement is shown in Figure 6.6. Only the mining face on the right was permitted to yield for reasons of economy, since the yield of the rock on the opposite face has negligible influence by
Figure 6.6 Arrangement of elements for base run of sensitivity analysis
virtue of its remoteness.

The results of the base analysis are shown in Figure 6.7, and the effect which each parameter had on the fracture depth is discussed in the sections which follow.

6.7.1 Depth of Mining

The depth of mining influences the virgin stress, but the presence of the earth's surface is not accounted for since the analysis is carried out at "infinite depth".

Mining depths of 0.5 km, 1.5 km, 2.5 km and 3.5 km were analyzed using base values for all other variables.

No face fracturing was obtained at the 0.5 km depth, since the stresses were below the yield strength of the rock. The depths of the fracture zones for the chosen depths are shown in Figure 6.8.

The analyses reveal a trend of increasing fracture zone size with depth, beyond some minimum depth, shallower than which no fracture zone forms. This is in qualitative agreement with observations of fracture zone size at different energy release rates (see Joughin and Jager, 1984).

6.7.2 Stope Span

Stope spans of 50, 100, 240 & 500 m were modelled while keeping the base values unchanged. Figure 6.9 shows how the depth of the
Figure 6.7 Results of base run of sensitivity analysis
fracture zone varied with span.

At spans of about 500 m and greater, complete closure occurs, i.e. the maximum convergence exceeds the mining stope width. It is thus to be expected that an equilibrium will be reached beyond which no further change in fracture zone size with span will be achieved.

The results reveal a fairly rapid initial increase in fracture depth with stope span, but only gradual increase beyond a span of about 200 m. Once again the results do not contradict observations of fracture zone size.

It should be pointed out that Figures 6.8 and 6.9 are based on a constant value of ESW, which itself almost certainly increases with increase in the depth and/or span. Figures 6.8 and 6.9 thus indicate the sensitivity of the method to depth and span, rather than absolute trends.

6.7.3 Effective Stope Width

Figure 6.10 shows results of analyses using base values of all parameters and effective stope widths of 2, 4, 7 and 10 m.

In practice, it is almost certainly true that the effective stope width increases as the span of a given stope is increased by mining, as discussed in Section 6.5.
The results suggest a fairly linear direct relationship between Effective Stope Width and fracture depth. This relationship is significant and will be referred to later.

6.7.4 Uniaxial rock strength

The strength of the intact rock can reasonably be expected to influence the size of the fracture zone and the results presented in Figure 6.11 are thus not surprising. A relatively linear negative variation in fracture depth with rock strength was found, in keeping with what one would intuitively expect.

6.7.5 Parting plane friction coefficient ($\tan \gamma_p = \mu_p$)

The friction coefficient of the parting planes is a particularly interesting parameter as it appears to have a fairly strong influence on fracture depth. Friction on parting planes has been suggested as one of the parameters which can influence the dynamic behaviour of the fractured face rock (Smart, 1983) and physical model tests of coal "bumps" have previously been described (Burgert & Lippmann, 1981).

Figure 6.12 suggests that below $\mu_p = 0.6$ a fairly large change in fracture depth can be effected by a change in friction coefficient. One possible way to change $\mu_p$ is by means of fluid injection (see Figure 6.1).

6.7.6 Shear plane friction coefficient ($\tan \lambda = \mu$)

As shown in Figure 6.13, this parameter does not appear to
Figure 6.11 Variation of fracture zone size with Uniaxial Compressive Strength
Figure 6.12 Variation of fracture zone size with parting plane friction coefficient.
Figure 6.13 Variation of fracture zone size with shear plane friction coefficient
Influence the fracture zone size as dramatically as does the parting plane friction coefficient, and suggests therefore that the friction properties of shear planes are less likely to be responsible for an increase in the fracture zone size should they be lubricated. Large scale water injection into mining faces would doubtless lubricate all fracture and slip planes in any case, and their combined effect can be expected to significantly increase the size of the fracture zone.

6.7.7 Shear fracture angle (θ)
Figure 6.14 Indicates that the depth of the fracture zone is relatively insensitive to the orientation of the shear fractures. Fracture angles between 10° and 30°, as are commonly observed, resulted in very similar fracture zone depths in the numerical model.

6.7.8 Horizontal confinement provided by hangingwall and footwall
The horizontal confinement provided by the hanging and footwall strata is a difficult parameter to measure. Estimates made by the author, based on the geometry and installed support at the Doornfontein site (Brummer, 1982), suggest that this stress should be relatively low, of the order of 0.6 MPa to 1.0 MPa, in the first two metres above and below the stope. For comparison, the average stress exerted on the hangingwall by a normal arrangement of hydraulic props is only of the order of 0.5 MPa. In addition, these values have been confirmed by back-analysis.
Figure 6.14 Variation in fracture zone size with shear fracture angle $\theta$.
Figure 6.15 shows that provided the confinement is less than 4 MPa, the depth of the fracture zone is not significantly affected by changes in the initial horizontal stress exerted by the hanging and footwall.
Figure 6.15 Variation in fracture zone size with initial horizontal confinement
7. EXAMPLE ANALYSES AND VALIDATION OF MODEL

In Section 6, appropriate values for the required parameters and input variables were proposed and justified. In order to demonstrate the capabilities of the numerical model, several example analyses which make use of certain of these parameters are now described. These analyses are intended to show, for several common situations, that the model produces results which agree with the observed behaviour of fracturing rock.

The actual numerical results produced by the model are dependent on the values of input variables used (some of which are not known exactly). For this reason, the behaviour of the model is assessed in some cases on a qualitative basis. This validation exercise is not intended to be a rigorous test of the numerical scheme and selected parameters; the intention rather is to demonstrate the validity of the mechanistic model on which the numerical scheme is based.

7.1 Size of fracture zone

The base analysis performed in Section 6.7 was intended to replicate face conditions at the Doornfontein Experimental Site, as discussed. The numerical model predicted a depth of fracture zone of 5.5 m as can be seen in Figure 6.7.

A typical series of petroscope observations at the Doornfontein site (where the ERR had a value of 26 MJ/m²) is shown in
Figure 7.1 (after Adams & Jager, 1980). It is interesting to note that at the boundary between the fractured rock and the unfractured rock (at 6 m depth into the original borehole), it became impossible to insert the petroscope after the face was advanced, apparently because of the formation of a shear fracture which displaced the borehole.

There is thus good agreement between observations of fracture zone depth and the results of the numerical model.

7.2 Dilation of fracture zone

The absolute dilation profile for the base analysis is shown in Figure 7.2, together with an absolute dilation profile derived from data published by Legge (1984) (see Section 4.1). As can be seen, the depth of fracturing as predicted by the model agrees well with Legge's observations. The amount of dilation is also in good agreement, except in the first 1.5 m of rock ahead of the face. This discrepancy can however be explained as outlined below.

Legge reported that in the first 1.5 m of rock exposed on the stope face, a large additional amount of dilation was observed to occur. Petroscope observations in the near face region revealed that vertical slabbing or splitting usually occurred in the first 1 - 1.5 m of rock on the face. It must thus be concluded that this additional dilation is associated with the formation of extension fractures, as was postulated in Section 3.3. The
Figure 7.1 A series of petroscope observations at a site with an ERR of 26 MJ/m (after Adams & Jager, 1980)
Figure 7.2 Absolute dilation profiles for base analysis and Legge's observations at Doornfontein
numerical model developed here does not account for this mode of fracturing, being based entirely on shear deformation mechanisms. Within the limitations imposed by the assumptions made in developing the model, agreement between observed and predicted dilation profiles is thus good.

7.3 Energy consumption in the fracture zone

As outlined in Section 5, SEAMS may be used to determine the energy absorbed in the fracture zone. The mining step analyses of Figures 5.14 to 5.16 were therefore used to calculate the work done by the elastic rock on the seam for typical conditions at the Doornfontein site. This calculation is shown in Table 7.1, and the results are presented graphically in Figure 7.3, which shows that the work done on the rock is a maximum some seven metres ahead of the face, i.e. at the boundary between the fractured and the unfractured rock. The total work done by the elastic rock on the seam in this example may be found by summing the work done on each element. In this case a total of 13,0 MJ of work (due to a face advance of 0,5 m) was done on the seam, part of which was stored elastically in the seam elements, but most of which was expended in doing work on the first 7 m of rock ahead of the face which is deforming inelastically. In this particular example, only a very small amount of stored strain energy is removed in the element which is mined from the face. The nominal Energy Release Rate for a slit of comparable span to the example presented is 25,7 MJ/m. For an advance of 0,5 m this corresponds to 12,85 MJ,
Table 7.1 WORK DONE ON SEAM BY ELASTIC ROCKMASS

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Total work done on seam during 0.5m mining step 13.006 MJ

This corresponds to 26.0 MJ/m of face advance.
Figure 7.3 Work done on seam ahead of the face by the elastic rockmass
in close agreement with the value calculated from the model. Thus ERR in a realistic scenario of rock fracture is seen to be accounted for in terms of energy absorption taking place stably in the fracture zone.

Patrick (1983) showed that microseismic events were concentrated ahead of the face as shown in Figure 7.4. The location accuracy of the events was between 2 m and 5 m but a clear trend is evident, with most events being concentrated up to 7 m ahead of the face, and within the vertical extent of the Effective Stope Width (7 m in this case). Assuming that these microseismic events are caused by slip movements on the many slip planes which make up the fracture zone, it is evident that there is good agreement between the locations of these energy-consuming processes in the model and ahead of a real mining face.

7.4 Load-deformation behaviour of model pillars

The load-deformation behaviour of rock pillars is not well understood, mainly because of the extreme difficulty involved in measuring both the load carried by a real pillar underground and the actual deformation of such pillars. Much of what is currently known about the strength and deformation of pillars has therefore been inferred from back analyses of well documented case histories and laboratory tests of model pillars, a notable exception being the Insitu coal pillar tests described by Wagner (1974). If, by fine-tuning of the parameters used in SEAMS, the results can be made to compare closely to the results of a series of laboratory
Figure 7.4 Microseismic event distributions near the stope face at the Doornfontein Experimental Site (after Pattrick 1983)
tests, then it should be possible to carry out realistic analyses of real pillars underground, by making appropriate adjustments to the parameters used.

Tests on model pillars are often criticized because the dissimilarity between the elastic properties of the pillar material and the steel of the loading platens can result in unrealistic stress distributions within the pillar. (A possibly more serious shortcoming of such tests is that foundation failure of the rock below and above a pillar is not possible in a model pillar test where the rock is confined between steel platens.) It is possible, using SEAMS, to evaluate the significance of differences in the elastic properties between the "pillar" (or model) and the "host rock" (or loading platens), since these properties are specified independently.

It is known from underground observations and laboratory studies that wide pillars do not shed load after failure, but that narrow pillars can shed load and even become unstable when the post peak negative slope of the pillars' load/deformation curve exceeds the "system stiffness" of the surrounding rock.

In order to investigate these aspects of pillar behaviour, a series SEAMS analyses is compared here in a qualitative way to a series of laboratory tests on model pillars carried out and described by Wagner and Madden (1984). This comparison is not ideal, since SEAMS considers only two dimensional plain strain
situations, and the model pillar tests were axisymmetric. However, no comparable truly plain strain results were available at the time of writing.

The numerical results were produced by analysing two metre high pillars (ESW = 2.0 m) with widths of 2, 4, 8 and 12 m which were "loaded" in a mining situation with 20 m mined "bords" on either side.

The geometry chosen was as shown in Figure 7.5, and the pillars were loaded by increasing the virgin (or field) stress in the host rock.

The average stress carried by each pillar was plotted against the average pillar deformation at each stage of loading. In this way, an averaged stress/deformation curve was obtained for each pillar. The results for each pillar are discussed below:

2 m x 2 m Pillar - Figure 7.6

Because of its relatively narrow width, this pillar is roughly equivalent to a uniaxial test specimen, and one would thus expect its average stress/deformation behaviour to be similar to that of such a lab test specimen.

In the analyses, it was found that on increasing the virgin stress from 40 MPa to 41 MPa (corresponding to a pillar stress equivalent to the uniaxial strength), the pillar dramatically failed,
Figure 7.6  Average stress/strain behaviour for 2m x 2m pillar
shedding virtually all load, as would a real pillar in a similar situation (e.g. Salamon, 1983, Brady, 1979). The reason for this behaviour is that the post peak stiffness of the pillar was less than that of the "system" of surrounding rock. The numerical model behaved entirely stably in this case - the instability apparent in the stress drop from A to B is a real, physical one, the start and end points of which the model is perfectly capable of following, as demonstrated by Figure 7.6.

4 m x 2 m Pillar - Figure 7.7
The behaviour of the 4 m x 2 m pillar was qualitatively very similar to that of the 2 m x 2 m pillar.

Note that the slope of the line from C to D which represents the "local mine stiffness" (See Hoek & Brown, 1981) appears to have been reduced, but this is as a result of the fact that average stress and not load has been plotted. The local mine stiffness in terms of load is relatively constant throughout the analyses presented in this section.

8 m x 2 m Pillar - Figure 7.8
The 8 m x 2 m pillar was sufficiently wide that significant confinement was generated by the fractured edges. This resulted in the pillar being capable of supporting a substantial load after the peak strength had been reached. In order to generate this confinement however, significant crushing of the pillar was necessary, as can be seen in Figure 7.8.
Figure 7.7  Average stress/strain behaviour for 4m x 2m pillar
Figure 7.8 Average stress/strain behaviour for 8m x 2m pillar
The behaviour of this pillar was significantly different from the two shown in Figures 7.6 and 7.7. The stress (load) drop after peak strength was far less severe. In practical terms, this suggests that the 1:1 and 2:1 pillars failed violently, but the 4:1 pillar yielded in a more controlled and stable manner.

12 m x 2 m Pillar - Figure 7.9
This pillar, with a 6:1 width to height ratio, did not fail violently. In the analysis using 250 MPa virgin stress the central core of the pillar was still elastic. At higher loads, the pillar can be expected eventually to yield throughout, but it is so wide that large confinement stresses are generated. This means that the "residual strength" of the pillar is large compared to the uniaxial strength of the rock and although the material itself is strongly strain softening, the overall behaviour of the pillar is stable because of the loading geometry and confinement forces generated.

Discussion
The pillar analyses are in qualitative agreement with observed pillar behaviour in the laboratory and insitu. A point which arose from the analyses carried out and which is worthy of discussion is the fact that as the pillars were made wider, the stress at which yield first occurred apparently was lower. At first glance this behaviour appears to be contrary to expectation.
Figure 7.9 Average stress/strain behaviour for 12m x 2m pillar
The explanation for this phenomenon is that the stress distribution within the pillars depends on the interaction between the elastic pillar and the surrounding elastic rockmass which has elastic modulus similar to the pillar. This interaction results in a highly non-uniform stress distribution within the pillar, with the edges carrying substantially more stress than the core. This effect worsens with increase in width to height ratio. The hangingwall rock tends to "sag" over the pillar, which promotes early failure of the pillar edges.

This situation is very different to the laboratory test, where the steel of the loading platen is usually much more rigid (higher elastic modulus) than the rock specimen, and the stress distribution is therefore much more uniform.

In order to confirm this explanation, the series of model-pillar simulations was redone using a host rock modulus considerably higher than that of the pillar. The analyses are thus similar to a laboratory pillar model with the "stiff" country rock representing the "stiff" steel loading platen.

These simulations revealed that the pillar strength did indeed increase with increasing width to height ratio. The simulated laboratory stress - deformation curves for the pillars are shown in Figure 7.10. As can be seen, the peak strength of the pillars increased with width to height ratio, as is commonly reported from laboratory tests e.g. Figure 7.11 (Wagner & Madden, 1984, Hudson,
Figure 7.10 Simulated laboratory stress/deformation curves for pillars
Figure 7.11 Stress/strain curves for pillars (after Wagner & Madden, 1984)
Brown & Fairhurst 1971). The simulated stress-deformation curves also bear a close resemblance to those derived from laboratory scale tests. The most notable difference between the curves shown in Figures 7.10 and 7.11 is that in the simulated curves, the shape of the curve is more sensitive to width to height ratio. The reason for this is that the numerical curves are derived for a two-dimensional plane strain situation, whereas the lab test results shown in Figure 7.11 are derived from axisymmetric pillars which are inherently less confined. The numerical model thus gives good qualitative agreement with laboratory data.

The most significant conclusion arising from this series of simulations is that the "draping" effect which occurs in the case of pillars which have elastic moduli the same as the host rock (as in a gold mine for example) results in a lower peak yield stress for increasing width to height ratios. This suggests that it is possibly dangerous to extrapolate results derived from laboratory tests conducted using relatively stiff loading platens to a real underground mining situation in cases where the pillars and host rock consist of similar material.
8. CRITICAL APPRAISAL OF THE METHOD

Several aspects of the principles demonstrated in this thesis have not been exhaustively tested and certain of the assumptions have, for lack of adequate data, been based on limited evidence and underground observations and therefore warrant further investigation. These aspects are discussed in the following sections:

8.1 Time-dependent behaviour

The time-dependent behaviour of the rockmass is ignored completely. It is known that deformation measurements taken for example in the stope behind an advancing face exhibit time dependent behaviour. The intention is not to suggest that this does not occur, but that this characteristic is beyond the scope of the method proposed in this thesis. The analyses are thus intended to apply sensibly and reasonably to "average" mining conditions, where the mining rates are neither excessively fast nor slow. The stopes where most of the measurements on which this thesis is based were mined at these rates, and the results and conclusions can thus be expected to apply to such "average" stopes.

8.2 Mining sequence and out-of-plane effects

While the iterative solution technique adopted for solving the equations is ideally suited to modelling successive effects, and most of the "mining" simulations presented are done in this way,
there are instances where it is not possible to model the history of excavation correctly. In particular, since the method is a two-dimensional idealization, it is not possible to model any "out-of-plane" effects, as in the case of the pillar simulations. Stabilizing pillar layouts are mined "out-of-plane", and some fractures are observed to occur which are not parallel to the edge of the pillar. This aspect can only be resolved by means of a three-dimensional model, which is beyond the original scope of this research.

8.3 Effective stope width

The "effective stope width" (ESW) was introduced into the analysis because it is observed that in real mining situations, the rock fractures and deforms over a vertical height considerably more than the actual stoping width. There is no doubt that the true behaviour of the horizontally layered rock is not as simple as described in the model (where only one parting plane in the hangingwall and one parting plane in the footwall slips, permitting relaxation into the stope). Rather what occurs is that severalpartings in both hanging and footwall slip, to a greater or lesser extent. This is commonly observed in any stope where fracturing occurs. The ESW is introduced as a computational convenience, and is intended to reasonably describe the notional vertical extent of the fracture zone which forms ahead of the stope face. The bedding plane separation commonly observed in the hangingwall behind the face is further proof that the rock in the immediate vicinity of the stope face is not part of the elastic
continuum, but is heavily fractured, and almost completely destressed.

8.4 Dilatancy and extension fractures

Dilatancy due to extension fractures is ignored in the analysis. The reason for this is that the extension fracturing has been shown to be a later phenomenon, which occurs at or just ahead of the stope face. In addition, shear mechanisms are capable of absorbing the large amounts of energy which must be consumed in the fracture zone while extension fracturing appears to result in very little energy loss. Attempts by Roering (1982) to explain the loss of ERR by the creation of fresh fracture surfaces (absorbed as fracture surface energy) were unsuccessful, since the best estimates of energy consumed by this mechanism in the fracture zone were two orders of magnitude below the energy available due to ERR. The existence of the extension fractures is thus acknowledged, but the role played by them in providing mechanisms for dilatation and energy absorption is omitted, since this does not appear to be crucial to the method.

8.5 Confinement provided by the hangingwall and footwall

The confinement provided by the hangingwall and footwall to the fractured rock ahead of the face has been inferred to exist. The actual amount of this confinement will be virtually impossible to measure in practice due mainly to the very discontinuous nature of the heavily fractured rock at the stope face. The most promising line of attack to determine the amount of this confinement is
probably to back-calculate it from observed situations. In the work described here, estimates of this confinement have been made based on limit equilibrium analyses of free body diagrams of the fractured rock. These have indicated surprisingly low values of this confinement, of the order of 1 MPa. This must be compared to the typical average stress applied by a layout of stope support to the hangingwall, which is typically much less than 1 MPa. This comparison puts the relatively low confinement so obtained into perspective. The analyses presented in this thesis have all been done with horizontal confinements of around 1 MPa, and have yielded reasonable results. Considerable scope thus exists for "fine tuning" the model parameters in order to realistically model any given situation.

8.6 Calibration of the model

Careful calibration of the model is required. The model analyses depend on a number of parameters, many of which are not known at a particular site, or will be almost impossible to measure in practice. In some respects, this is actually an advantage, since the model can be "fine tuned" to model the conditions at such a site. The strength of a model such as this lies not in its ability to model in absolute terms, but rather to examine the likely effect on the rock behaviour of a proposed change in mining pattern or other model parameters.

8.7 Rake type displacements

The method as described here is capable of modelling horizontal
seams only; ride type displacements between hangingwall and footwall have not been allowed for in its development. This is not a very great disadvantage, since it is commonly believed that the maximum principal stress direction is normal to the very pronounced bedding which occurs. The introduction of ride type displacements would destroy symmetry about the horizontal plane, and would thus complicate the analysis, possibly beyond the benefits obtained at this stage.

8.8 Lack of adequate data

Several of the arguments used in this thesis have been based on intuitive reasoning where appropriate data was not available. Justification for some of these arguments is that the answers obtained by the model have been qualitatively and quantitatively satisfactory. This has been necessitated by the extreme difficulty of measuring several of the key characteristics underground and the consequent lack of suitable published data; stress for example is almost impossible to measure in all but the most competent and lightly stressed rock at depth, and displacements cannot by their nature be measured except in a relative way.

8.9 Computer hardware

The numerical model as described here has been implemented on a small computer by modern standards, and the analyses are thus slow to perform. The objective of this research was however only to
demonstrate the principles involved in the development of numerical techniques to describe the severe inelastic deformations which occur at the edges of tabular excavations, and not to develop full-scale analytical tools.
9. CONCLUSIONS

Investigation of the fracture zone by means of underground mapping revealed that on a macroscopic scale, the processes which occur there are relatively simple to understand. The high edge stresses induced by the mining cause the rockmass to fracture in such a way that the high vertical stresses are relieved. The most natural way for this to occur is for inclined shear fractures to form, which permit the rock to dilate horizontally into the mined area. This horizontal dilation is facilitated by the horizontal bedding or parting planes which occur at 0.3 - 1.0 m spacing. Near the mining face, low confinement or extension fractures form, similar to those observed in a uniaxial compression test.

The relatively regular spacing of the shear fractures results in an apparent "banding" of fractures ahead of the mining face, as has been reported by previous researchers.

In the fracture zone, the reef rock shortens vertically but dilates horizontally in such a way that the rock volume is almost constant (since the strains are large compared to those easily simulated in the laboratory). Based on laboratory data it must be inferred that some volumetric increase occurs as the fractures first form. The deformations which occur are substantially larger than can be easily obtained during laboratory testing of triaxial, plane strain or extension test specimens, and it was thus necessary to develop a constitutive law based on certain
assumptions about the post-failure behaviour of rock. The constitutive law developed in this thesis is in agreement with observed rock behaviour in the laboratory, but extends the range of available data far into the nonlinear zone.

The overall deformations of the fractured rock as described above manifest themselves on a small scale as discrete shear displacements on shear fractures and as horizontal slip on bedding planes. Since these processes were observed to occur in a region of rock whose vertical extent is usually greater than the actual stoping width, the existence of an "Effective Stope Width" was postulated. This Effective Stope Width can be identified in suitable boreholes as the vertical height between the highest and lowest parting planes on which substantial horizontal slip occurs.

The dilation of the fractured zone must influence the stope hangingwall and footwall, and the closure of footwall dip gullies and occasional upward buckling of footwall strata are proposed as consequences of the Effective Stope Width concept.

The most important feature of the geology of the near reef rock is its bedded nature. The parting planes, from underground observations and from numerical model studies, appear to exert a strong controlling influence on the deformation processes which occur near the edges of the tabular excavations.

A relatively simple numerical model which is capable of capturing
the essential features of the observed phenomena has been developed. This model realistically describes the elastic behaviour of the host rock by means of boundary element (Displacement Discontinuity) techniques. The reef horizon is regarded as a yielding seam. The vertical stress which each seam element can support is related to the minimum horizontal confinement provided by adjacent elements. In this way, relatively large deformations are permitted to occur when the seam elements fracture and deform. Horizontal dilations predicted by the model are in good quantitative agreement with observed dilations. The model also yields fracture zone depths and vertical stress profiles which agree with available underground observations.

The numerical model was applied to the analysis of the load-deformation behaviour of pillars of various width to height ratios. A potentially very significant result was obtained from this series of simulations. It was found that the "draping" or "sag" effect which occurs when a pillar is stressed by country rock of comparable modulus to that of the pillar, significantly reduces the peak strength of the pillar. This implies that it may be dangerous to extrapolate pillar strengths derived from model tests carried out in the laboratory to the underground mining situation. This effect is probably not significant in soft seam or coal mining geologies, but can be extremely significant where hard-rock pillars are left.
The algorithm describing yield of the fractured rock is potentially sufficiently simple and general enough for it to be incorporated into a three-dimensional tabular mining stress analyzer. This will enable rock mechanics engineers to design three-dimensional mine layouts for tabular excavations using analytical tools which simply but realistically account for the way in which the rock near the edges of their excavations fractures and deforms.

The numerical model provides a satisfactory explanation for the energy "released" by mining. An element on the face, if mined or removed, contains relatively little strain energy. Its removal however, disturbs the equilibrium established between it, the elements ahead of it, and the elastic rockmass. In re-establishing the equilibrium condition, the elastic rockmass deforms the seam elements and does work on them. This work is equal to the Energy Release Rate, less the energy removed in the mined element. In most cases, virtually all of the "released" energy will be expended in crushing the rock ahead of the mining face, and virtually none of this energy will be actually "removed" in the mined rock, nor will a balance of energy necessarily be available for release in the form of crippling seismic events or rockbursts.

Certain mining parameters are controllable. It was established by means of the numerical model that it should be possible to influence the size of the fracture zone by reducing the friction
coefficients on the parting planes or shear fractures (for example by means of water infusion). Altering the Effective Stope Width should also produce a change in size of the fracture zone. This is not necessarily true if one attempts to alter the actual stope width. The practice of cutting dip slots in the hanging and footwall as is routinely done on some mines, will almost certainly produce a change in size of fracture zone. This practice (i.e. initiating "caving") should tend to move the stress peak further ahead of the face, as has been proposed by various researchers.
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APPENDIX

LISTING OF COMPUTER PROGRAM 'SEAMS'

DEVELOPED TO TEST THE PROPOSED METHOD

OF ANALYSIS

HARDWARE REQUIRED:

1. Hewlett Packard Model 86 Desktop Computer
2. Graphics ROM
3. HP 7475 or similar plotter
4. Printer
5. Matrix ROM
6. 192k Memory
10 ! PROGRAM SEAMS FOR ANALYZING SEAMS WITH FINITE STRENGTH
20 ! WRITTEN BY R. K. BRUMMER
30 !
40 !
50 ! INITIALIZE THE SYSTEM
60 !
70 !
80 ALPHALL
90 DEG
100 CLEAR
110 PRINTER IS 701
120 CRT IS 1
130 OPTION BASE 1
140 DIM A(100,100), D(100), STRESS(100), STIFF(100), TYPE(100), YD(100), IYD(100), O00, STRESSH(100), BOUNDS(100), STATEFLAG(100), Q1*(1), Q2*(1), Q3*(1), HSTRAIN(100)
150 DIM DILAT(100), Q4*(1), D5*(1)
160 !
170 !
180 ! NOW SETUP THE PROBLEM
190 !
200 !
210 GOSUB SETUP
220 !
230 !
240 ! NOW CONSTRUCT THE INFLUENCE COEFFICIENTS MATRIX
250 !
260 !
270 GOSUB AMAT
280 !
290 !
300 ! NOW SOLVE FOR THE DISP DISCONTs
310 !
320 !
330 GOSUB ITER
340 !
350 !
360 ! NOW PLOT THE RESULTS
370 !
380 !
390 DISP "PLOT? " @ INPUT Q1$
400 IF Q1$="Y" THEN GOSUB PICTURE
410 !
420 !
430 ! NOW PRINT THE RESULTS OUT
440 !
450 !
460 DISP "PRINT ? " @ INPUT Q2$
470 IF Q2$="Y" THEN GOSUB RESULTS
480 !
490 !
500 ! NOW UPDATE THE VIRGIN STRESS
510 !
520 !
530 DISP "UPDATE THE VIRGIN STRESS " @ INPUT Q3$
540 IF Q3$="Y" THEN GOSUB UPDATEVIR:
550 !
560 !
570 DISP "MINE AN ELEMENT? " @ INPUT Q4$
580 IF Q4$="Y" THEN GOSUB MINE
590 !
600 !
610 GOTO 330 ! REDO WITH CHANGED PARAMETERS
620 !
630 !
BEGINNING OF THE SUBROUTINES***************************************

END OF PROG *******************************************************

VALUES FOR THE CONSTANTS USED

VIRG=2.5*27
IHS=1 ! AN IHS OF 1 MPA
G=30000
ITERKOUNT=0
ITOT=0
TANLAM=.5
UNAX=200
m=16
TOL=.0001
S=1
ESW=7
THETA=20
LAMBDA=31

BETA=TAN (THETA+LAMBDA)/TAN (THETA)

DELTA=2.8
ACC=1.5
mu=.2
LSF=.5
E=70000
EDASH=E/(1-mu^2)
GDASH=(G/PI/(1-mu))
IVSTRAIN=VIRG/EDASH

NOW INPUT THE PROBLEM GEOMETRY

DO YOU WISH ABS DISP OF SEAM ELEMENTS ? @ INPUT OS*

HOW MANY MINED REGIONS ARE THERE ? @ INPUT NREGIONS

LOOP OVER THE NUMBER OF REGIONS

FOR IREGIONS=1 TO NREGIONS

FOR I=1 TO NELTS

END FOR I

END FOR IREGIONS
1270 ITOT=ITOT+1
1280 X(ITOT)=XBEG+XGAP*(I-.5)
1290 STRESS(ITOT)=VIRG
1300 STRESSH(ITOT)=VIRG*LSR*TYPE+IHS ! 1MPa FOR STABILITY REASONS (REMOVED)
1310 a(ITOT)=XGAP/2
1320 STIFF(ITOT)=EDASH/ESW*TYPE
1330 TYPE(ITOT)=TYPE
1340 IYDIR(ITOT)=0
1350 D(ITOT)=0
1360 STATEFLAG(ITOT)=0
1370 BOUNDS(ITOT)=0
1380 DILAT(ITOT)=0
1390 HSTRAIN(ITOT)=0
1400 !
1410 !
1420 NEXT I ! END OF ELT LOOP
1430 !
1440 !
1450 NEXT IREGIONS ! END OF REGION LOOP
1460 !
1470 !
1480 N=ITOT ! RECORD THE NO OF ELTS
1490 RETURN
1500 !
1510 !
1520 ! END OF SUBROUTINE TO SETUP THE PROBLEM
1530 !
1540 !********************************************************************~***
1550 ! SUBROUTINE TO SETUP THE INF COEFT MATRIX
1560 !
1570 ! SUBROUTINE TO SETUP THE INF COEFT MATRIX
1580 !
1590 !
1600 AMAT:
1610 FOR I=1 TO N
1620 FOR J=1 TO N
1630 A(I,J)=GDASH*a(J)/((X(I)-X(J))^2-a(J)^2)
1640 NEXT J .
1650 NEXT I
1660 RETURN
1670 !
1680 ! END OF INF COEFT MATRIX
1690 !
1700 !
1710 !
1720 !********************************************************************~***
1730 ! SUBROUTINE TO SOLVE THE PROBLEM ITERATIVELY
1740 !
1750 !
1760 IETER;
1770 FLAG=0 ! FLAG TO CHECK FOR CONVERGENCE OF SOLUTION
1780 !
1790 FOR I=1 TO N ! LOOP OVER THE NO OF ELEMENTS
1800 !
1810 !
1820 ! FIND THE DIRECTION OF YIELD FOR TYPE 1 (YIELDING ELEMENTS)
1830 !
1840 !
1850 IF TYPE(I)=1 THEN GOSUB YIELD_DIR
1860 !
1870 !
1880 ! FIND THE STIFFNESS For TYPE 1 ELEMENTS
1890 ! USING A SECANT MODULUS SELF EFFECT APPROACH
1900!
1910!
1920 IF TYPE(I)=1 THEN GOSUB CONSTIT!
1930!
1940!
1950! SOLVE BY GAUSS SEIDEL ITERATION
1960!
1970!
1980 DUMMY=STRESS(I)
1990 FOR J=1 TO N.
2010 DUMMY=DUMMY-A(I,J)*D(J).
2020 NEXT J.
2030 DUMMY=DUMMY/(A(I,I)+STIFF(I)).
2040 DIFF=DUMMY-D(I).
2050 D(I)=D(I)+ACC*DIFF.
2060 BOUNDS(I)=STIFF(I)*D(I).
2070 IF ABS (DIFF) > TOL THEN FLAG=FLAG+1.
2080!
2090!
2100! DISP THE TEMP RESULTS FOR DIAGNOSTICS
2110!
2120 DISP USING "DDD,DDD.DD,DDD.DDDD,DDD.DDDD,D,DDD.DDDD.D,D,DDD.DDDD.D,D,DDD.DDDD.D.";
2130 I,X(I),D(I),BOUNDS(I),STRESSH(I),STIFF(I),STATEFLAG(I),IYDIR(I),DILAT(I).
2140 DISP "OUT OF TOL IN ";FLAG;" ELEMENTS".
2150 NEXT I.
2160 I=TERKOUNT=TERKOUNT+1.
2170! **************END FORWARD LOOP
2180! IF FLAG=0 THEN DISP "CONVERGED IN ";TERKOUNT;" ITERATIONS" @ GOTO 2270.
2190!
2200!
2210!
2220 IF Q5$ = "Y" THEN GOSUB HORMOVE FIND THE HORIZ MOVEMENTS OF SEAM ELEMENTS.
2230!
2240!
2250 GOTO 1770! DO ANOTHER ITERATION - GO BACK TO BEG OF SUBROUTINE.
2260!
2270 RETURN.
2280!
2290!
2300! END OF SOLVE SUBROUTINE
2310!
2320!
2330! SUBROUTINE TO PRINT THE RESULTS OUT.
2340!
2350!
2360! RESULTS:
2370 PRINT ": I X(I) Converg Dilat StressV StressH Stiff State Dir"
2380 PRINT " m m m MPa MPa mpa"
2390 PRINT
2400 FOR I=1 TO N.
2410 PRINT USING "DDD,DDDD.DD,DDDD.DD,DDDD.DD,DDDD.D,DDDD.D,DDDD.D,DDDD.D.DDDD.D.
2420 DDDD";
2430 I,X(I),D(I),DILAT(I),BOUNDS(I),STRESSH(I),STIFF(I),STATEFLAG(I),IYDIR(I).
2440 NEXT I.
2450 PRINT CHR$ (12).
2460 RETURN.
2470! END OF OUTPUT SUBROUTINE
2480!
2490!
2500!
2510 ! SUBROUTINE TO PLOT THE RESULTS
2520 !
2530 !
2540 PICTURE:
2550 DISP "START PLOT FROM ELT NO. @ INPUT FIRST
2560 DISP "END PLOT AT ELT NO. @ INPUT LAST
2570 F=FIRST
2580 L=LAST
2590 PLOTTER IS 805
2600 LIMIT 40,240,25,175
2610 SPAN=X(L)-X(F)+a(F)+a(L)
2620 SCALE X(F)-a(F),-.1*SPAN,X(L)+a(L),.1*SPAN,-.8,.8
2630 PEN 5
2640 FRAME
2650 IF F#1 THEN GOTO 2680
2660 MOVE X(F)-a(F),-.01
2670 PLOT X(F)-a(F),.01,-1
2680 FOR I=F TO L
2690 MOVE X(I)-a(I),-.01
2700 PLOT X(I)+a(I),-.01,-1
2710 MOVE X(I)-a(I),.01
2720 PLOT X(I)+a(I),.01,-1
2730 NEXT I
2740 IF L#N THEN GOTO 2770
2750 MOVE X(L)+a(L),-.01
2760 PLOT X(L)+a(L),.01,-1
2770 FOR I=F TO L
2780 PEN STATEFLAG(I)+1
2790 OI=.04
2800 MOVE X(I)-a(I),-D(I)-OI
2810 PLOT X(I)+a(I),-D(I)-OI,-1
2820 PLOT X(I)+a(I),0-0I,-1
2830 PLOT X(I)-a(I),0-0I,-1
2840 PLOT X(I)-a(I),-D(I)-OI,-1
2850 NEXT I
2860 FOR I=F TO L
2870 PEN STATEFLAG(I)+1
2880 MOVE X(I)-a(I),BOUNDS(I)/2000+OI
2890 PLOT X(I)+a(I),BOUNDS(I)/2000+OI,-1
2900 PLOT X(I)+a(I),0+OI,-1
2910 PLOT X(I)-a(I),0+OI,-1
2920 PLOT X(I)-a(I),BOUNDS(I)/2000+OI,-1
2930 NEXT I
2940 PEN UP
2950 MOVE 0,0
2960 CSIZE 2
2970 FOR I=0 TO 14 STEP 2
2980 MOVE X(F)-a(F),-.1*SPAN,+(.05*I)-OI
2990 PLOT X(F)-a(F),.095*SPAN,+(.05*I)-OI,-1
3000 LABEL 50*I
3010 NEXT I
3020 MOVE X(F)-a(F),-.03*SPAN,-.5
3030 LABEL "Convergence (mm)"
3040 FOR I=0 TO 14 STEP 2
3050 MOVE X(F)-a(F),-.1*SPAN,.05*I+OI
3060 PLOT X(F)-a(F),.095*SPAN,.05*I+OI,-1
3070 LABEL 100*I
3080 NEXT I
3090 MOVE X(F)-a(F),-.03*SPAN,.5
3100 LABEL "Stress (MPa)"
3110 MOVE 0,0
3120 MOVE X(F)-a(F)-SPAN/50,+(0I*.8)
3130 LABEL X(F)-a(F)
3140 MOVE X(L)+a(L)-SPAN/50,-(0I*.8)
SUBROUTINE TO RETURN THE STIFFNESS GIVEN THE CONFINEMENT AND STRAIN
ALSO RETURNS THE HORIZONTAL STRAIN AND DISPLACEMENT

CONSTITUT:
STRESS(I) = STRESS(I+IYDIR(I)) + 4*a(I)*TANLAM*BOUNDS(I)/ESW ! AT END OF ELT
STRESSH(I) = (STRESS(I+IYDIR(I)) + STRESSH(I))/2 ! CONF IN MIDDLE OF ELEMENT
STRESS = STRESSH(I+IYDIR(I))

VERTRAIN = D(I)/ESW + IVSTRAIN
SIGV = STRESS + SOR*(UNAX*STRESS + S*UNAX^2)

ELASTLIM = SIGV/EDASH
SLOPELIM = (-BETA*STRESS + 2*SIGV)/(EDASH + DELTA*UNAX)

IF VERTRAIN > ELASTLIM THEN GOTO 3420
SIGV = EDASH*VERTRAIN
IF STATEFLAG(I+1) = 1 THEN IYDIR(I) =I
RETURN
IF STATEFLAG(I-1) = 1 THEN IYDIR(I) = 0
RETURN

VERTRAIN = D(I)/ESW + IVSTRAIN
SIGV = STRESS + SOR*(UNAX*STRESS + S*UNAX^2)

ELASTLIM = SIGV/EDASH
SLOPELIM = (-BETA*STRESS + 2*SIGV)/(EDASH + DELTA*UNAX)

IF VERTRAIN > ELASTLIM THEN GOTO 3420
SIGV = EDASH*VERTRAIN
IF STATEFLAG(I+1) = 1 THEN IYDIR(I) =I
RETURN
IF STATEFLAG(I-1) = 1 THEN IYDIR(I) = 0
RETURN

VERTRAIN = D(I)/ESW + IVSTRAIN
SIGV = STRESS + SOR*(UNAX*STRESS + S*UNAX^2)

ELASTLIM = SIGV/EDASH
SLOPELIM = (-BETA*STRESS + 2*SIGV)/(EDASH + DELTA*UNAX)

IF VERTRAIN > ELASTLIM THEN GOTO 3420
SIGV = EDASH*VERTRAIN
IF STATEFLAG(I+1) = 1 THEN IYDIR(I) =I
RETURN
IF STATEFLAG(I-1) = 1 THEN IYDIR(I) = 0
RETURN

END OF CONSTIT LAW SUBROUTINE

SUBROUTINE TO FIND THE DIRECTION OF YIELDING

YIELD_DIR:
IF I=1 THEN IYDIR(I) = 0 @ RETURN
IF I=N THEN IYDIR(I) = 0 @ RETURN
IF STRESS(I+1) > STRESSH(I-1) THEN IYDIR(I) = -1 @ GOTO 3710

RETURN

RETURN
SUBROUTINE TO FIND THE HORIZONTAL MOVEMENT OF ELASTIC SEAM ELEMENTS

ALSO FINDS THE HORIZONTAL STRAINS

DISP " WHICH ELEMENT DO YOU WANT TO MINE " @ INPUT ELTNO

TYPE(ELTNO)=0

STIFF(ELTNO)=0

IYDIR(ELTNO)=0

STATEFLAG(ELTNO)=0

BOUNDS(ELTNO)=0

DILAT(ELTNO)=0

HSTRAIN(ELTNO)=0

STRESSH(ELTNO)=0 ! THE MINIMUM HORIZONTAL STRESS (MPA PUT BACK)

RETURN

HORMOVE:

FOR I=1 TO N

IF TYPE(I) IS NOT 1 THEN GOTO 4220

IF STATEFLAG(I) IS NOT 1 THEN GOTO 4220

DUMMY1=DUMMY1+D(J)*A(I,J)

DUMMY=DUMMY-D(J)*LOG (ABS ((X(I)-X(J)-a(J))/(X(I)-X(J)+a(J)))))

NEXT J

DILAT(I)=(1-2*mu)/(1-mu)*DUMMY

HSTRAIN(I)=(DUMMY1*(-(1-2*mu)))/2/PI/(1-mu)/E2ASH

RETURN

END OF THE SUBROUTINE.

END OF SEAM MINING SUBROUTINES

UPDATEVIRG:

DISP "NEW VIRGIN STRESS " @ INPUT VIRG

FOR I=1 TO N

STRESS(I)=VIRG

NEXT I

RETURN